Color I: trichromatic theory

CS 178, Spring 2009

Begun 5/12/09, finished 5/14/09.

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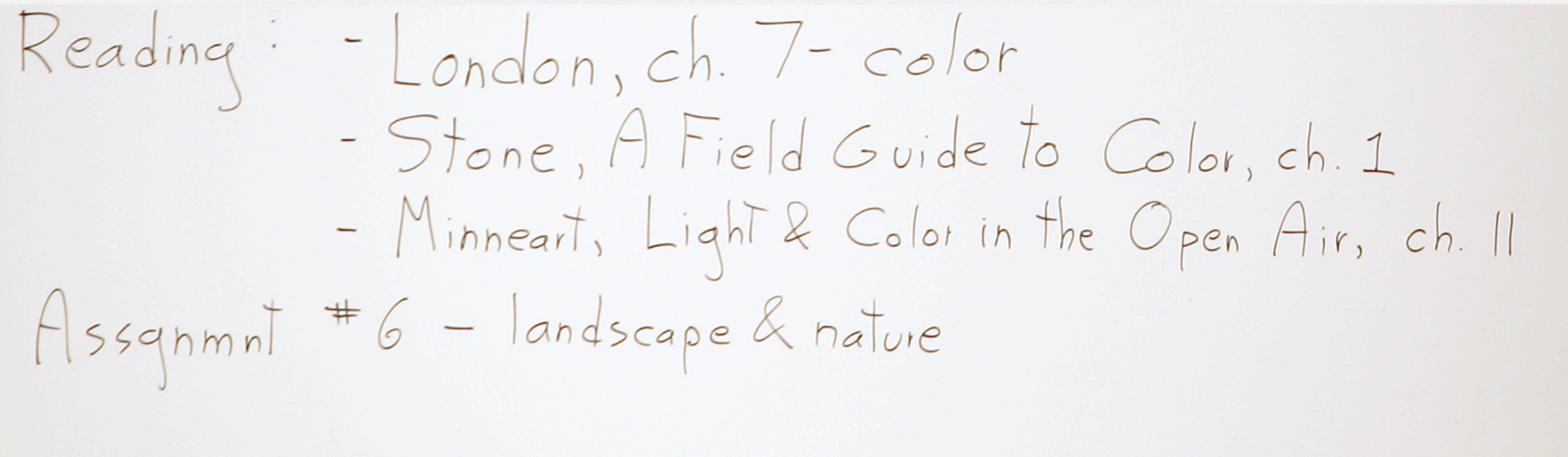
Announcements (from whiteboard)

- see also "Midquarter pep talk" slides

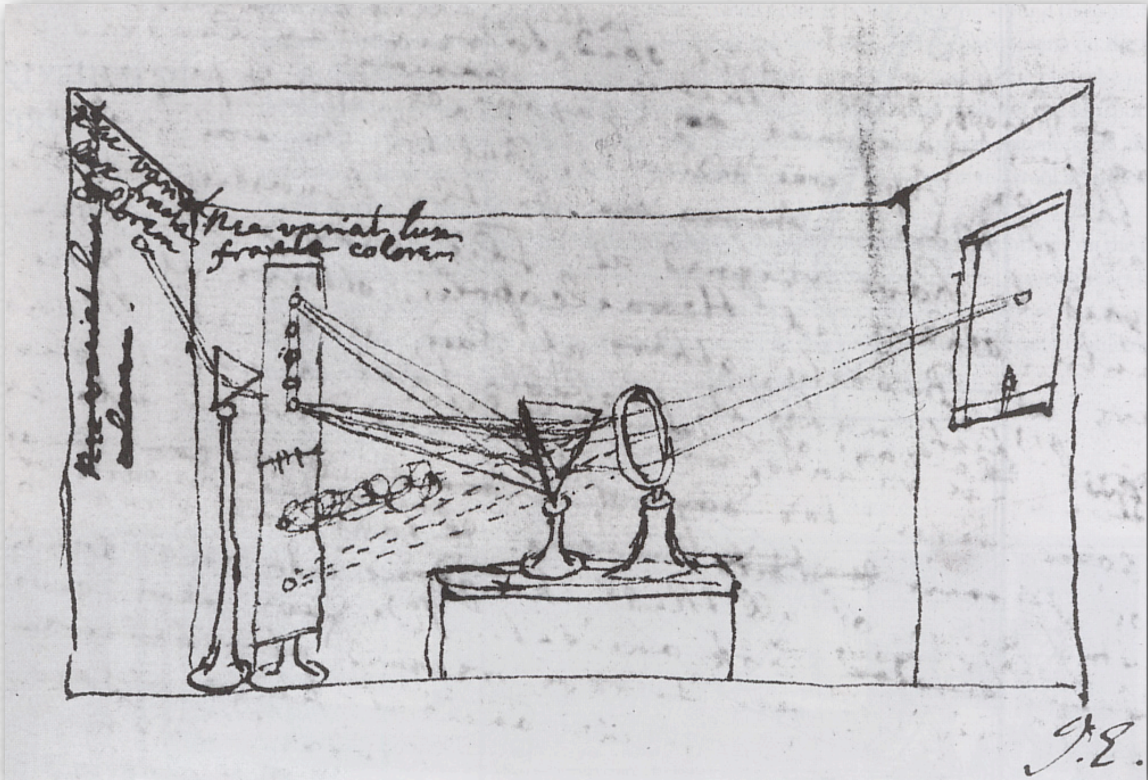


Outline

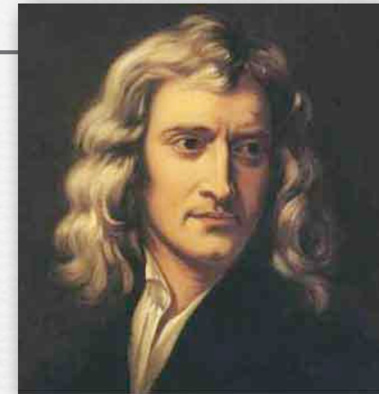
- spectral power distributions
- color response in animals and humans
- 3D colorspace of the human visual system
- reproducing colors using three primaries
- additive versus subtractive color mixing

- cylindrical color systems used by artists (and Photoshop)
- chromaticity diagrams
  - color temperature and white balancing
  - standardized color spaces and gamut mapping
  - uniform perceptual spaces and opponent colors

Newton's Experimentum Crucis



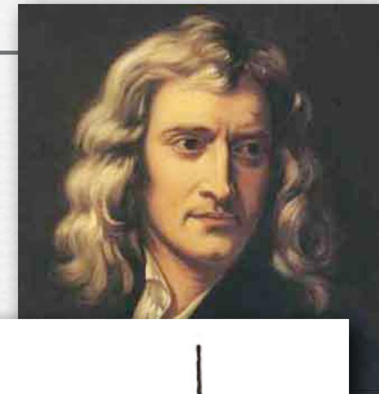
(Robin)



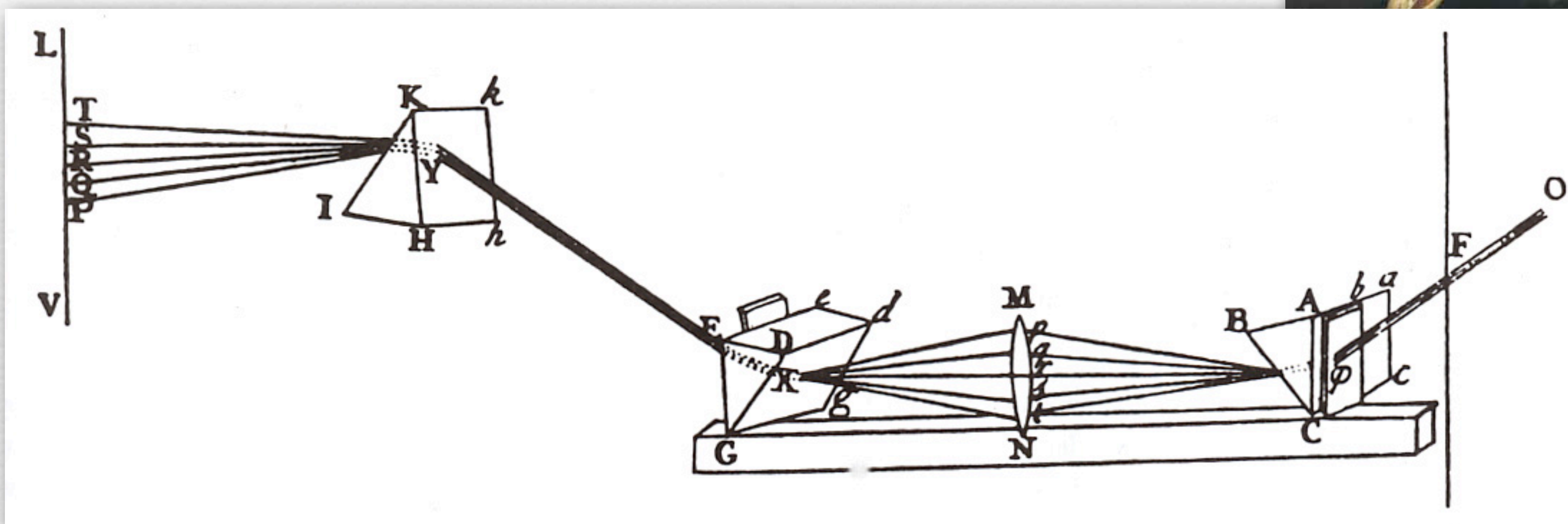
Isaac Newton
(1643-1727)

- sunlight can be divided into colors using a prism
- these colors cannot be further divided using a 2nd prism
- experiment performed 1665, drawing made in 1672

Newton's Experimentum Crucis

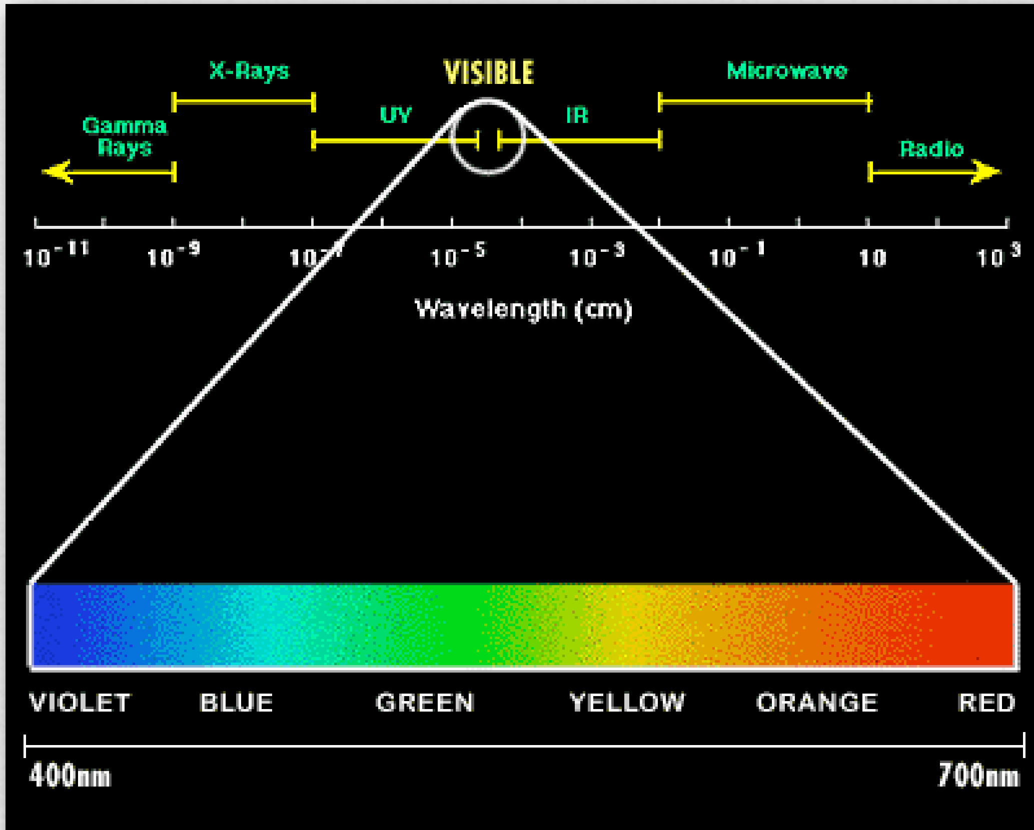


(Robin)



- alternatively, the divided colors can be recombined using a lens and 2nd prism into a new beam that has exactly the same properties as the original

The visible light spectrum



(sciencebuddies.org)

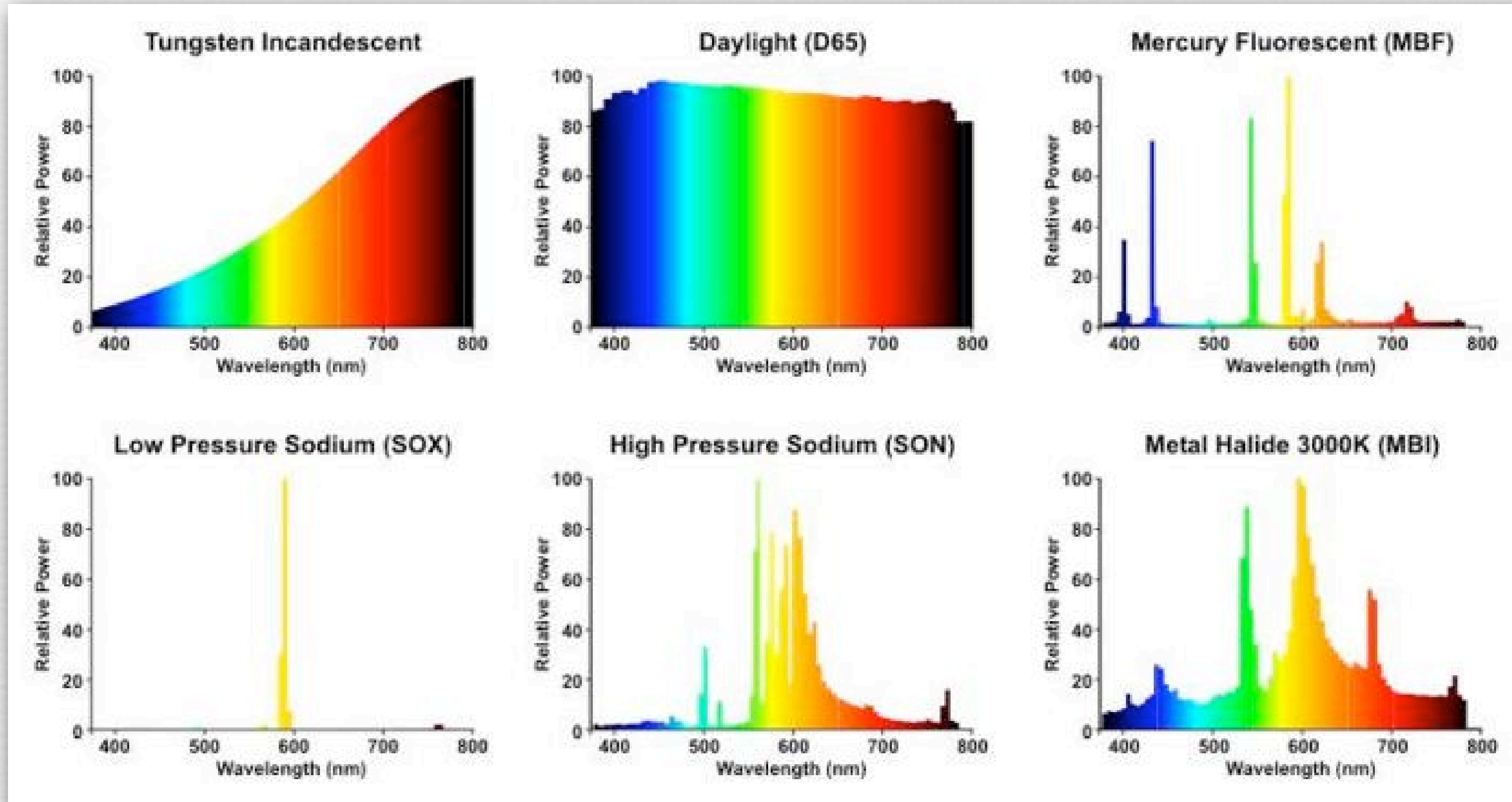
- wavelengths between 400nm and 700 nm (0.4μ - 0.7μ)
- exactly the colors in a rainbow

The visible light spectrum

(Dan Bush)

- wavelengths between 400nm and 700 nm (0.4μ - 0.7μ)
- exactly the colors in a rainbow

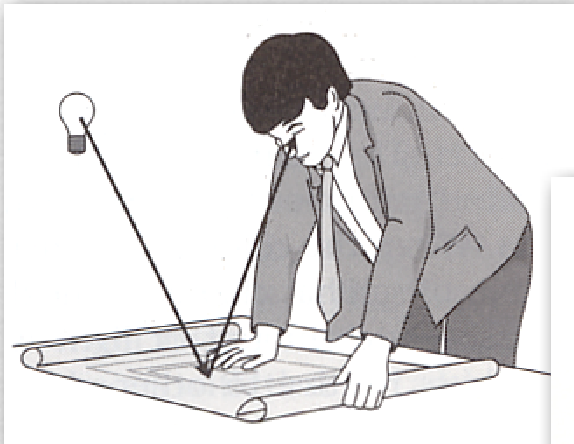
Spectral power distribution (SPD)



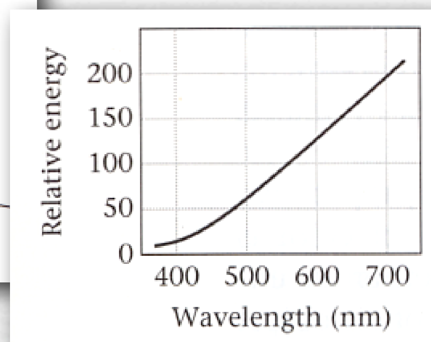
(LampTech)

- units of power are watts (joules per second)
- relative amount (%) of each wavelength present
- shown here are spectra of common illumination sources

Interaction of light with matter

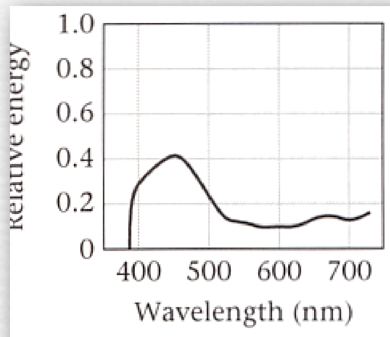


light is reflected by an object



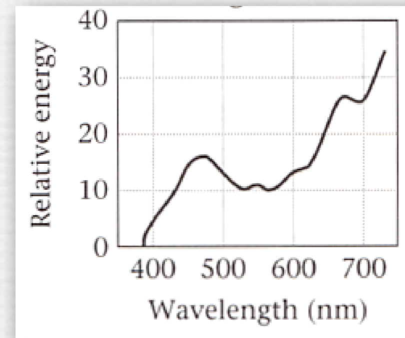
illumination

×



reflectance

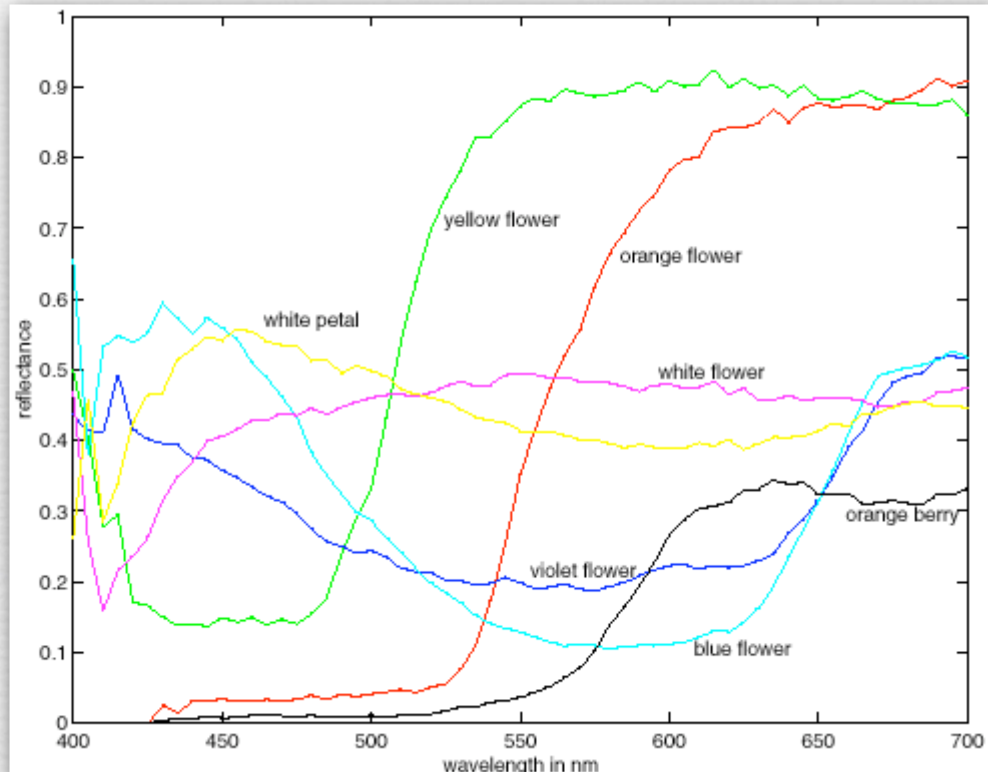
=



stimulus that enters your eye

- ✦ spectrum of illumination is multiplied wavelength-by-wavelength by reflectance spectrum of object
  - cause is absorption by the material
  - so the spectrum you see depends on the illumination!
- ✦ transmittance operates the same way

Examples of reflectance spectra



- two different spectra may appear alike to us
  - white petal and white flower above
  - these are called *metamers*
- Newton observed this, but could not explain it

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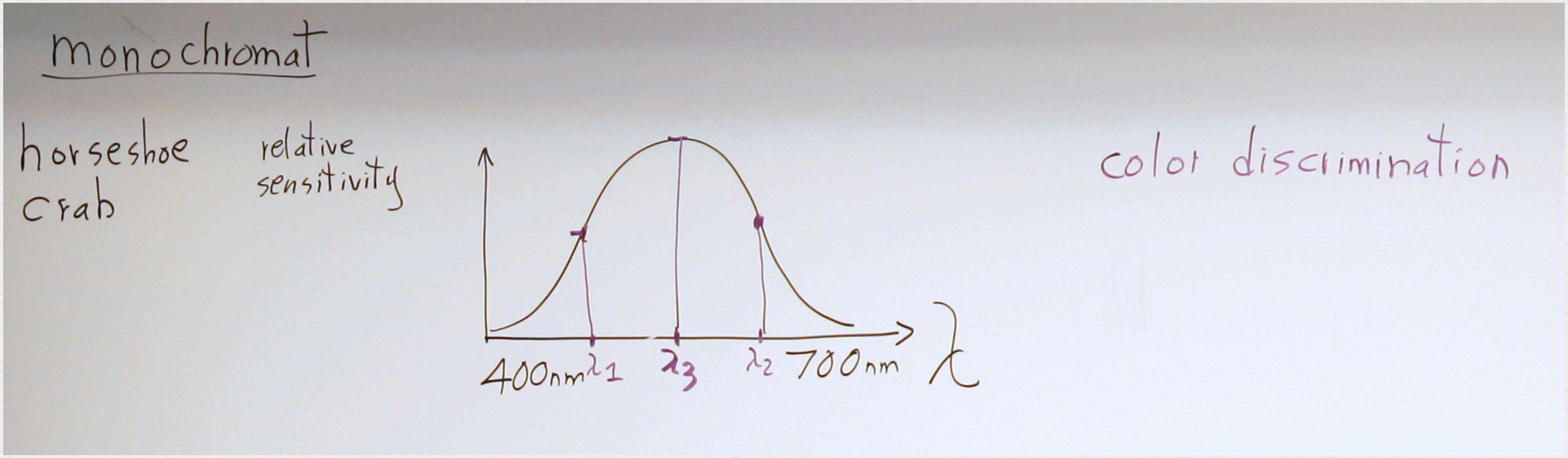
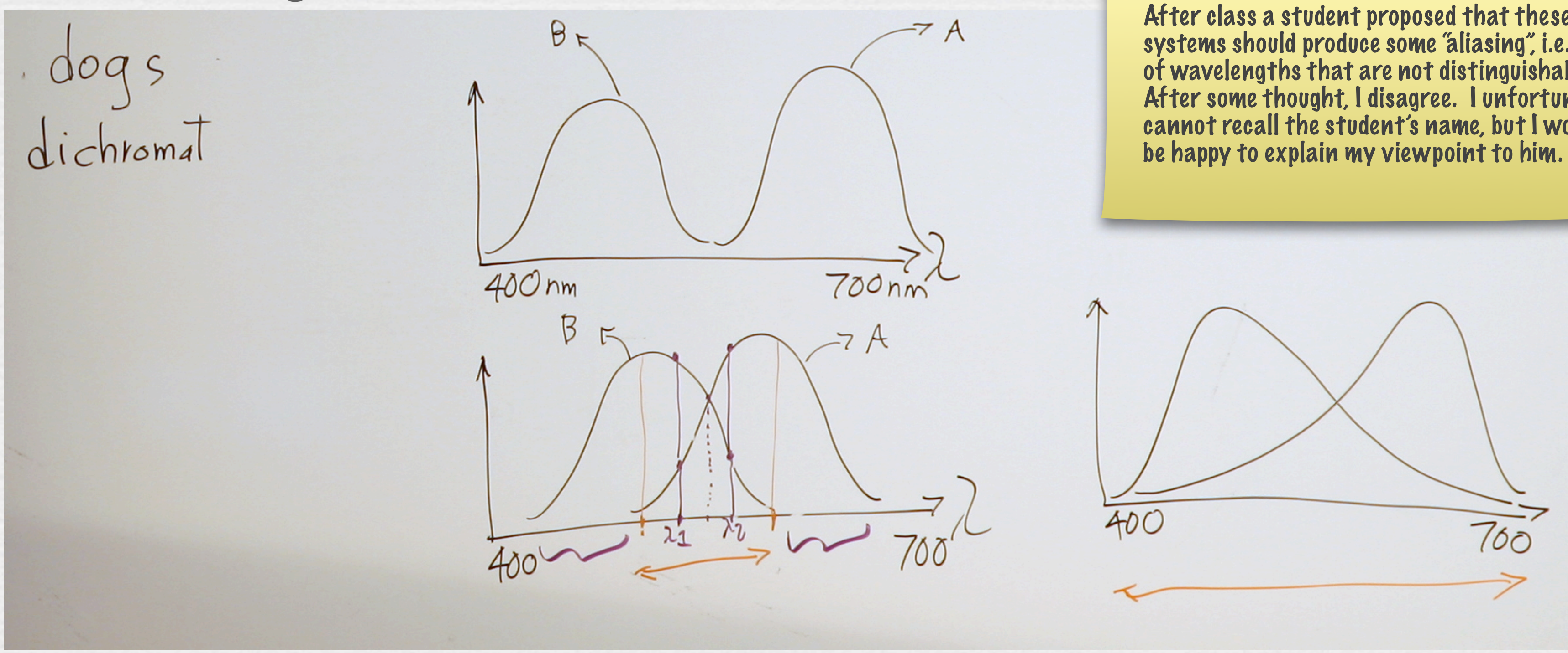
Monochromats

- organisms having only one kind of retinal receptor cannot distinguish changes in intensity from changes in wavelength, hence they have no *color discrimination*
  - for example a unit amount of λ_1 versus λ_2 below
  - or a unit amount of λ_1 versus half as much of λ_3 (assuming the sensitivity to λ_3 is twice the response to λ_1)



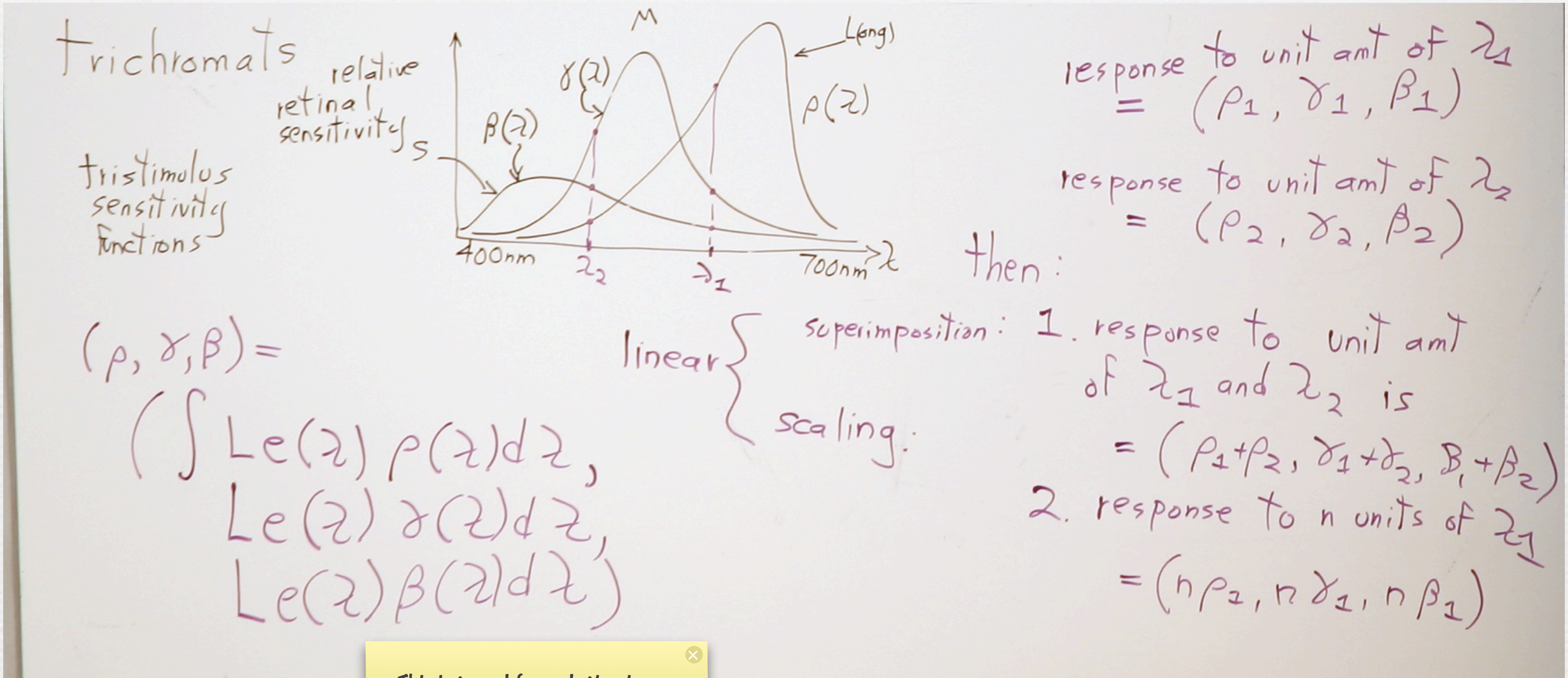
Dichromats

- the lower two systems have color discrimination over the range of wavelengths denoted by the orange arrows
  - for each wavelength within that range, the ratio of responses of receptors A and B is unique; hence the organism can reliably identify which wavelength it's looking at



After class a student proposed that these systems should produce some "aliasing", i.e. pairs of wavelengths that are not distinguishable. After some thought, I disagree. I unfortunately cannot recall the student's name, but I would be happy to explain my viewpoint to him.

Trichromats (including humans)



This integral formulation is summarized pictorially on the next slide.

Summary of human color response (1 of 2)

spectrum of stimulus arriving
at one spot on retina

×

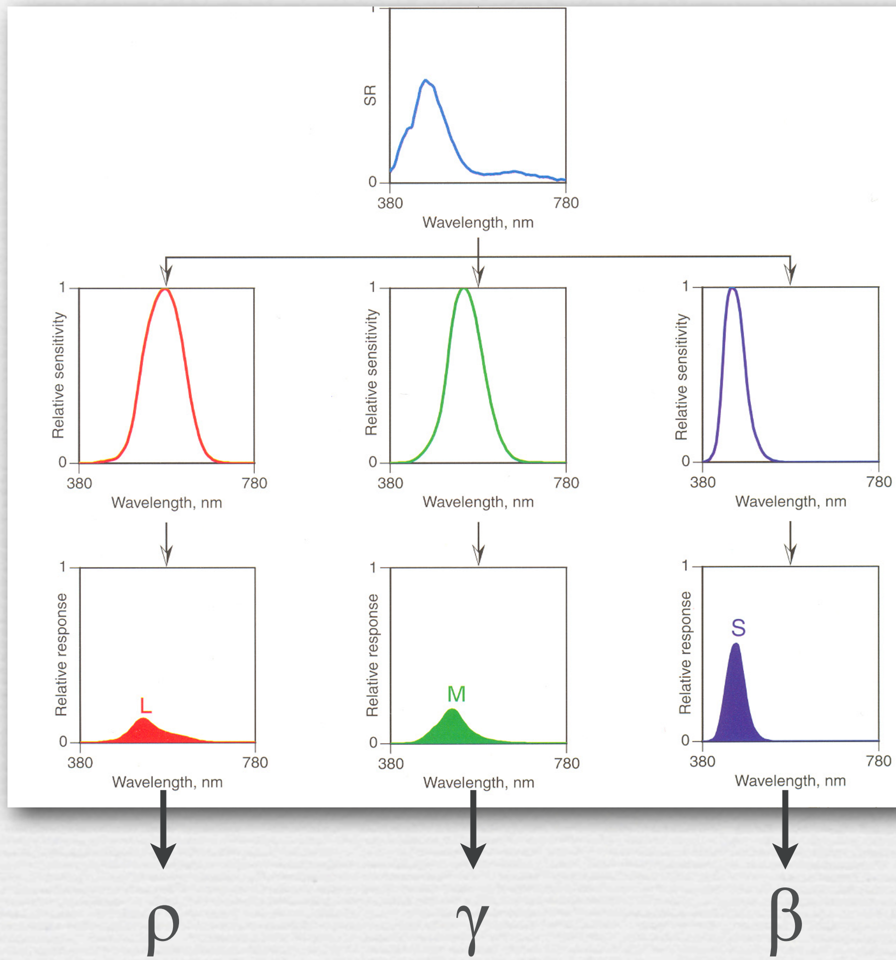
spectral sensitivity of each
type of cone (L,M,S)

=

multiply wavelength-by-
wavelength to get response spectra

∫

integrate over wavelengths to get
total response for that type of cone



- output is three numbers (ρ, γ, β) per spot on retina

Summary of human color response (2 of 2)

- in other words, given a stimulus spectrum $L_e(\lambda)$, the human response to it (ρ, γ, β) are the integrals over all visible wavelengths of our responses $\rho(\lambda)$, $\gamma(\lambda)$, and $\beta(\lambda)$ to each constituent wavelength λ, i.e.

$$(\rho,\gamma,\beta) = \left(\int_{400\,nm}^{700\,nm} L_e(\lambda)\,\rho(\lambda)\,d\lambda,\ \int_{400\,nm}^{700\,nm} L_e(\lambda)\,\gamma(\lambda)\,d\lambda,\ \int_{400\,nm}^{700\,nm} L_e(\lambda)\,\beta(\lambda)\,d\lambda \right)$$

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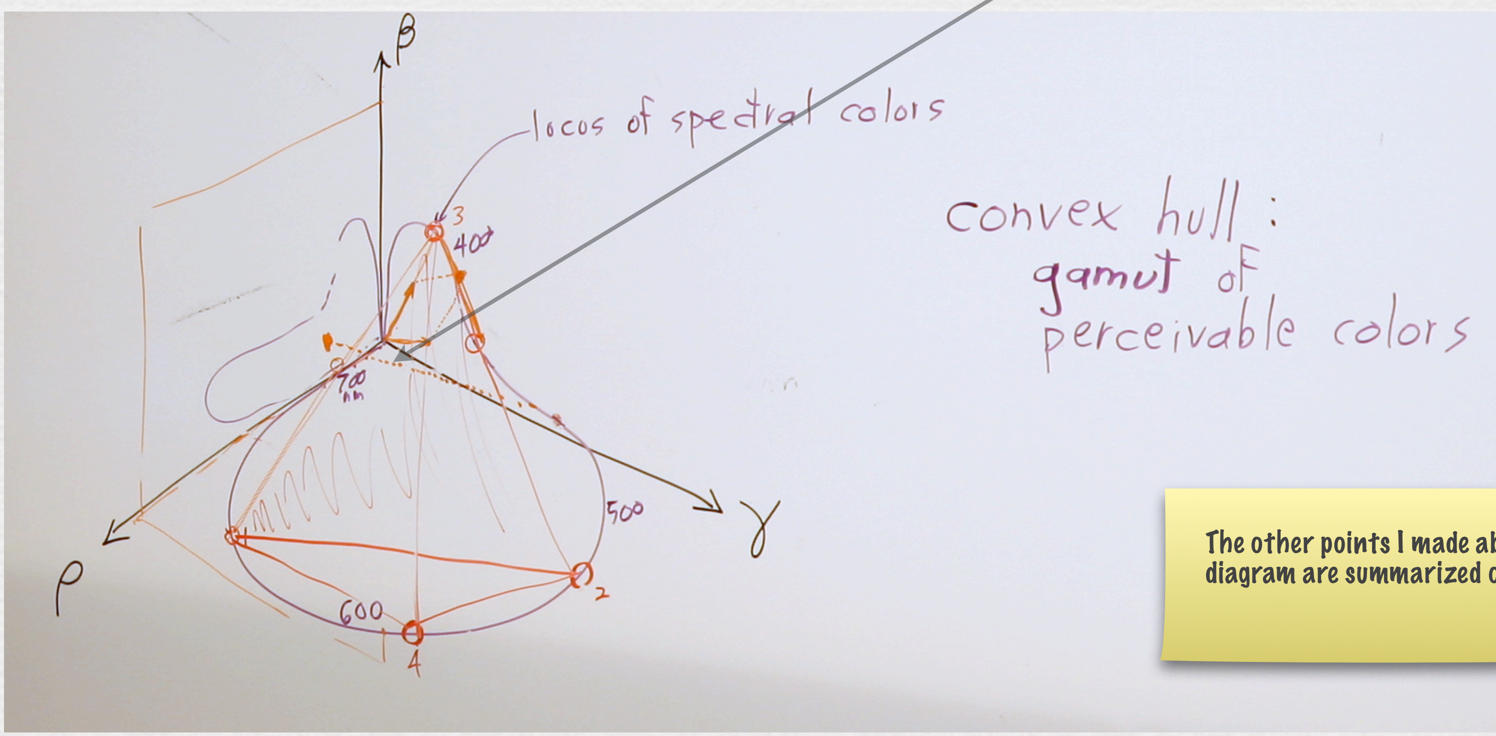
3D gamut of perceivable colors

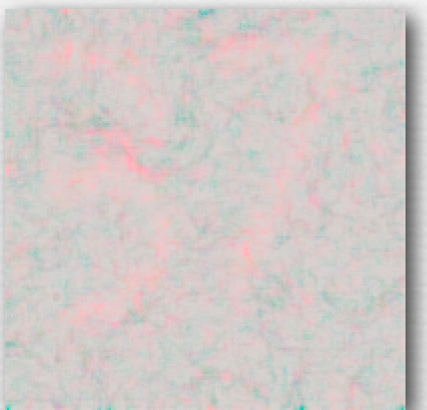
- to a deuteranope - a color-blind person who is missing their medium-wavelength receptor, i.e. their gamma receptor - this diagram is squashed into the rectangle shown below on the rho-beta plane
  - as a result, spectra whose (ρ, γ, β) responses lie along this dotted line cannot be distinguished; they will appear as the same color, i.e. as metamers
  - by a similar argument, many spectra distinguishable to pentachromats (e.g. Mallard ducks) are indistinguishable to trichromats (humans)



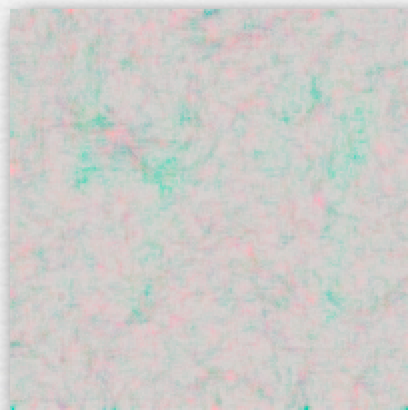
The other points I made about this diagram are summarized on slide #21.

Color blindness

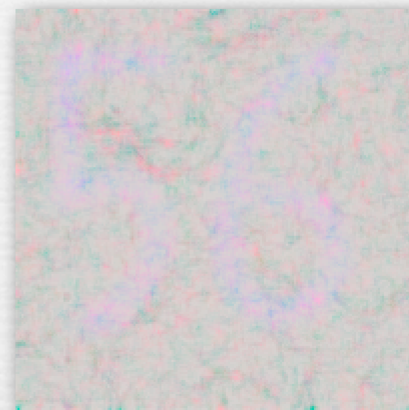
37?



44?



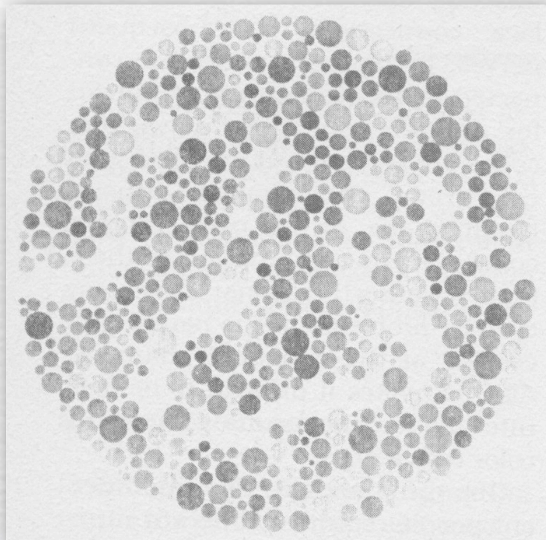
56?

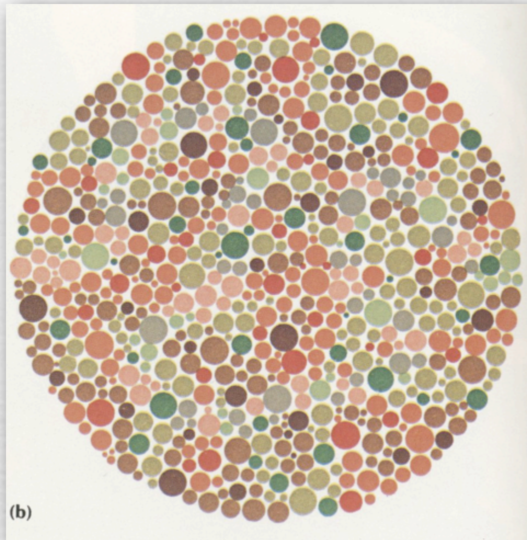


(wikipedia)

- protanopia (1% of males)
- deuteranopia (1% of males)
- tritanopia (< 1% of both genders)
- protanomaly (1% of males)
- deuteranomaly (6% of males)
- tritanomaly (< 1% of both genders)

The advantage of being color blind





- the maze (at left) is recreated (at right) using subtle intensity differences, but overridden by stronger red-green color differences
- only a deuteranope can see the maze at right

Summary of human 3D colorspace

- the three types of cones in our retina (Long, Medium, Short wavelength) define the axes of a three-dimensional space
- our response to any stimulus spectrum can be summarized by three numbers (ρ, γ, β) and plotted as a point in this space
- our responses to all visible single-wavelength spectra (a.k.a. pure wavelengths λ, i.e. positions along the rainbow), if connected together, form a curve in this space, called the *locus of spectral colors*; the sequence of (ρ, γ, β) numbers form the *tristimulus sensitivity functions* ρ(λ), γ(λ), and β(λ)
- our response to any mixture (Σ = 1) of two pure wavelengths falls on a line connecting the responses to each wavelength
- our responses to all possible mixtures (Σ ≤ 1) of all visible wavelengths forms an irregular volume called the *gamut of perceivable colors*, equal to the convex hull of the spectral locus

The convex hull of a shape is the "shrink wrap" of that shape. In 3D it is defined as the smallest convex polyhedron that fully encloses the shape.

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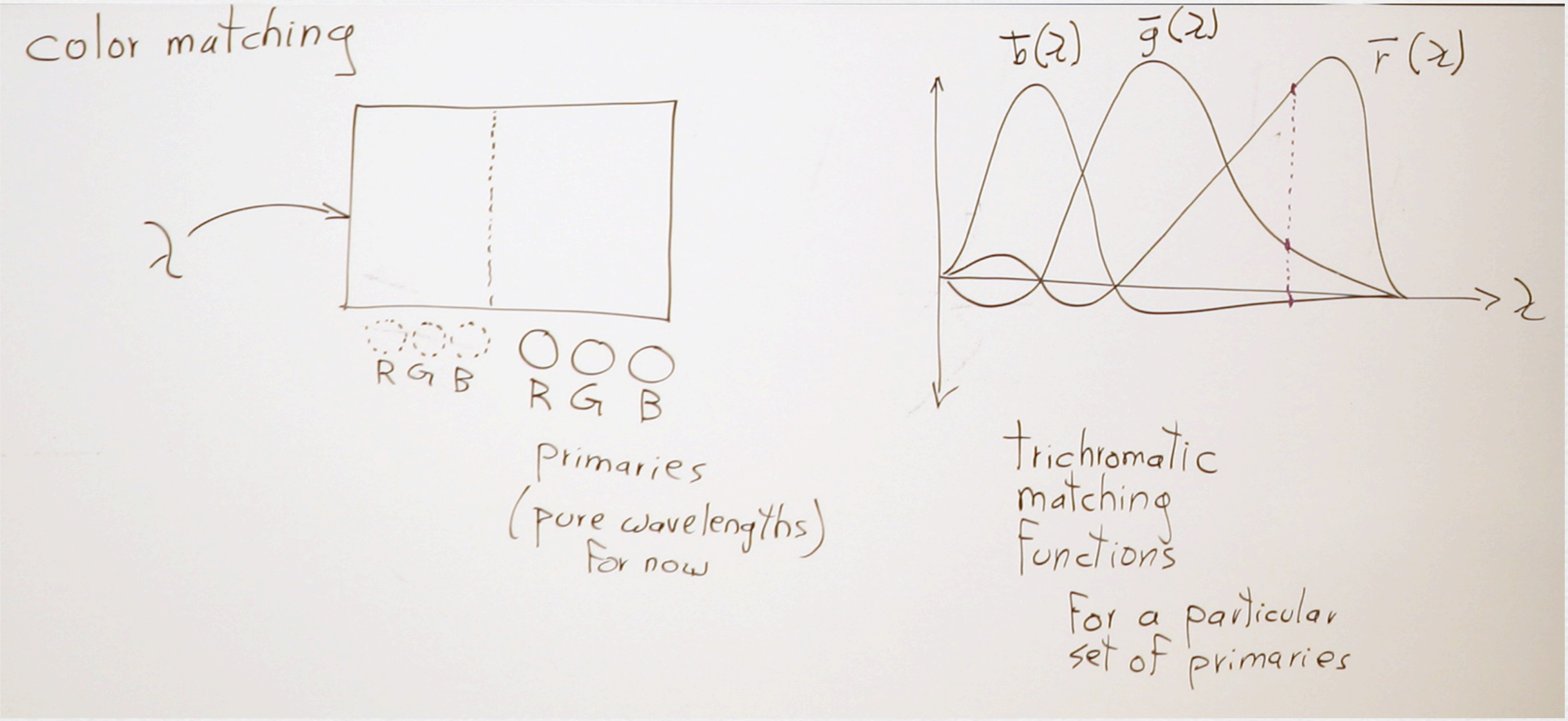
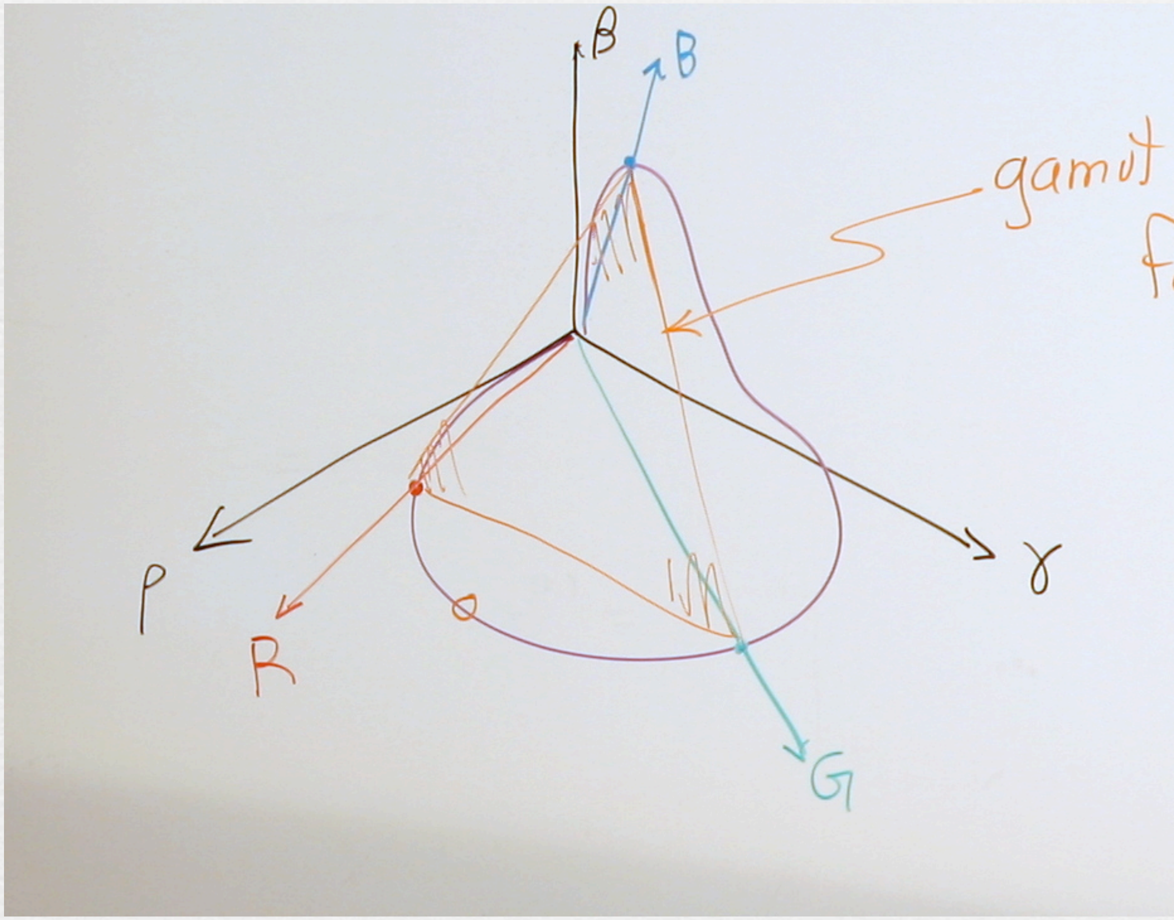
Maxwell's color matching experiment

- Maxwell actually used a slightly different procedure
  - see http://www.handprint.com/HP/WCL/color6.html for details
  - the procedure is often used in modern versions of the experiment



3D gamut of reproducible colors for a particular choice of primaries



See the next three slides for a text summary of this diagram.

Summary of Young-Helmholtz trichromatic theory of color vision (1 of 3)

Thomas Young
(1773-1829)

James Clerk Maxwell
(c. 1860)

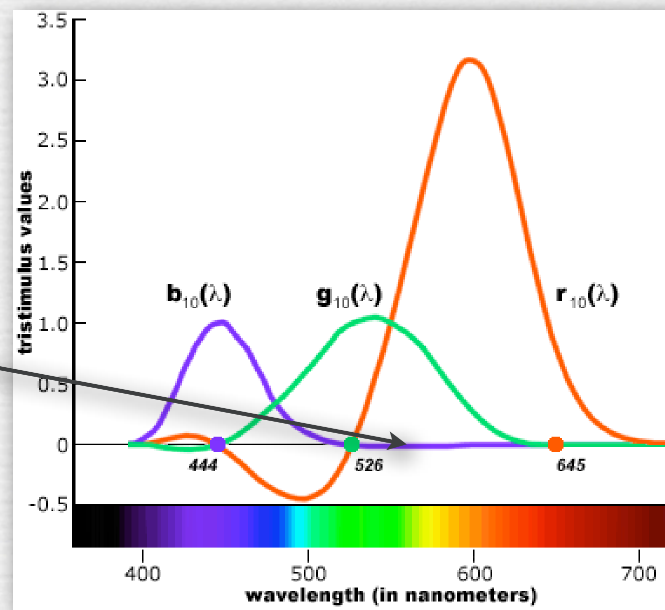
Hermann von Helmholtz
(1821-1894)

- spectra can be visually matched using mixtures of *primary colors*; such matches are called *metamers*
- a primary can be a pure wavelength or a mixture of wavelengths
- our response to varying amounts of a primary forms a vector in (ρ, γ, β) space, rooted at the origin
- to provide a normal range of color vision, three primaries are required, and their vectors must not lie on a plane

Summary of Young-Helmholtz trichromatic theory of color vision (2 of 3)

- given a stimulus wavelength, the amount of each primary required to match it is given by three numbers $(\bar{r}, \bar{g}, \bar{b})$
- some stimuli cannot be matched unless first desaturated by adding a primary to it before matching; the amount added is denoted by negative values of $\bar{r}$, $\bar{g}$, or $\bar{b}$
- the sequence of $(\bar{r}, \bar{g}, \bar{b})$ values, some negative, required to match the locus of spectral colors across all λ, form the *trichromatic matching functions* $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$ for a particular set of 3 primaries

this segment goes slightly negative as well



(http://www.handprint.com/HP/WCL/color6.html)

Summary of Young-Helmholtz trichromatic theory of color vision (3 of 3)

- given a stimulus spectrum $L_e(\lambda)$, the amounts of each primary R, G, and B required to match it, for any particular choice of 3 primaries, are the integrals over all visible wavelengths of the amounts $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$ required to match each constituent wavelength λ, *i.e.*

$$(R,G,B) = \left(\int_{400nm}^{700nm} L_e(\lambda)\,\bar{r}(\lambda)\,d\lambda,\ \int_{400nm}^{700nm} L_e(\lambda)\,\bar{g}(\lambda)\,d\lambda,\ \int_{400nm}^{700nm} L_e(\lambda)\,\bar{b}(\lambda)\,d\lambda \right)$$

- our responses to all possible mixtures ($\sum \leq 1$) of three primaries form a tetrahedron called the *gamut of reproducible colors* for these primaries

- adding more primaries increases the volume of this gamut; making them pure spectral colors also increases this volume

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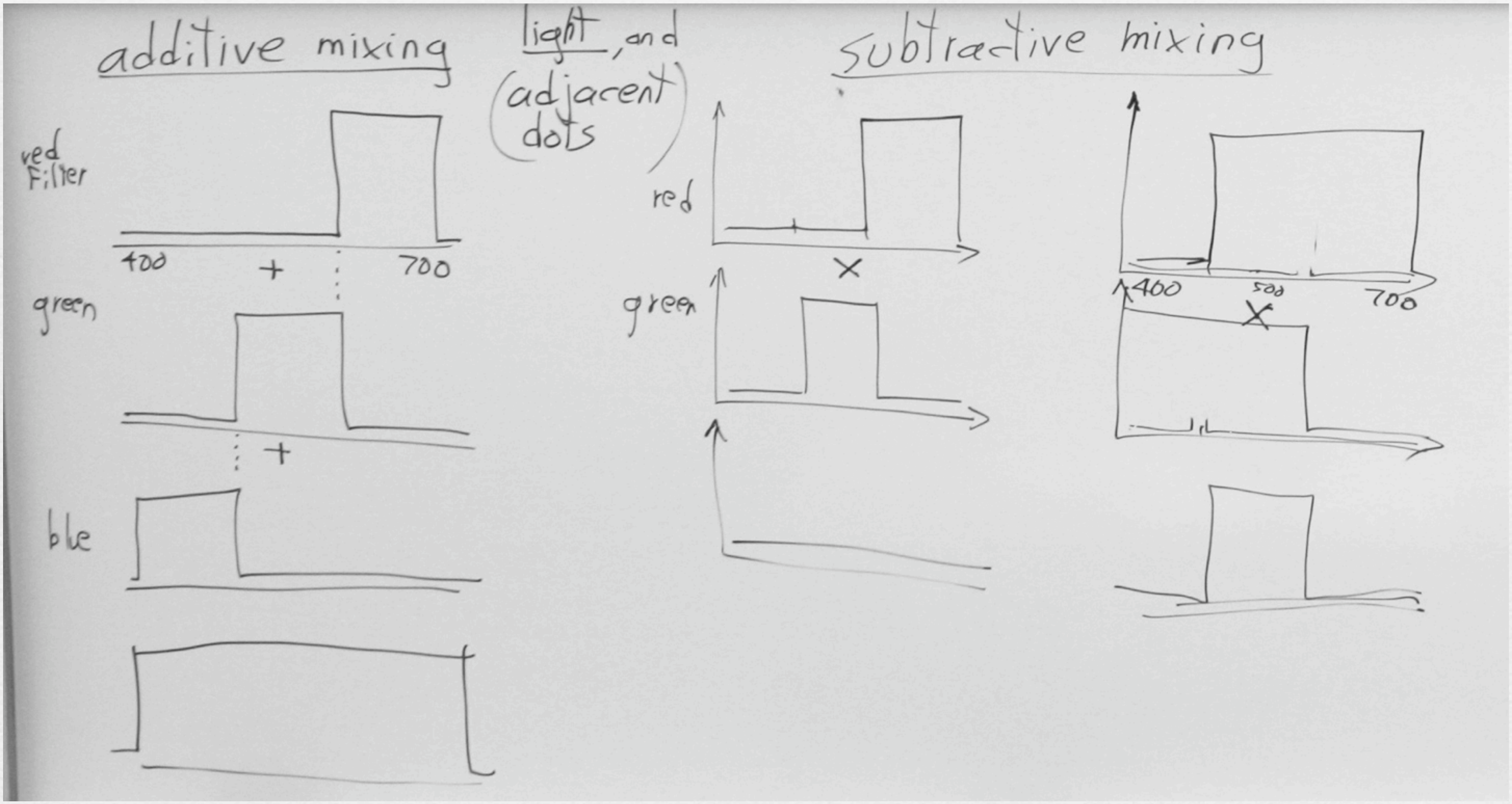
Additive versus subtractive mixing

- ✦ accompanied by live demo using color guns & filters



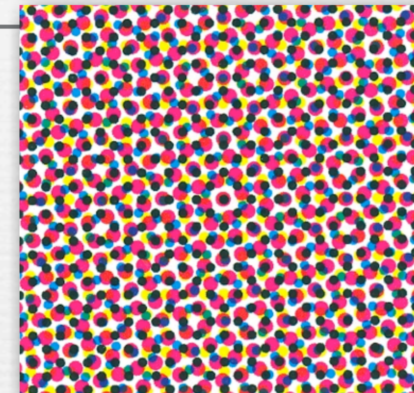
Summary of additive versus subtractive color mixing (1 of 2)

Point #2 originally said "reflectance spectra". For layers or sequences, this is obviously incorrect; I meant to say "transmittance spectra".

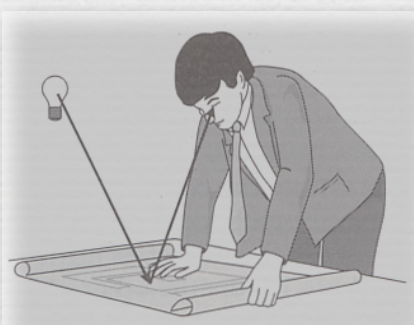
- superimposed colored lights or small adjacent dots combine additively - by adding their spectra wavelength-by-wavelength
- layered dyes or sequenced color filters combine subtractively - by multiplying their transmittance spectra wavelength-by-wavelength
- narrow spectra, widely spaced in wavelength, are best for primaries that are to be combined additively
- wide spectra that overlap are best for primaries that are to be combined subtractively, but product of all three must be black
- the particular spectra chosen is flexible; additive primaries need not be R,G,B, nor subtractive primaries C,M,Y
- additional primaries may be added to either system, resulting in a larger gamut of reproducible colors; adding black to a subtractive system (CMYK) ensures a deep black

Summary of additive versus subtractive color mixing (2 of 2)

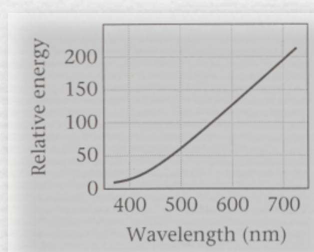
Continues with 5/14/09 lecture.



- in color printing, patches of the 3 subtractive primaries (C,M,Y) overlap only partially on the page, leading to patches of 8 meta-primaries (Wh,C,M,Y,CM,CY,MY,CMY), which combine additively in the eye when viewed from a distance
  - these effects are modeled by the *Neugebauer equations*
- two spectra that match (i.e. are metamers) under one illuminant may not match under another
  - clothes that match in the store may not match outdoors

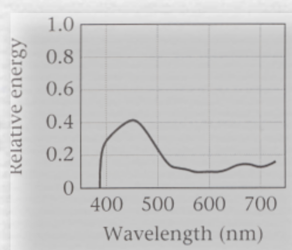


light is reflected by an object



illumination

×



reflectance

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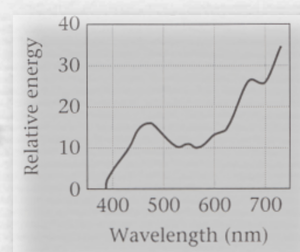

stimulus that enters your eye