Optics I: lenses and apertures

CS 178, Spring 2009

Begun Tuesday, April 7, finished Thursday, April 9. Note added to slide 61 on 5/4/09, and to slide 56 on 6/4/09.

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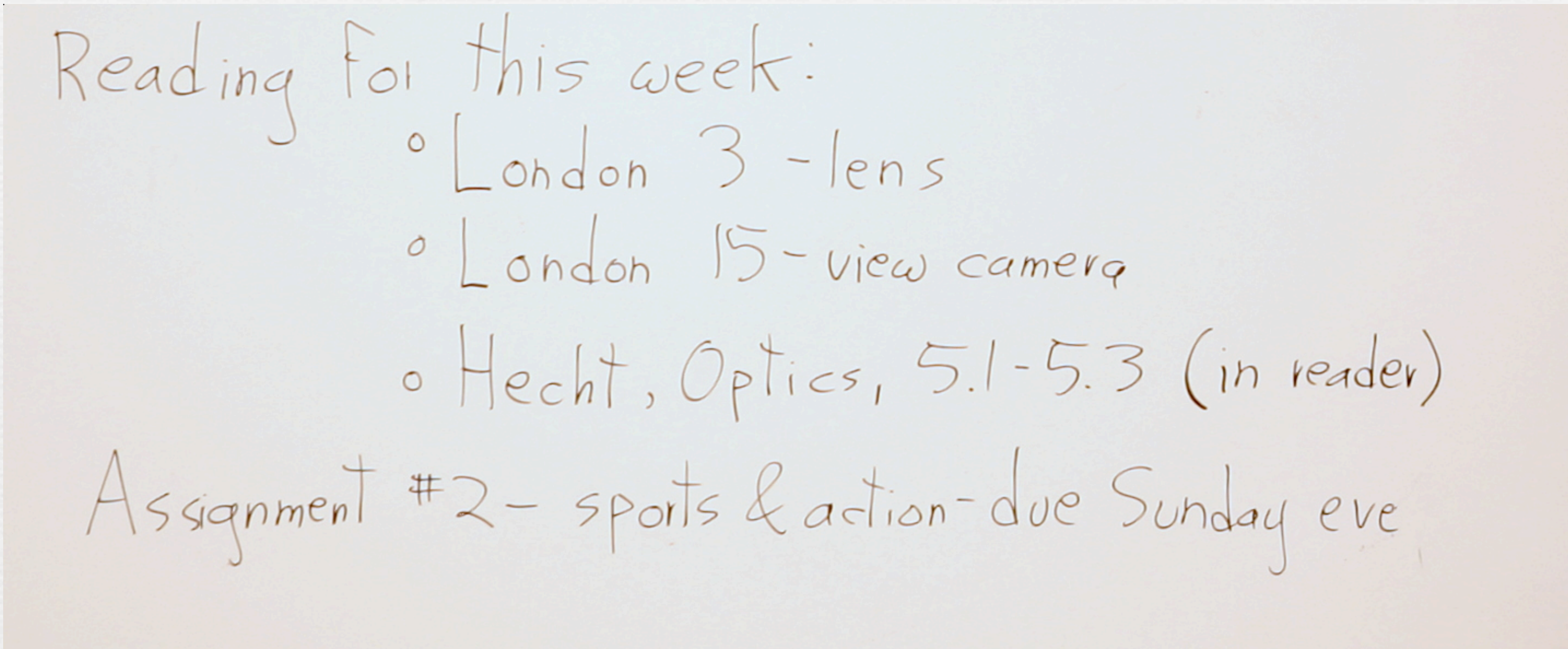
Announcements (from whiteboard)



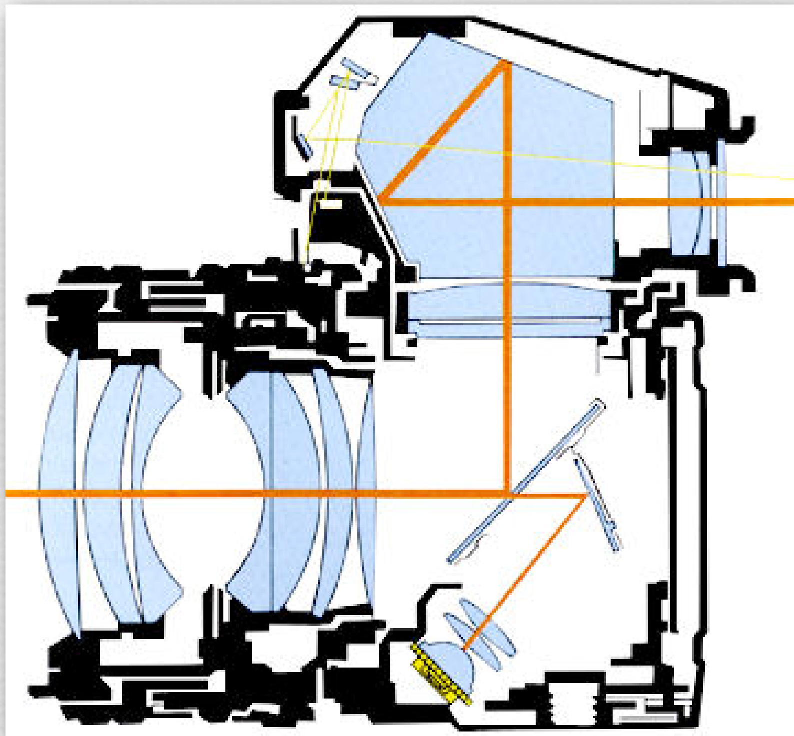
Outline

- why study lenses?
- geometrical optics
- depth of field

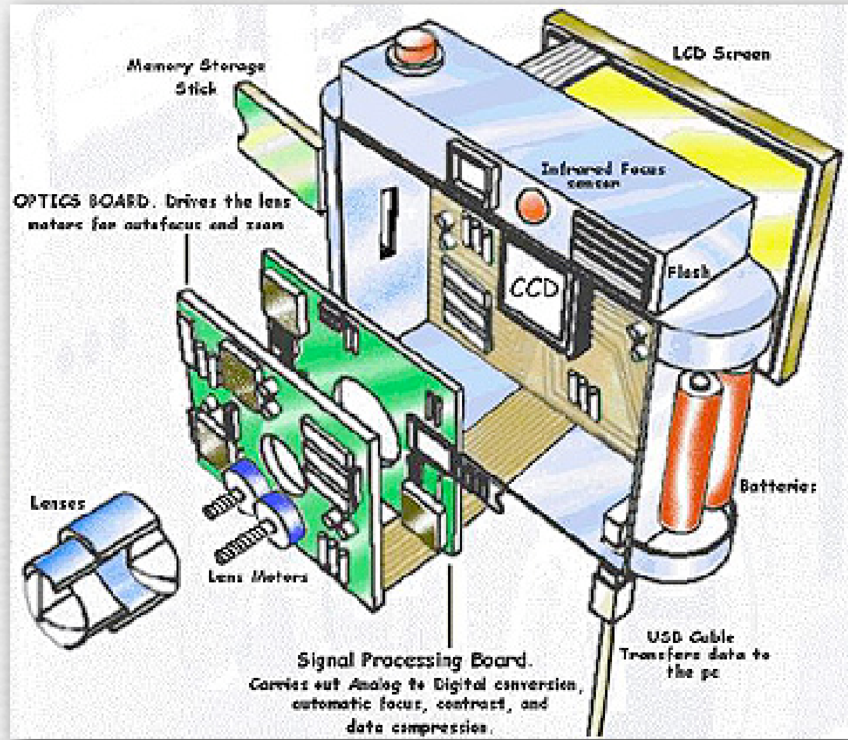
--------------------------------------- ?

- aberrations
- vignetting, glare, and other lens artifacts
- diffraction
- measuring lens quality

Cameras and their lenses



single lens reflex
(SLR) camera



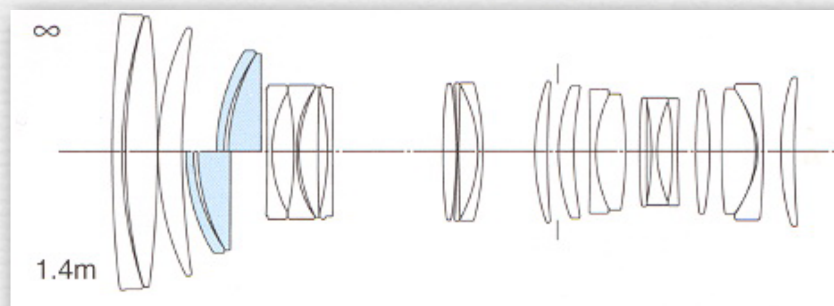
digital still camera (DSC),
i.e. point-and-shoot

Lens quality varies

- Why is this toy so expensive?
  - EF 70-200mm f/2.8L IS USM
  - $1700

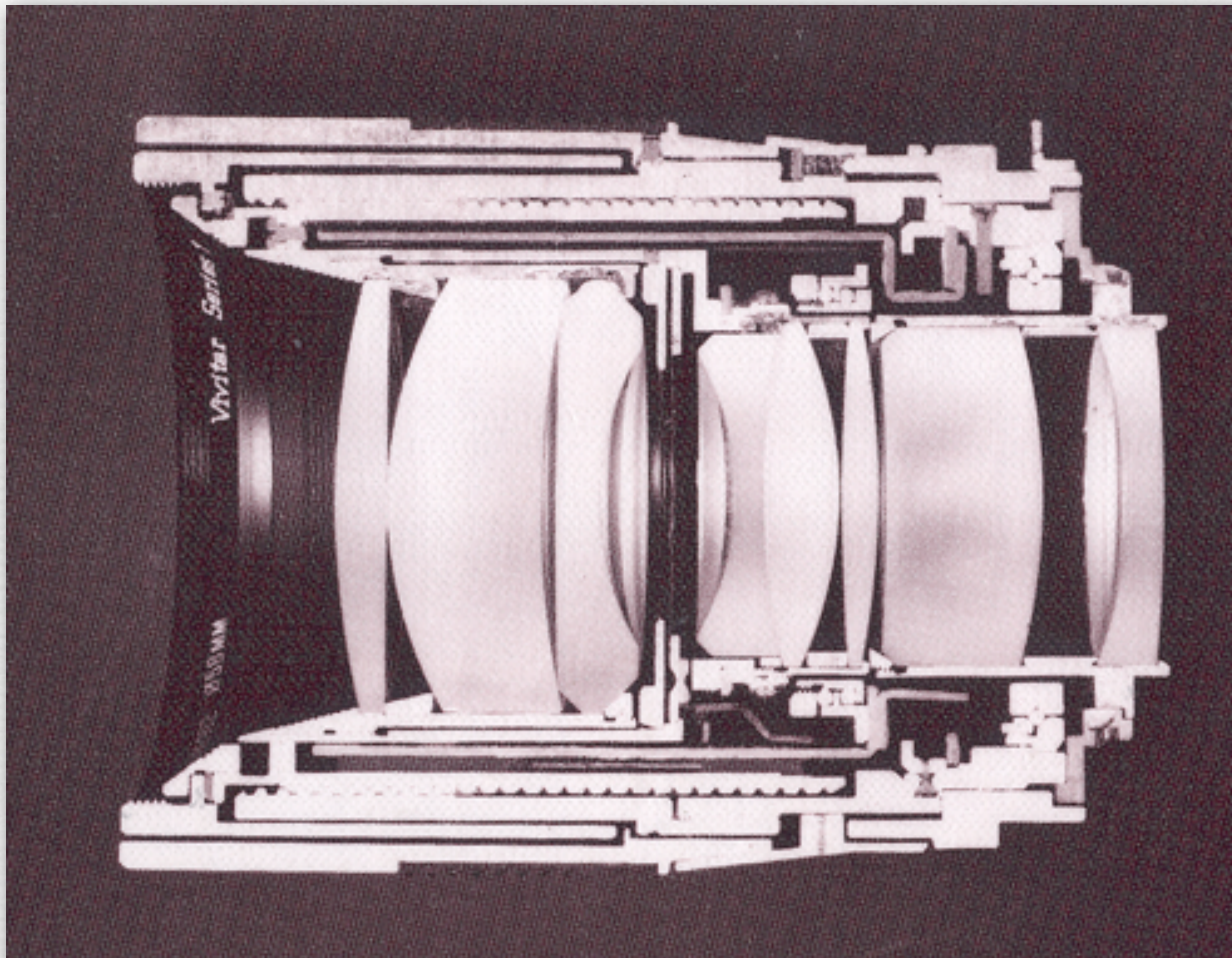
- Why is it better than this toy?
  - EF 70-300mm f/4-5.6 IS USM
  - $550

- Why is it so complicated?



(Canon)

Cutaway view of a real lens



Vivitar Series 1 90mm f/2.5
Cover photo, Kingslake, *Optics in Photography*

Image quality varies

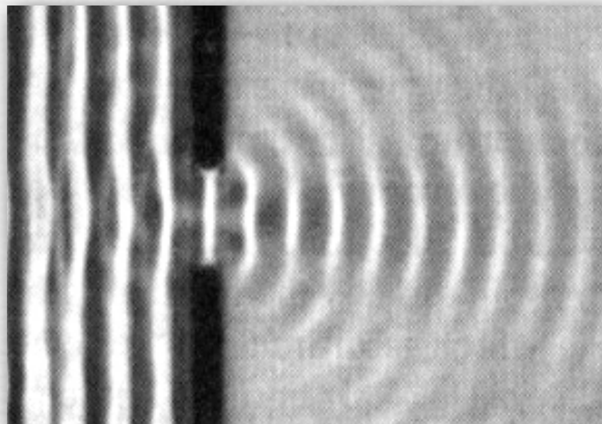
(luminous-landscape)

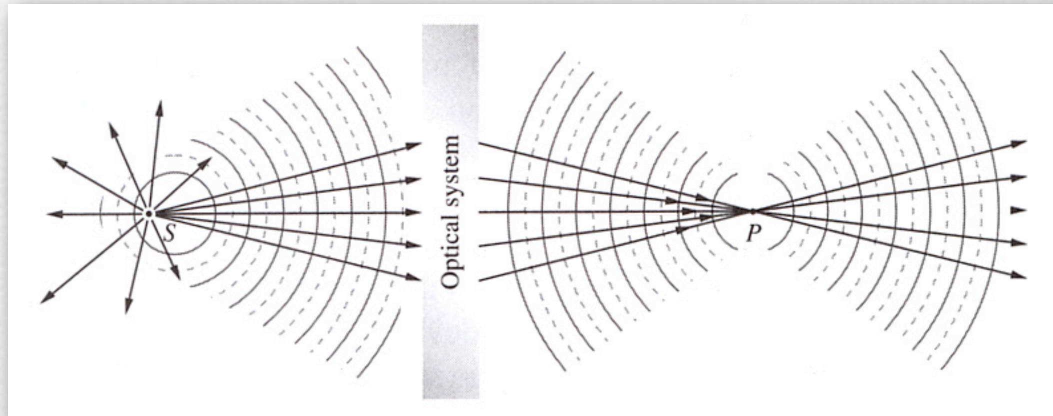
Zoom lens versus prime lens

Parameters of lenses

- zoom versus prime
- focal length (field of view)
- maximum aperture (minimum F-number, like f/2.8)
  - varies with focal length in a zoom lens
- image stabilization, faster autofocus, etc.
- minimum focusing distance
- other quality issues
- special-purpose lenses
  - fisheye
  - macro (1:1)
  - perspective control (a.k.a. tilt-shift)

Physical versus geometrical optics

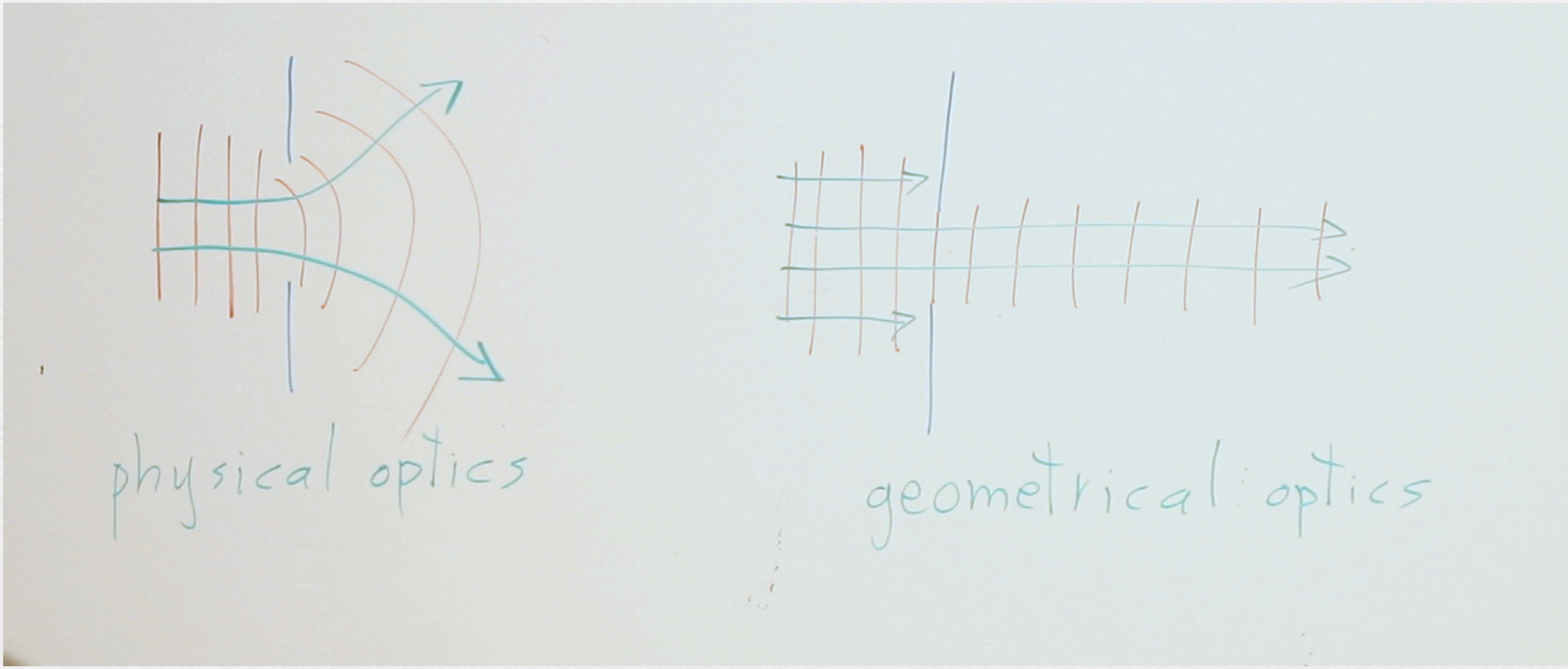




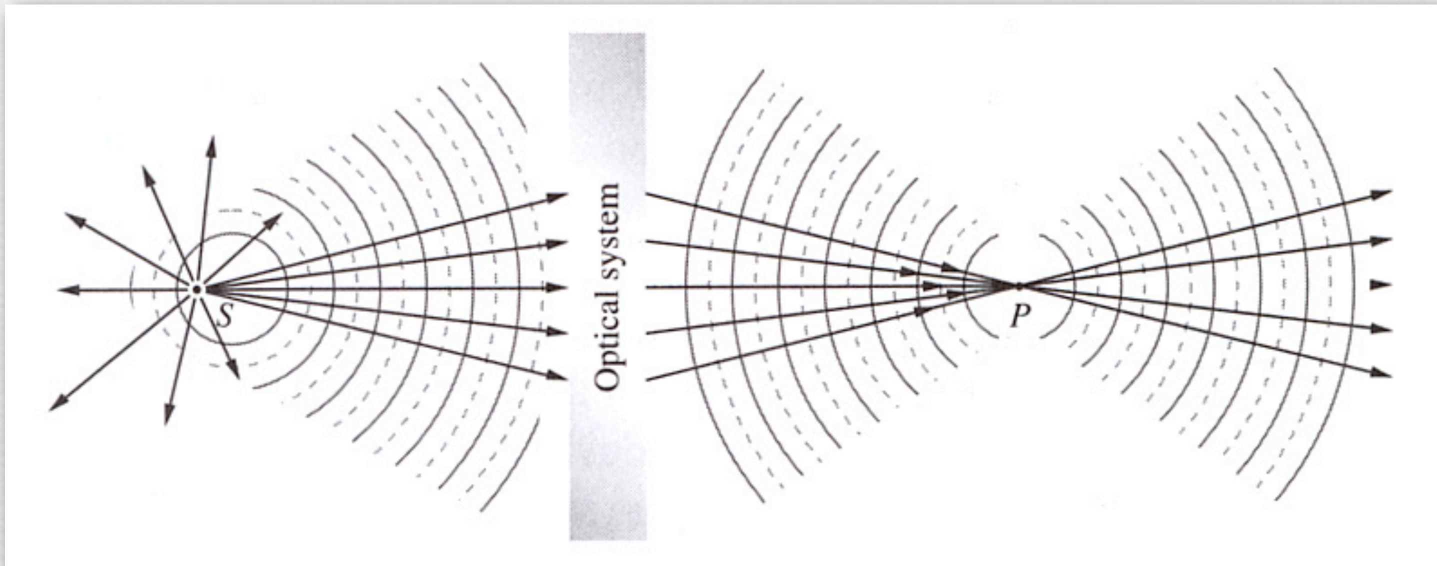
(Hecht)

- light can be modeled as traveling waves
- the perpendiculars to waves can be drawn as rays
- diffraction causes these rays to bend, e.g. at a slit
- *geometrical optics* assumes
  - $\lambda \to 0$
  - no diffraction
  - straight rays in free space (a.k.a. rectilinear propagation)

Physical versus geometrical optics



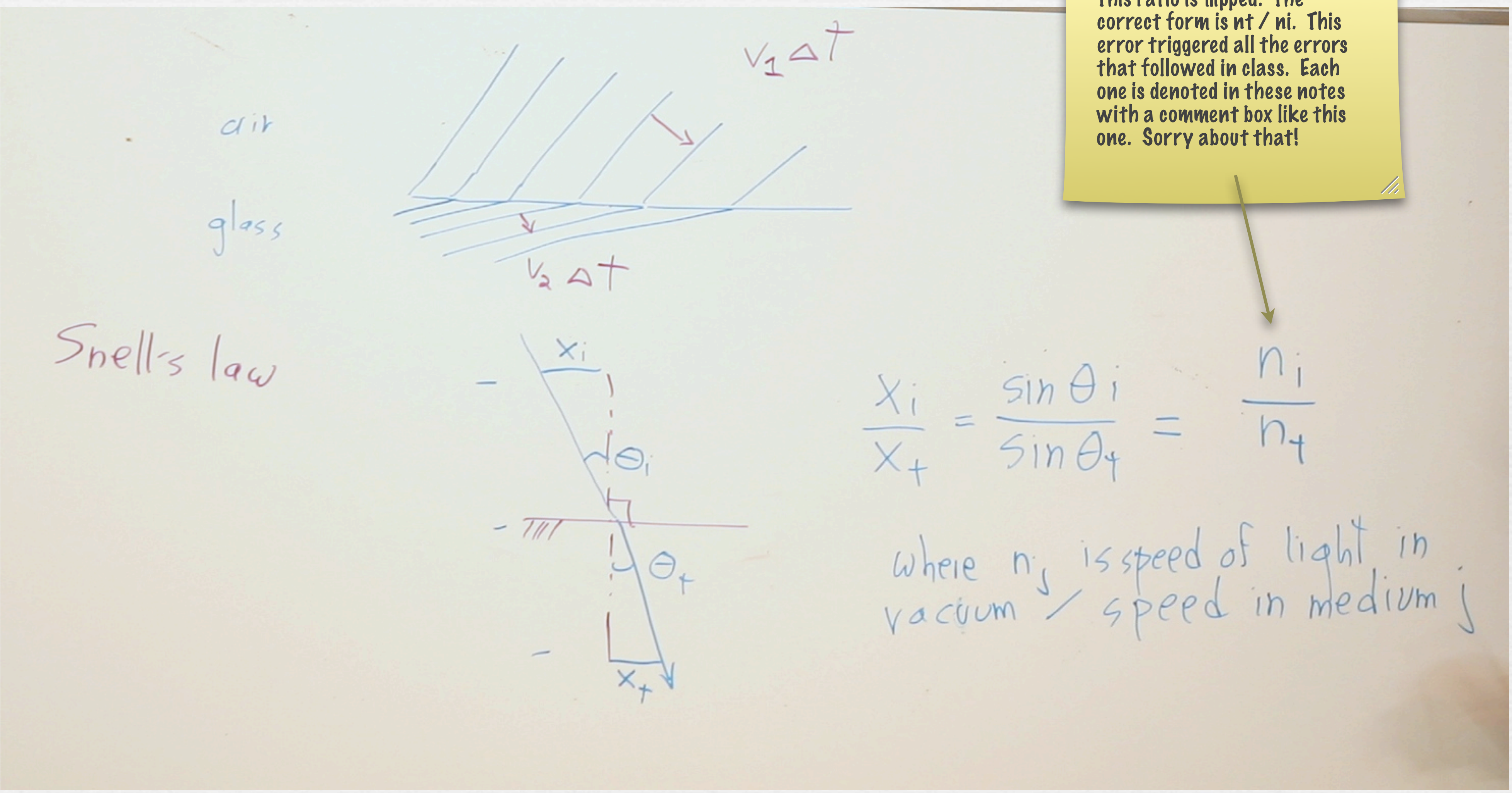
Some definitions



(Hecht)

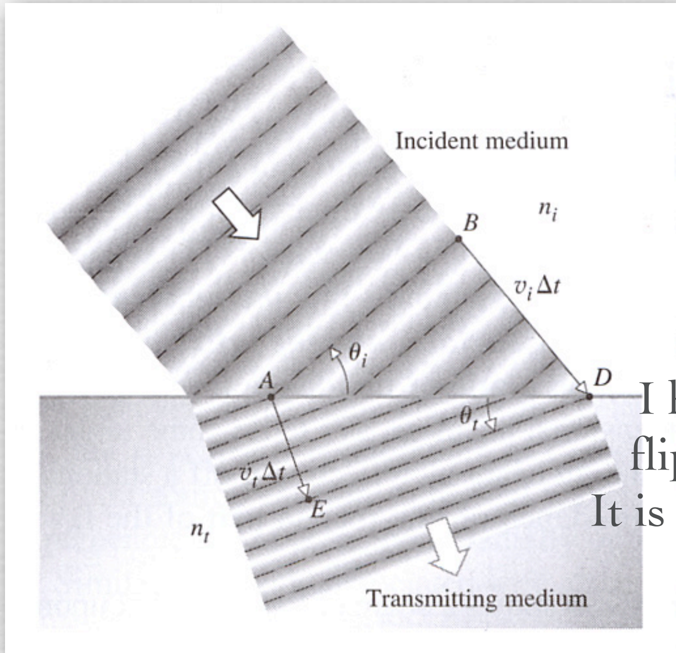
- *object space* on the left; *image space* on the right
- if rays leaving a point arrive at another point (as shown), the optical system is called *stigmatic* for these two points
- *S* and *P* are called *conjugate points*

Snell's law of refraction



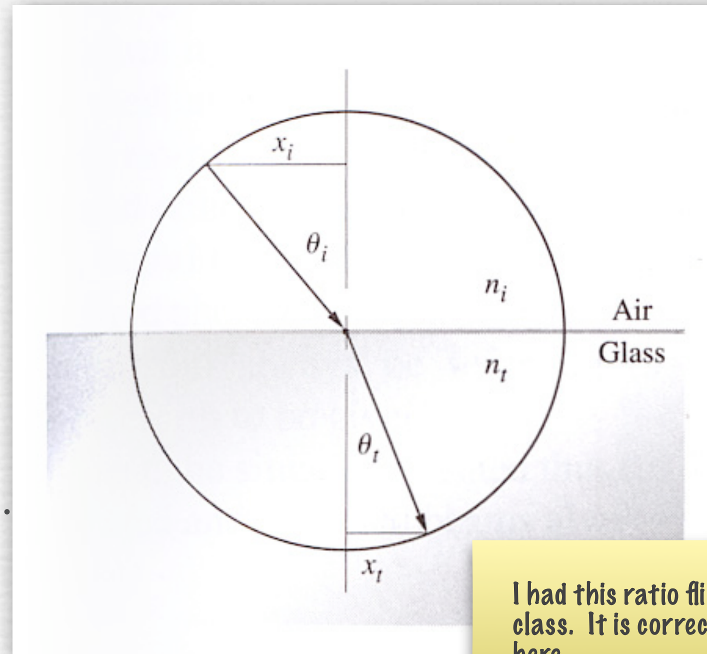
This ratio is flipped. The correct form is nt / ni. This error triggered all the errors that followed in class. Each one is denoted in these notes with a comment box like this one. Sorry about that!

Snell's law of refraction



(Hecht)

I had this ratio flipped in class. It is corrected here.



I had this ratio flipped in class. It is corrected here.

- as waves change speed at an interface, they also change direction

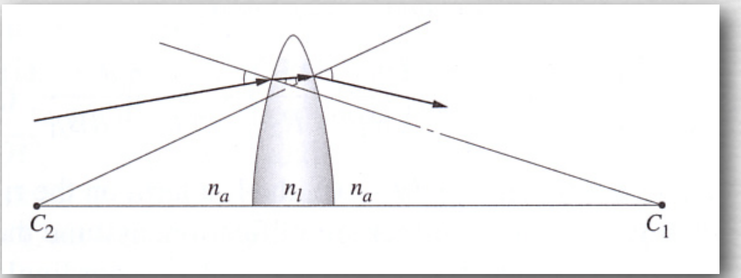
$$\frac{x_i}{x_t} = \frac{\sin\theta_i}{\sin\theta_t} = \frac{n_t}{n_i}$$

- index of refraction is defined as the ratio between the speed of light in a vaccum / speed in some medium

Typical refractive indices (*n*)

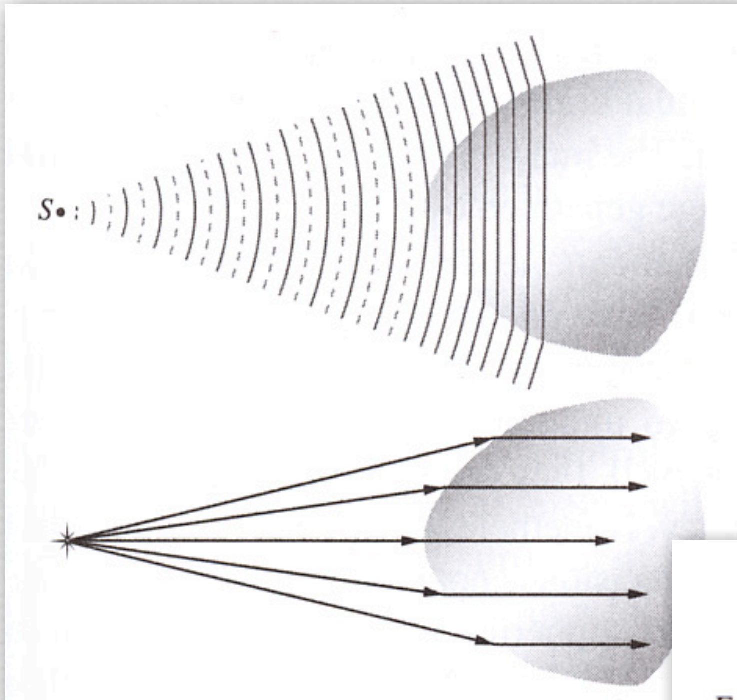
- air = 1.0
- water = 1.33
- glass = 1.5 - 1.8
- microscope immersion oil = 1.52

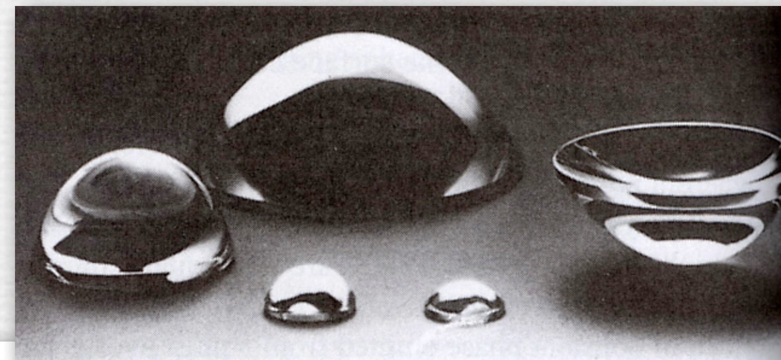
- when transiting from air to glass, light bends towards the normal
- when transiting from glass to air, light bends away from the normal

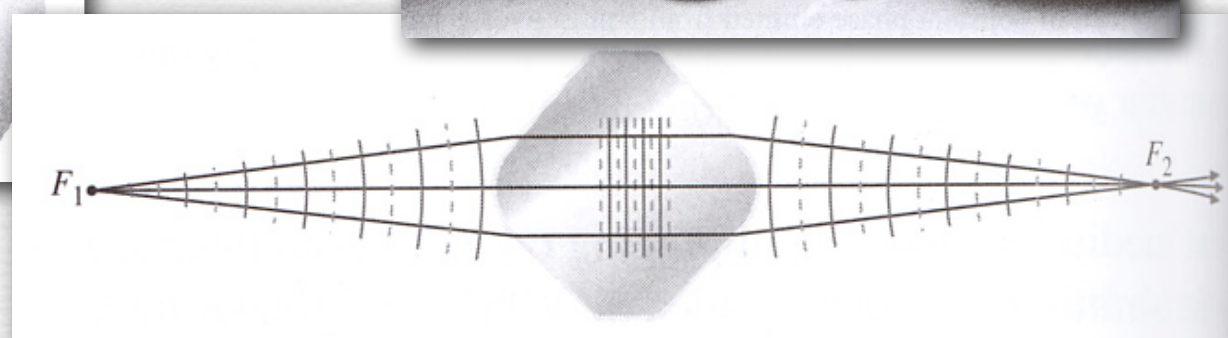


- light striking a surface perpendicularly does not bend

Q. What shape should an interface be to make parallel rays converge to a point?







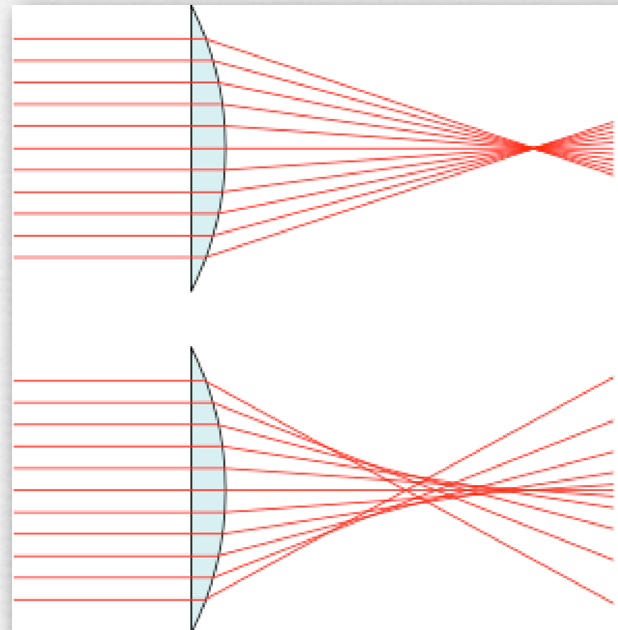
(Hecht)

A. a hyperbola

- so lenses should be hyperbolic!

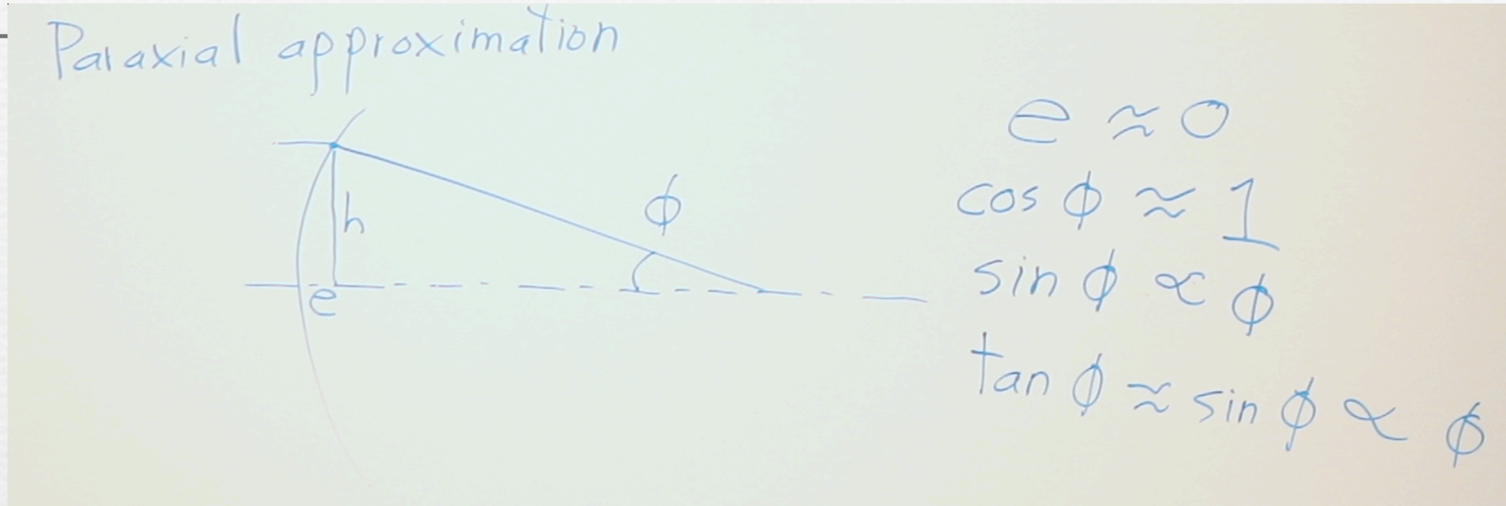
Spherical lenses

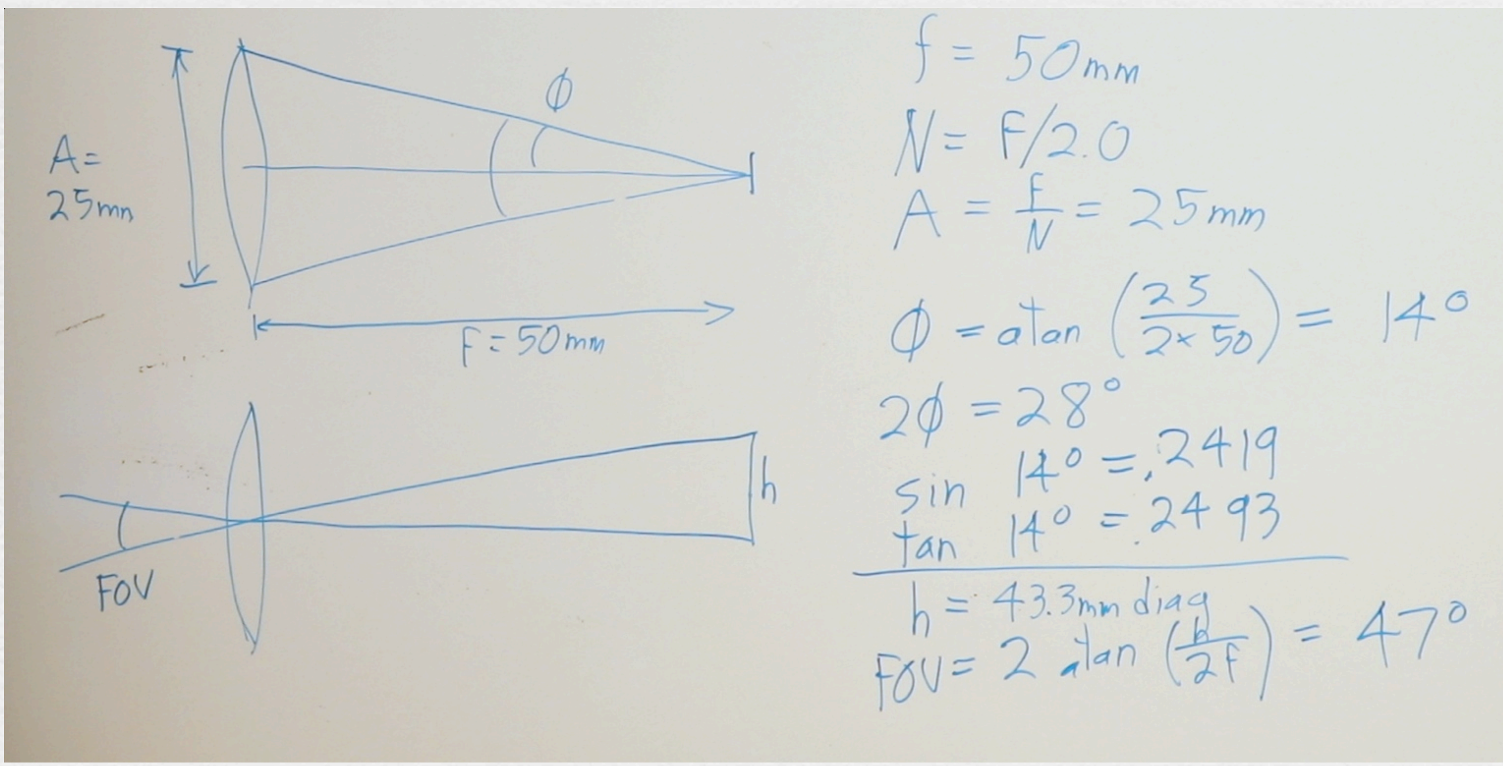

(Hecht)


(wikipedia)

- two roughly fitting curved surfaces ground together will eventually become spherical
- spheres don't bring parallel rays to a point
  - this is called *spherical aberration*
  - nearly axial rays behave best

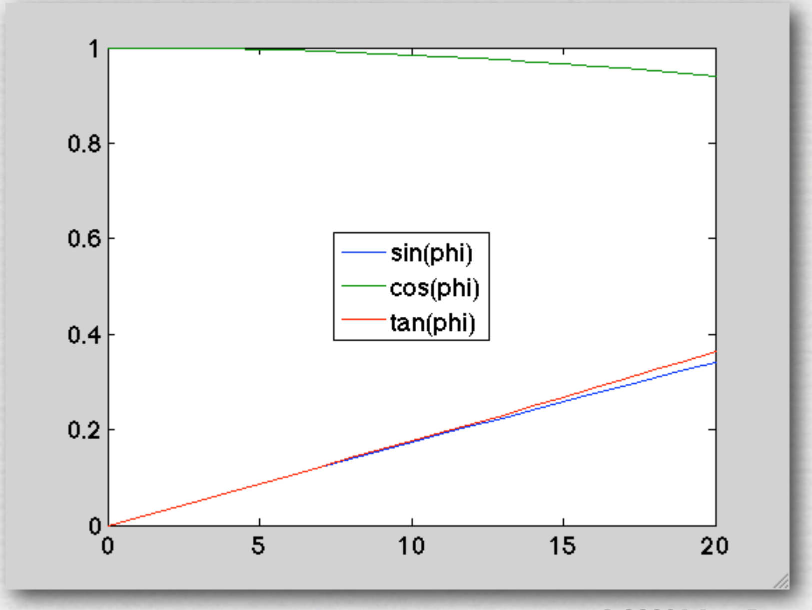
The paraxial approximation





The paraxial approximation is a.k.a. first-order optics

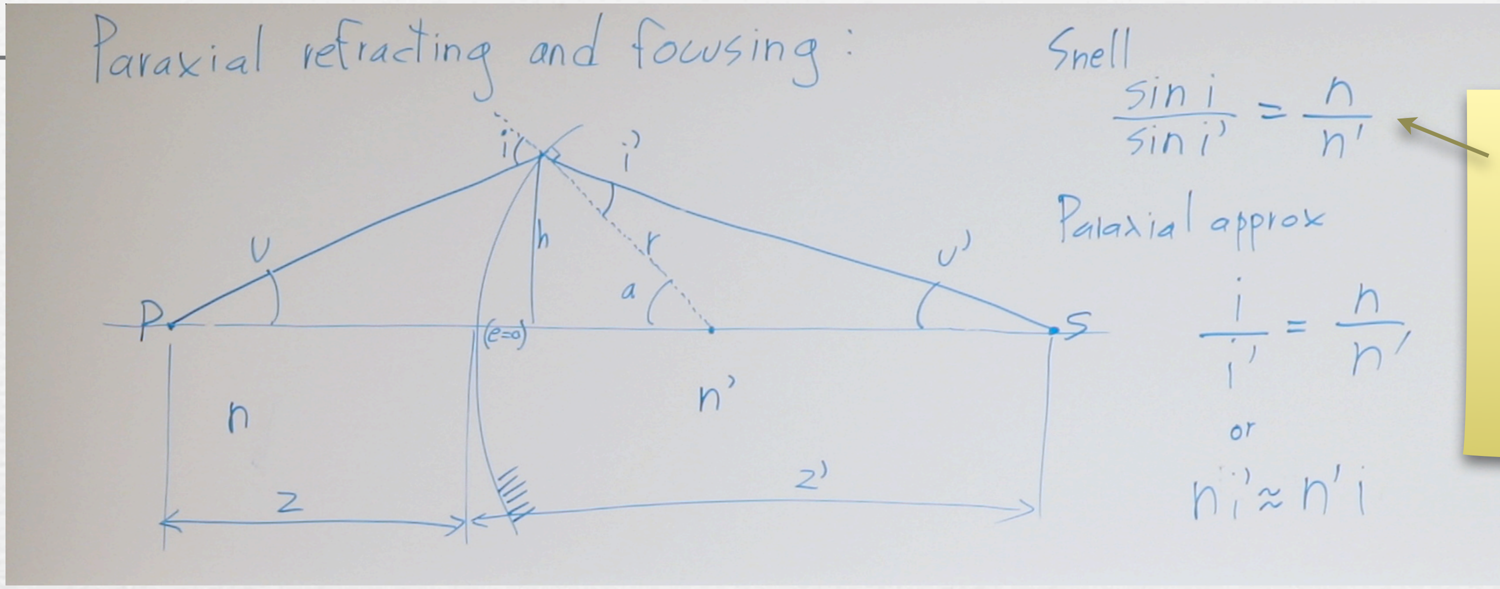
- assume first term of $\sin\phi = \phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + \ldots$
  - i.e. $\sin\phi \approx \phi$
- assume first term of $\cos\phi = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \frac{\phi^6}{6!} + \ldots$
  - i.e. $\cos\phi \approx 1$
  - so $\tan\phi \approx \sin\phi \approx \phi$



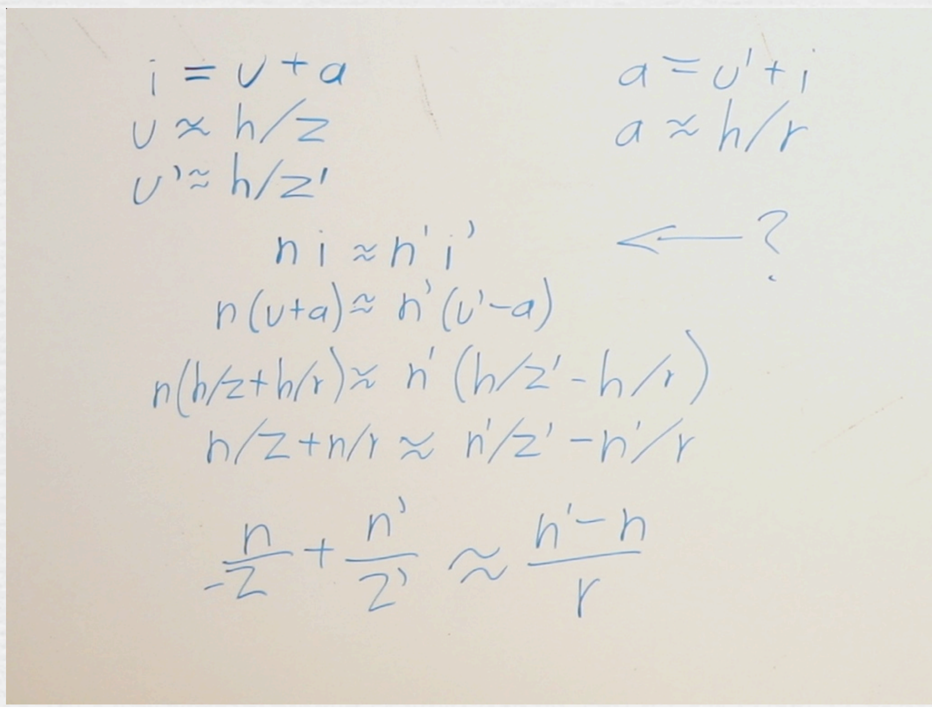
Paraxial refraction and focusing

- this derivation uses classical paraxial notation (letters for angles, instead of Greek symbols)
- Hecht's derivation uses Fermat's principle instead of Snell's law, but the result is the same

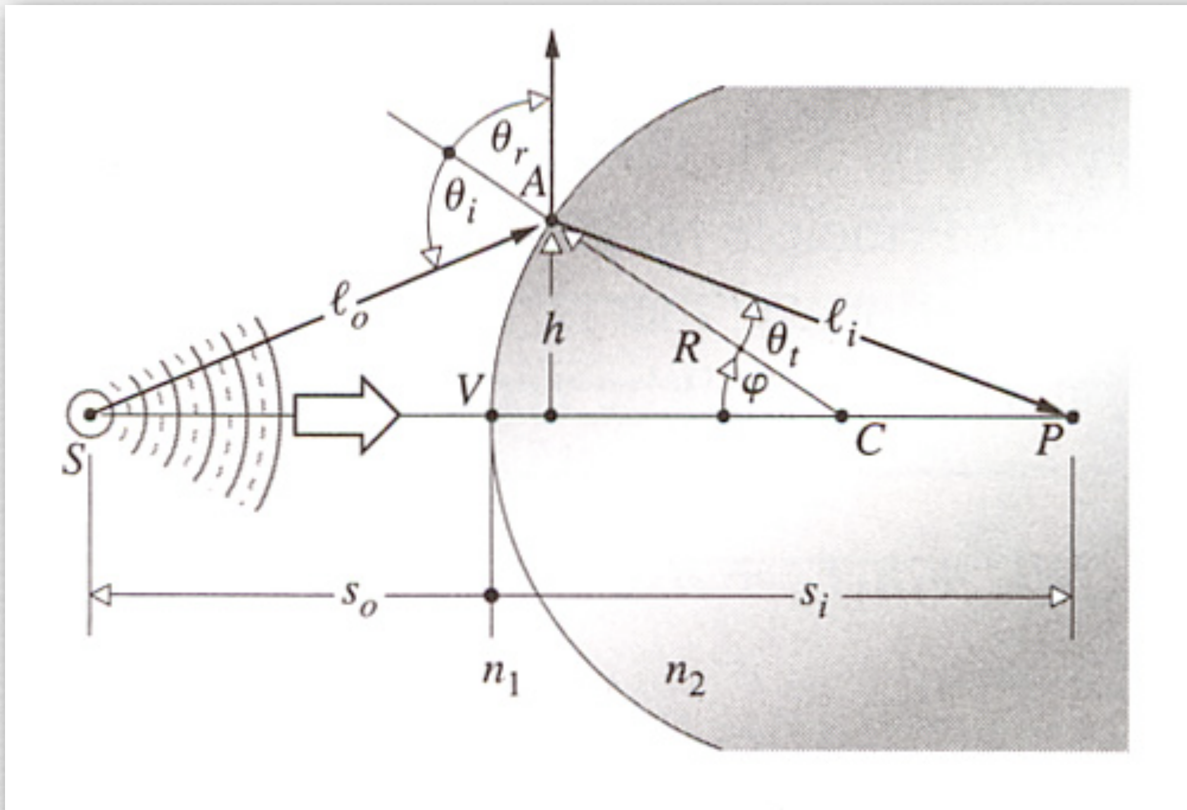
Paraxial refraction and focusing



This is flipped too. It should be n′ / n. Then the paraxial approximation should read i/i′ ≈ n/n′, or ni ≈ n′i′ (as I had originally written). The rest of the derivation (below) is correct. Ignore the question mark I wrote on the whiteboard (below).



Paraxial refraction and focusing

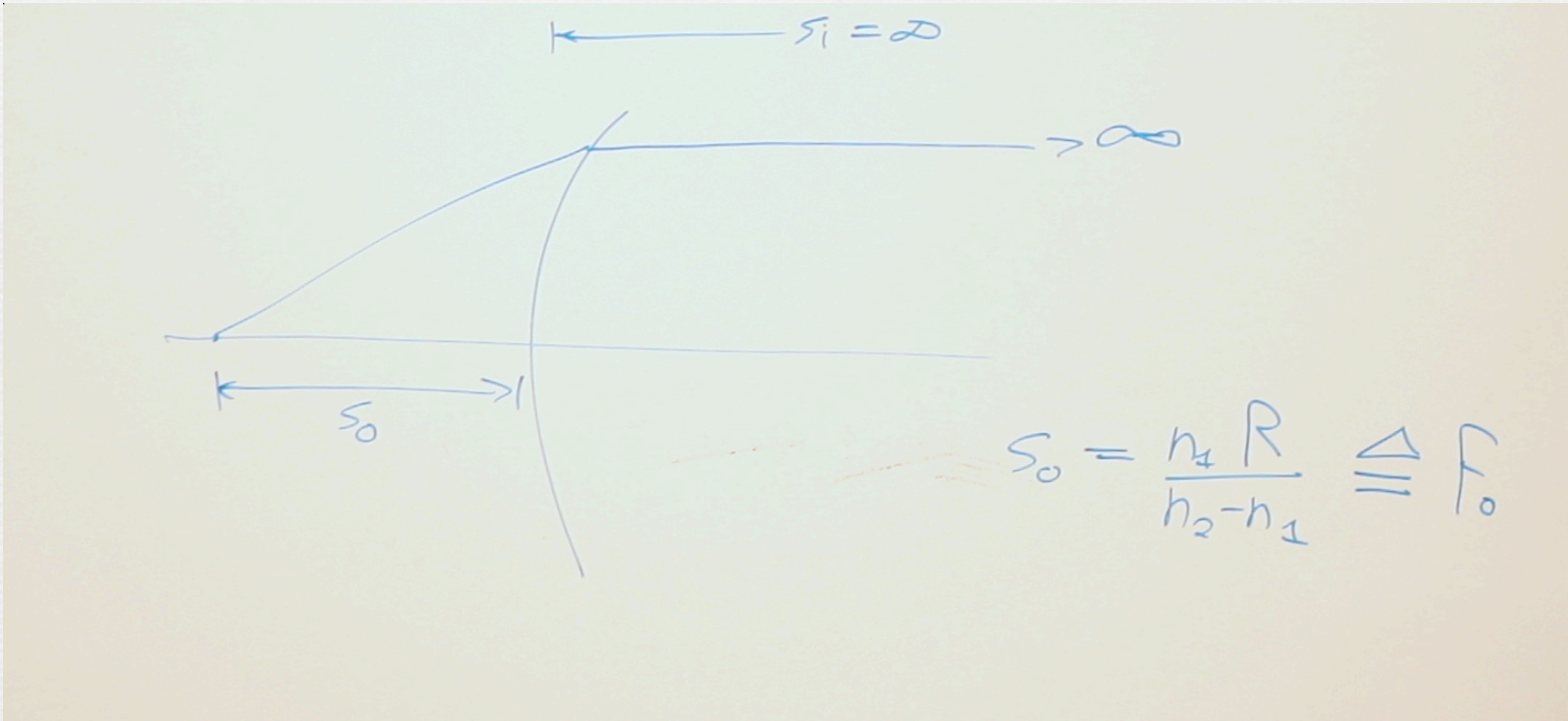


(Hecht)

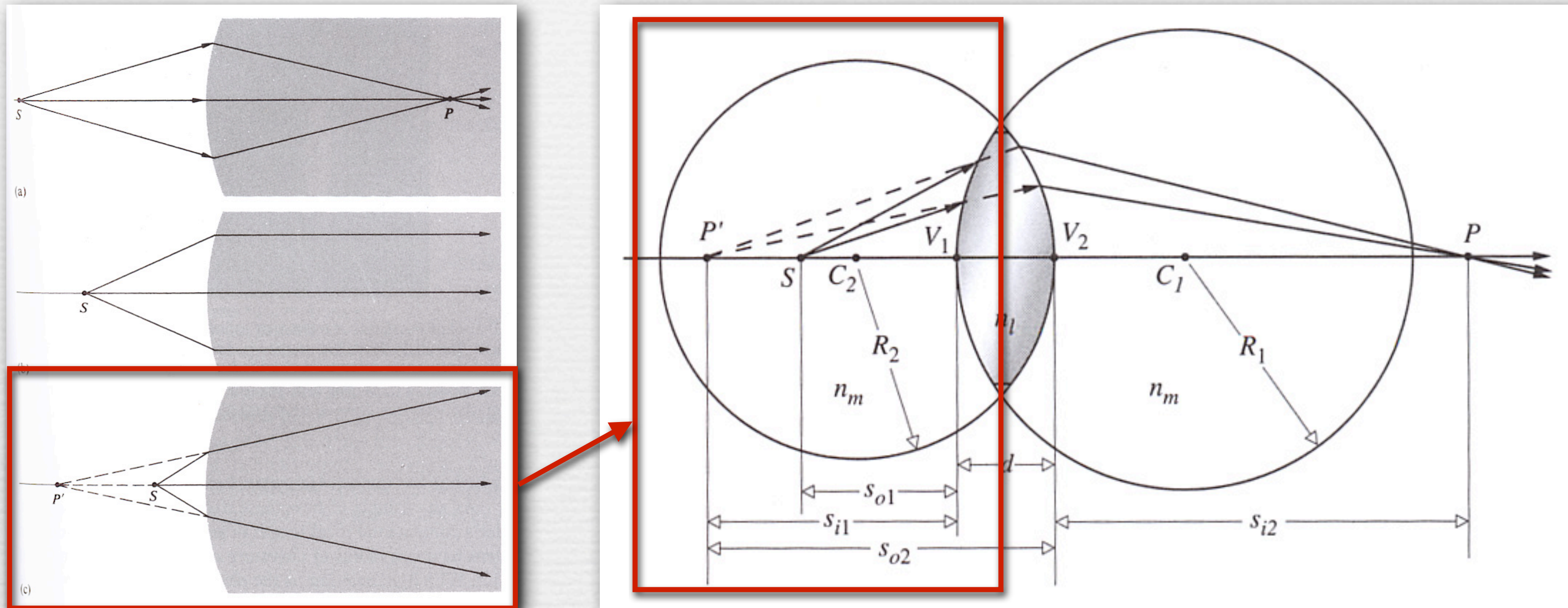
$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

Paraxial refraction and focusing

- the case where s_i is at infinity...



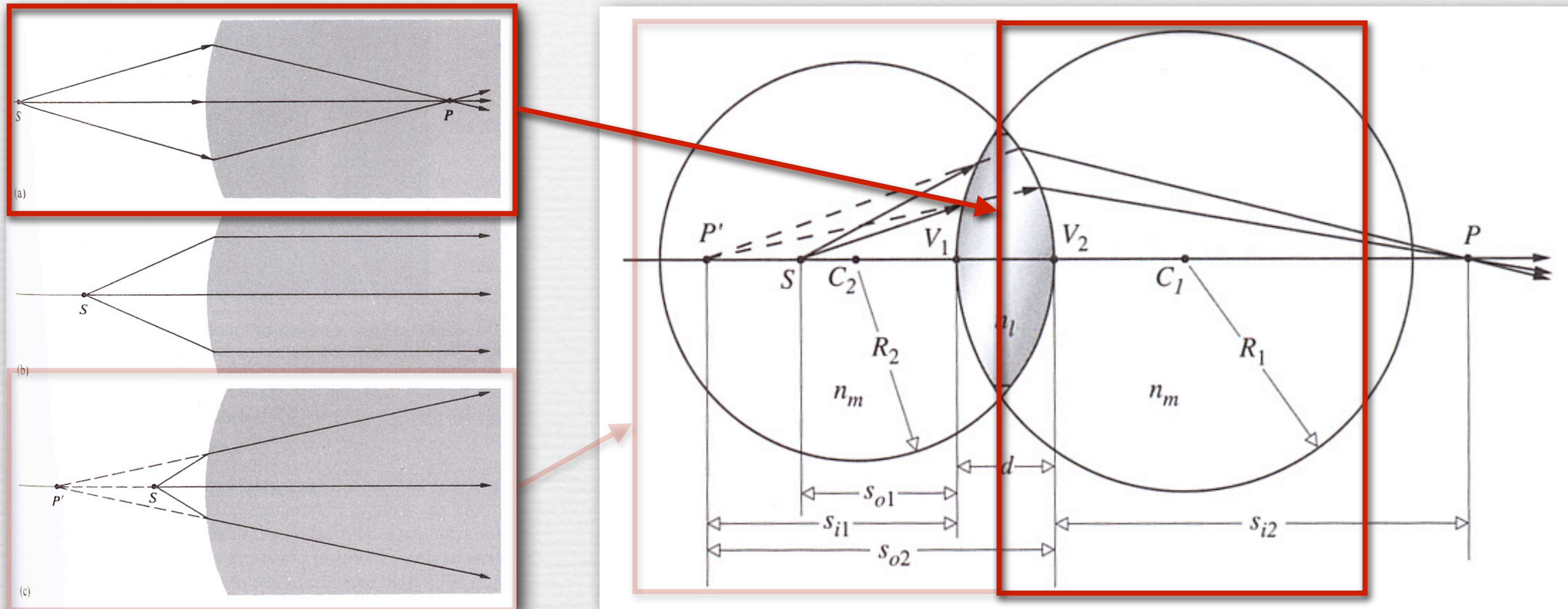
Thin lens equation, a.k.a. lensmaker's formula



(Hecht)

- we just derived cases (a) and (b)
- for a thin lens in air, apply (c), then (a) with air and glass reversed, then set d = 0

Thin lens equation,
a.k.a. lensmaker's formula



(Hecht)

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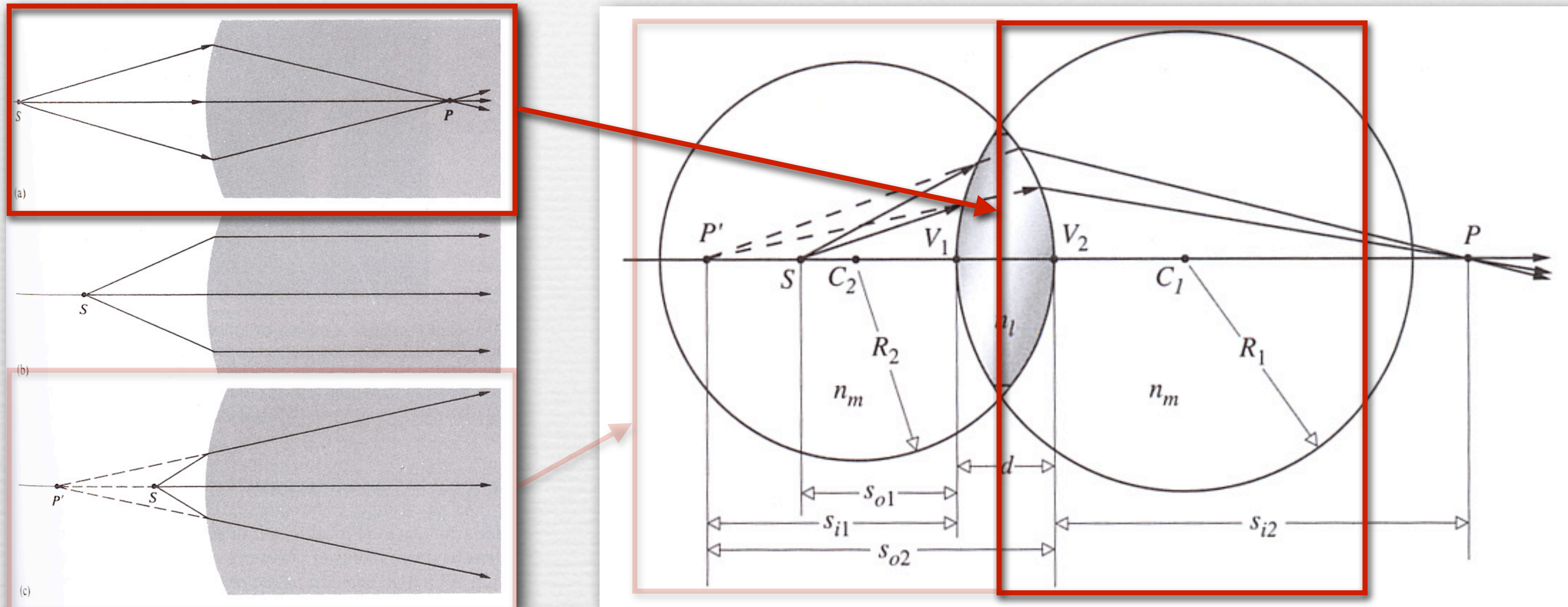
Thin lens equation, a.k.a. lensmaker's formula

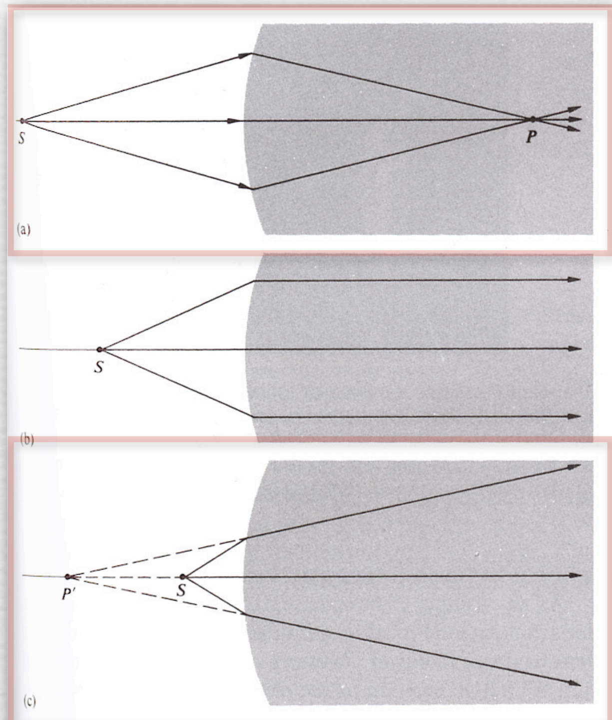


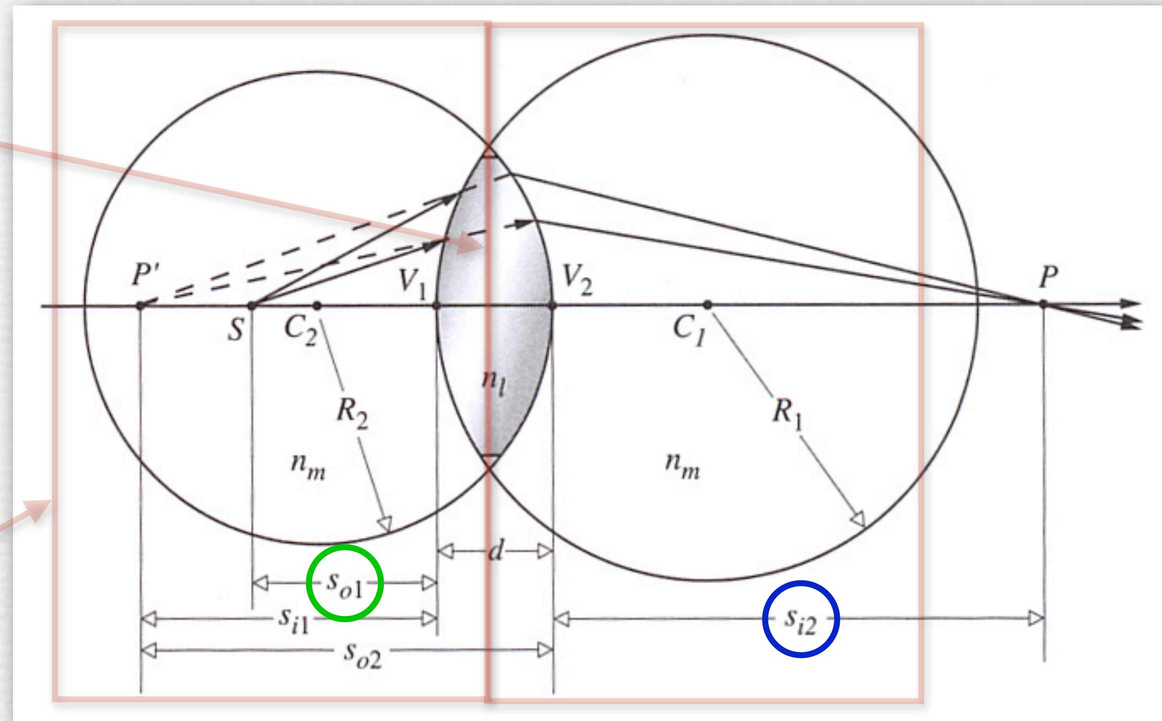
(Hecht)

$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

(Hecht, eqn 5.15)

Thin lens equation, a.k.a. lensmaker's formula

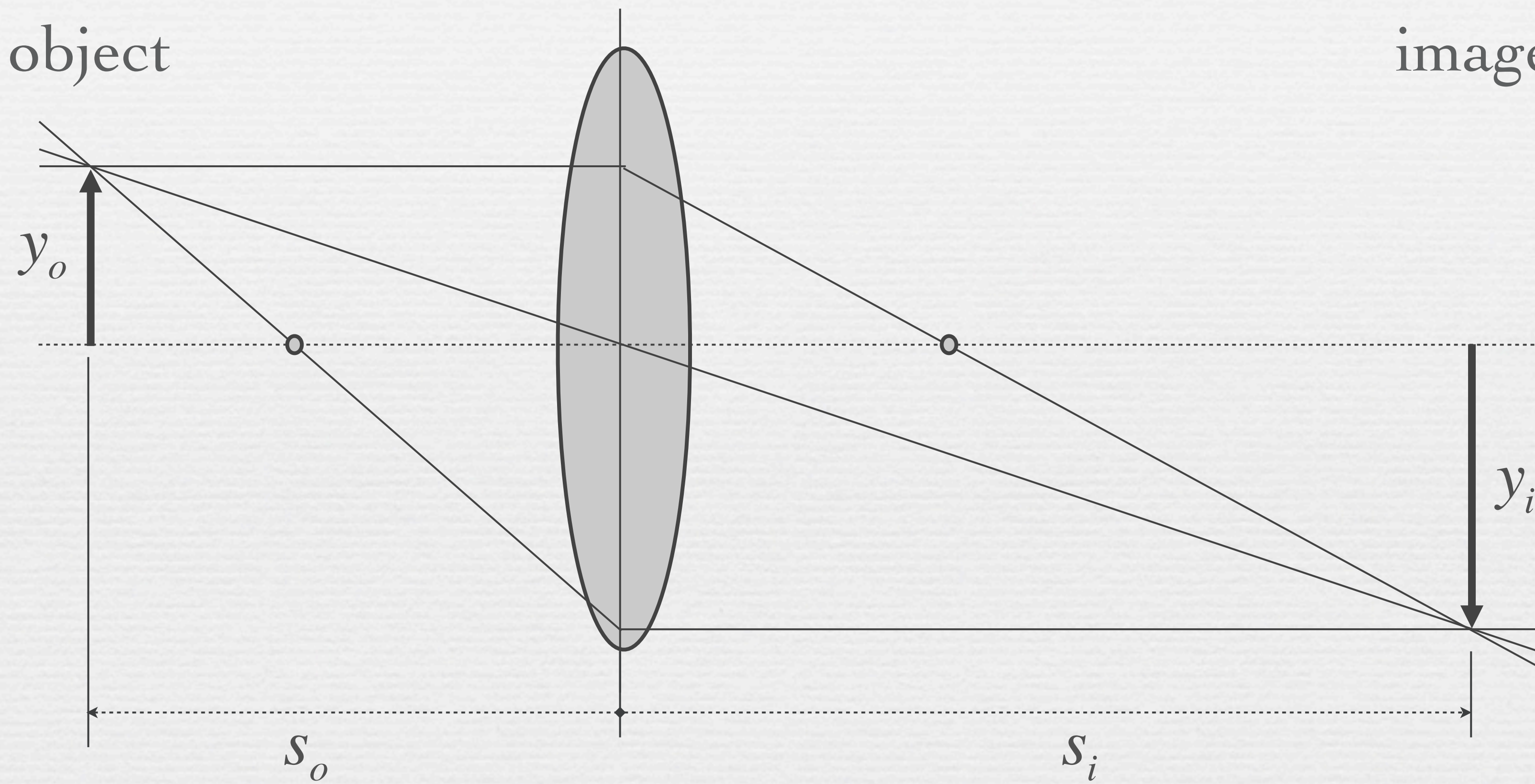




(Hecht)

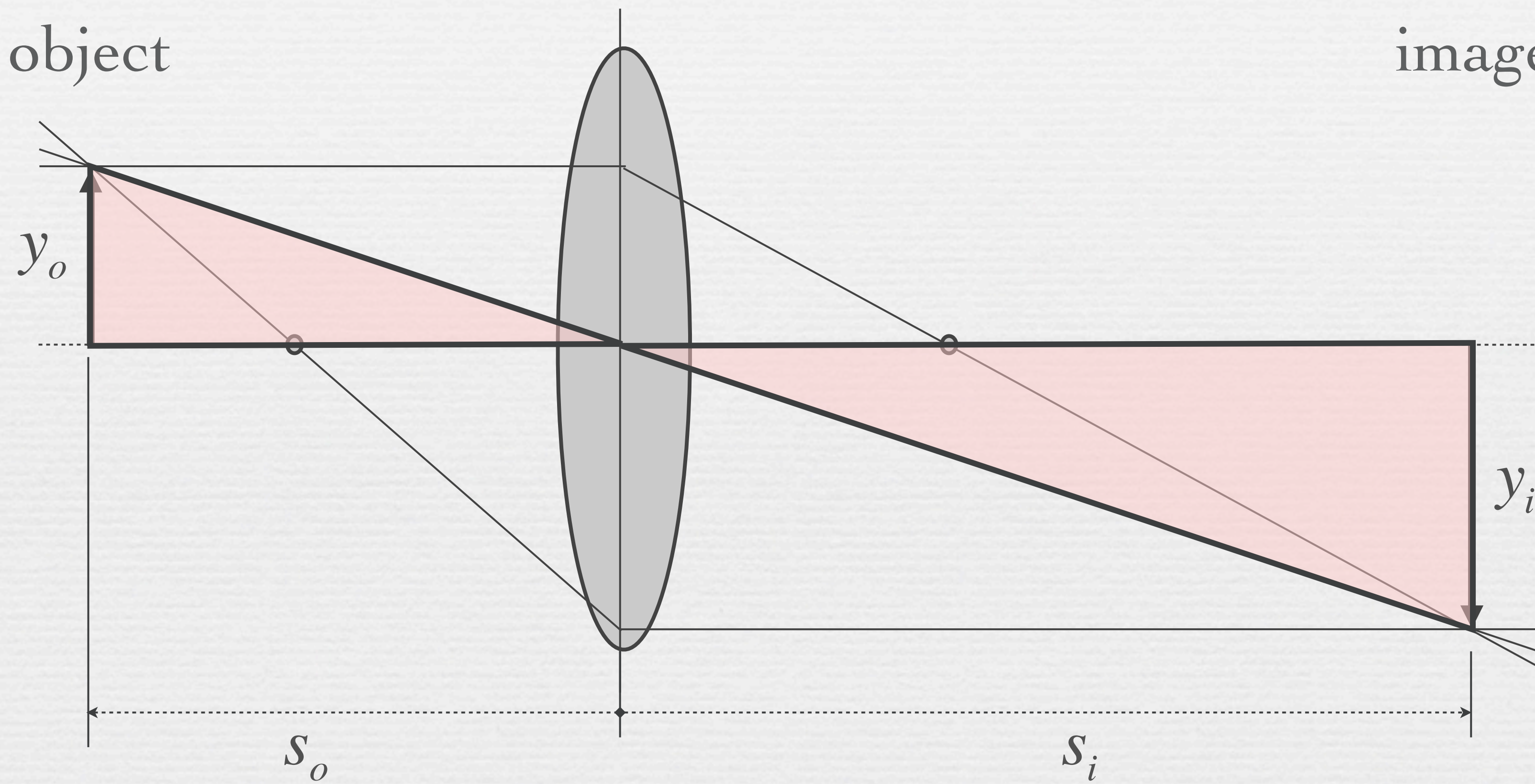
$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$ (Hecht, eqn 5.15)

Gaussian lens formula



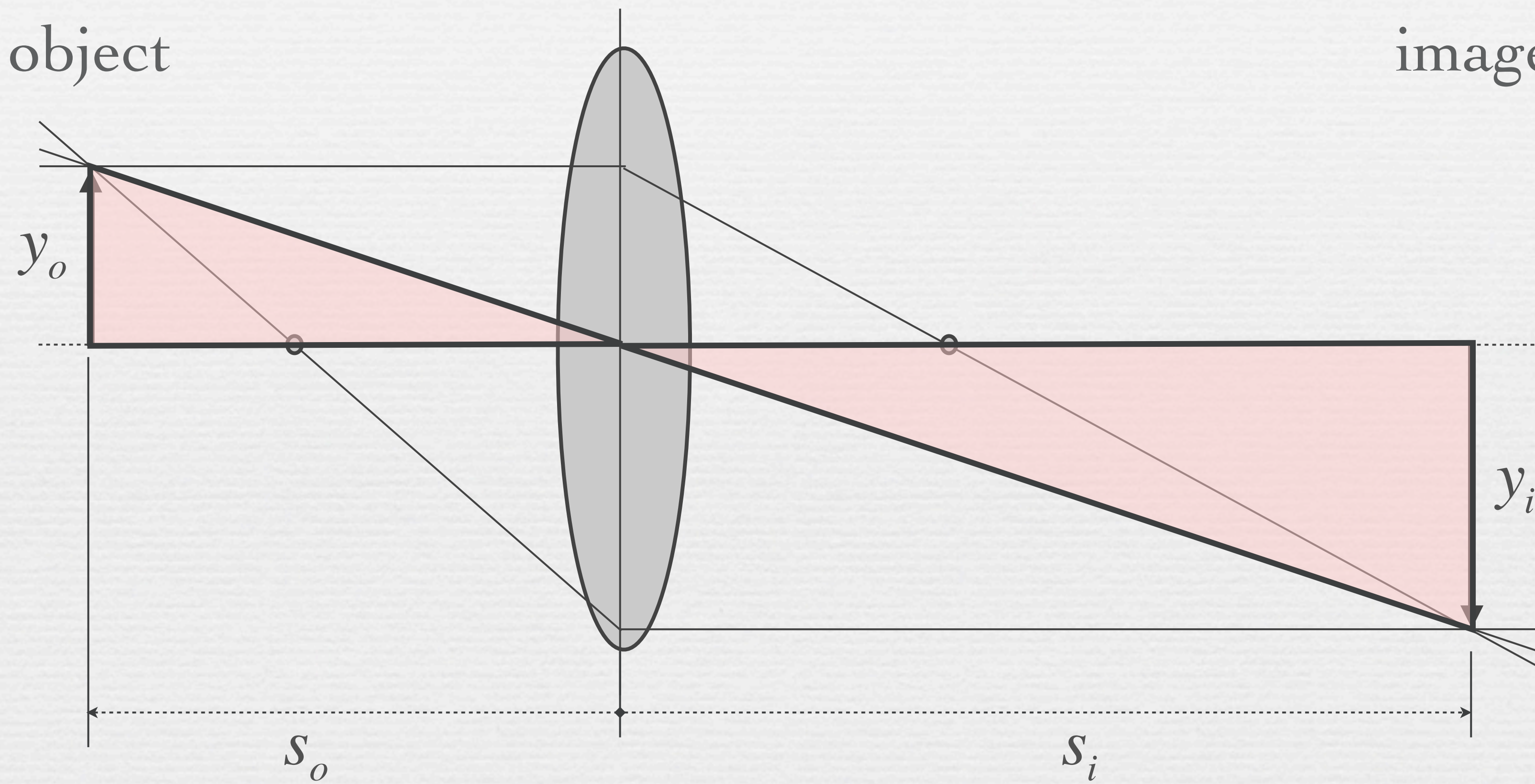
- a geometrical derivation
- begins with Gauss' ray diagram

Gaussian lens formula



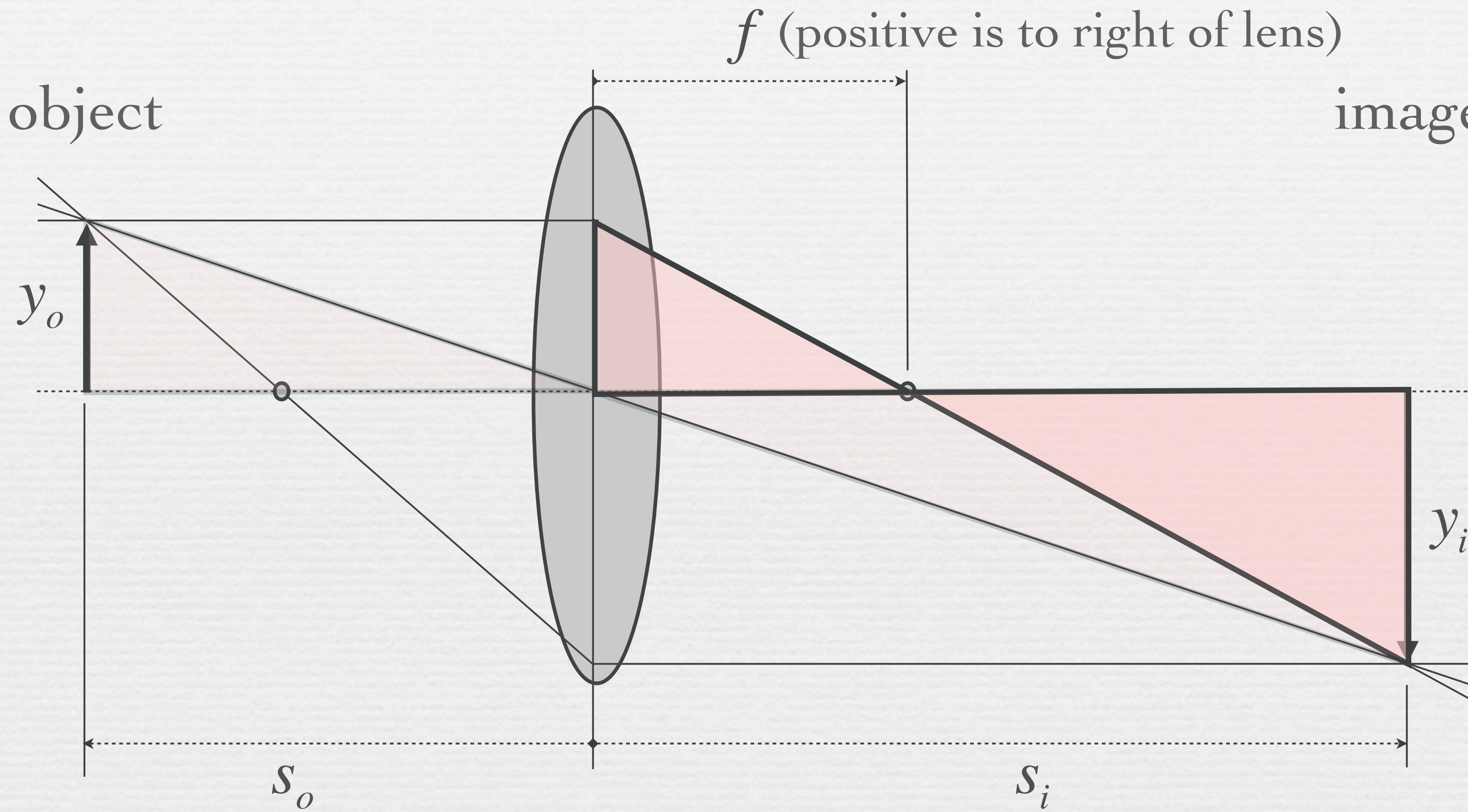
- positive s_i is rightward, positive s_o is leftward
- positive y is upward

Gaussian lens formula



$$\frac{|y_i|}{y_o} = \frac{s_i}{s_o}$$

Gaussian lens formula



$$\frac{|y_i|}{y_o} = \frac{s_i}{s_o} \quad \text{and} \quad \frac{|y_i|}{y_o} = \frac{s_i - f}{f} \quad \ldots\ldots \quad \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

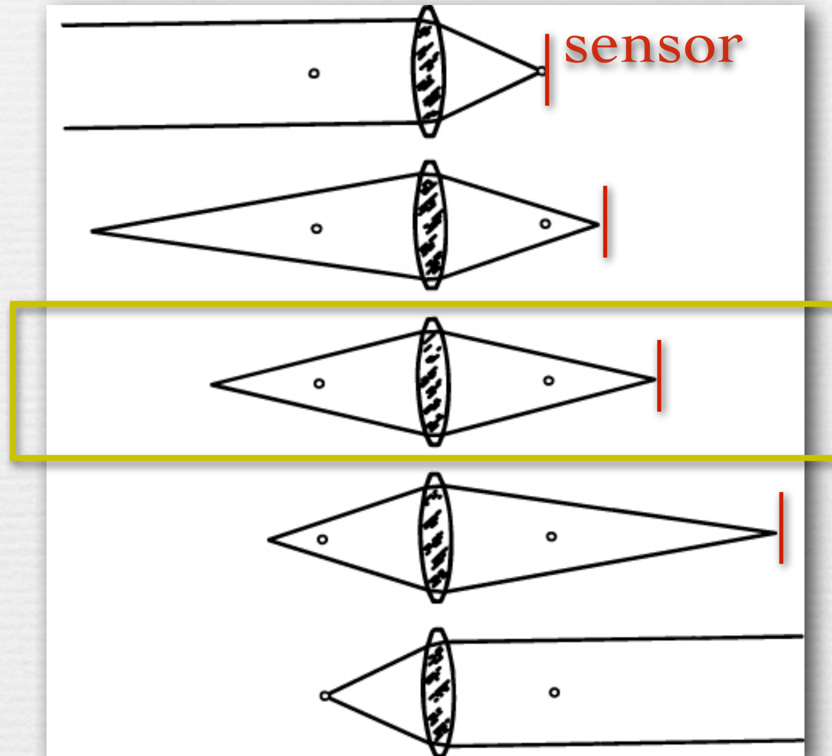
Changing the focus distance (again)

- note that at $s_o = s_i = 2f$,

  we have 1:1 imaging,

  because

$$\frac{1}{2f} + \frac{1}{2f} = \frac{1}{f}$$

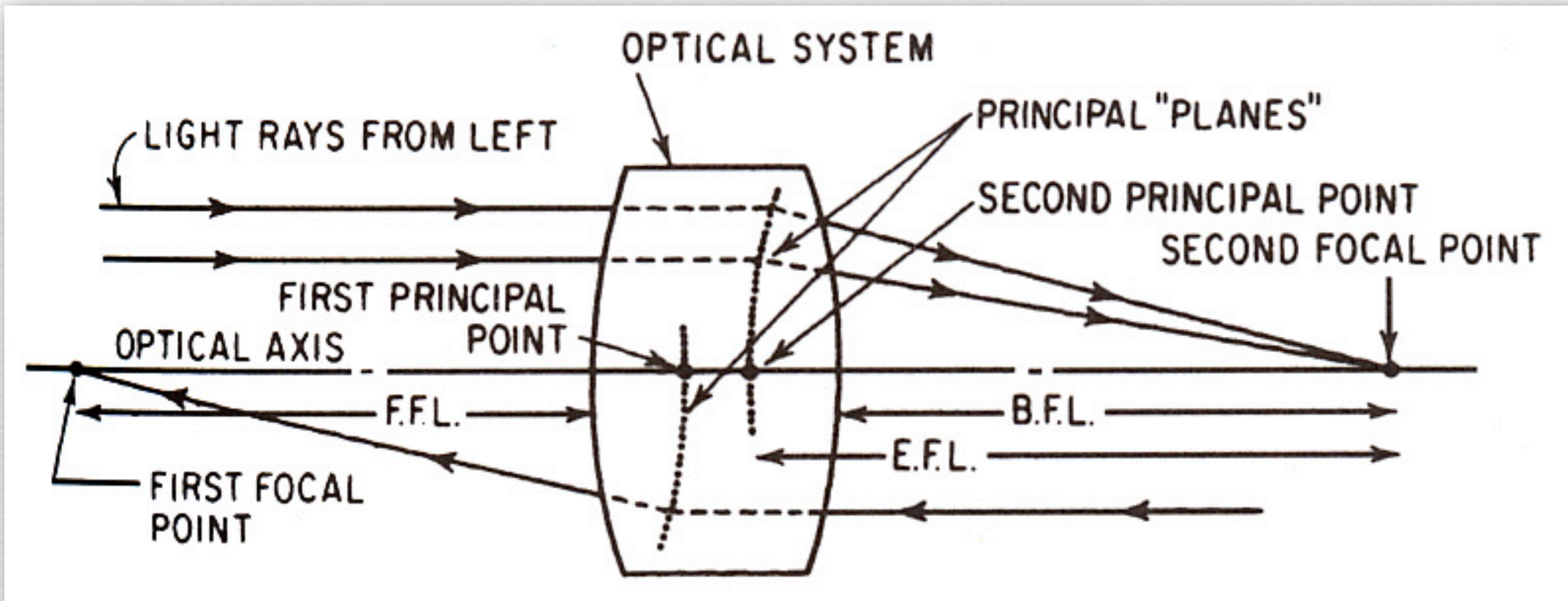


- in 1:1 imaging, if the sensor is 36mm wide, an object 36mm wide will fill the frame

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

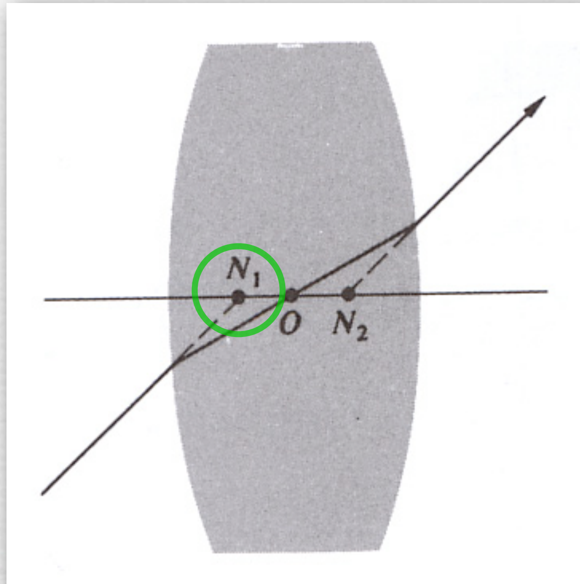
Thick lenses

- an optical system may contain many lenses, but can be characterized by a few numbers



(Smith)

Center of perspective

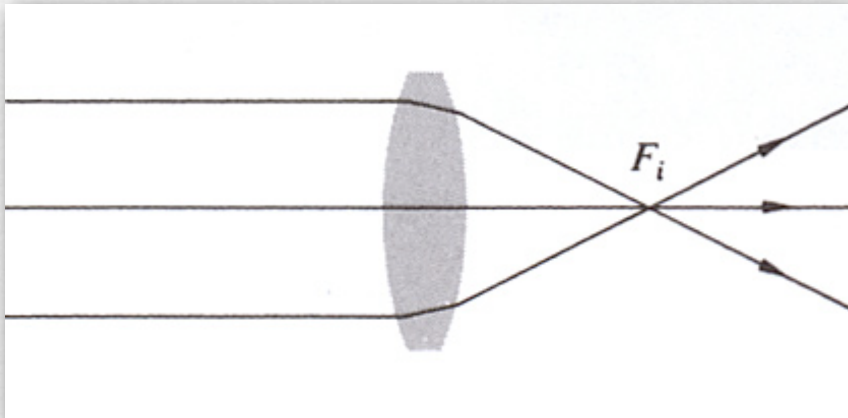


(Hecht)

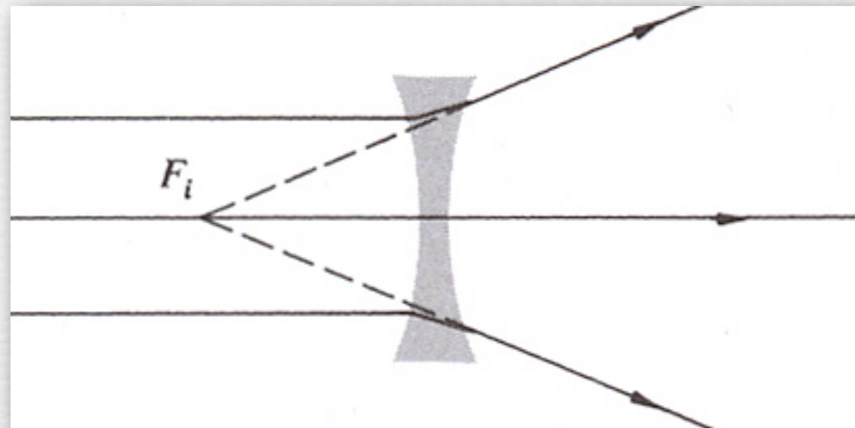
- in a thin lens, the *chief ray* traverses the lens (through its optical center) without changing direction
- in a thick lens, the intersections of this ray with the optical axis are called the *nodal points*
- for a lens in air, these coincide with the *principal points*
- the first nodal point is the *center of perspective*

Convex versus concave lenses

(Hecht)



rays from a convex lens converge

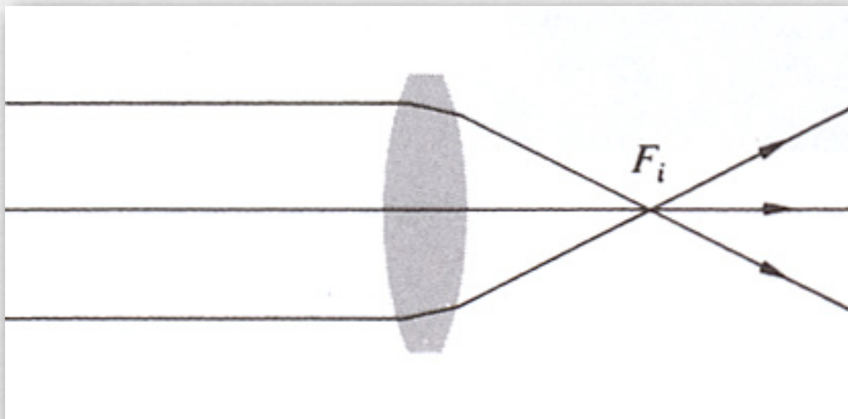


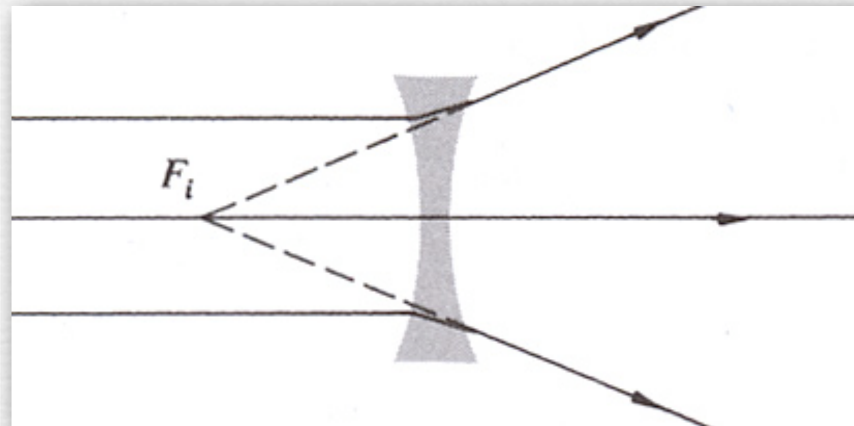
rays from a concave lens diverge

- positive focal length *f* means parallel rays from the left converge to a point on the right
- negative focal length *f* means parallel rays from the left converge to a point on the left (dashed lines above)

Convex versus concave lenses

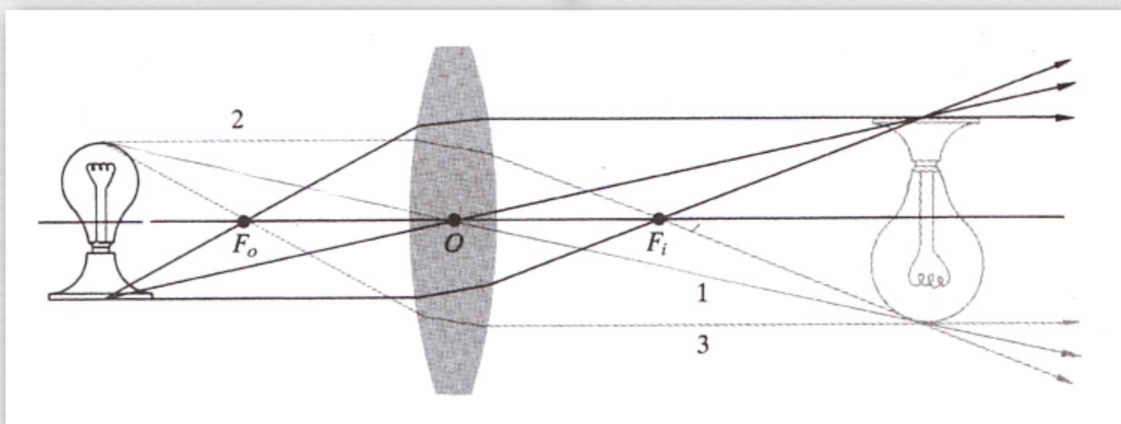
(Hecht)

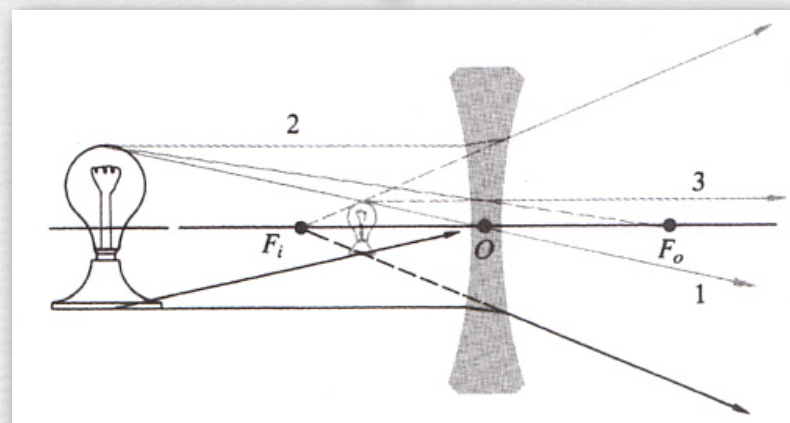




rays from a convex lens converge

rays from a concave lens diverge

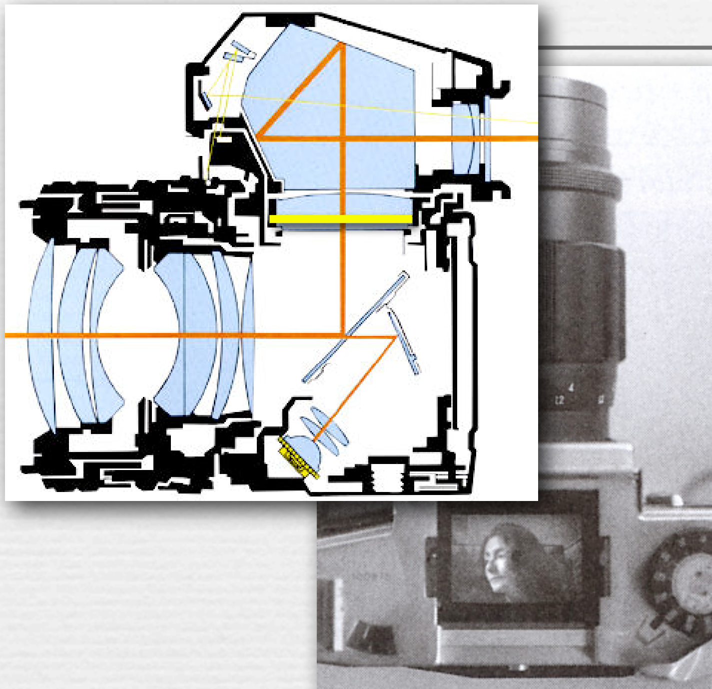


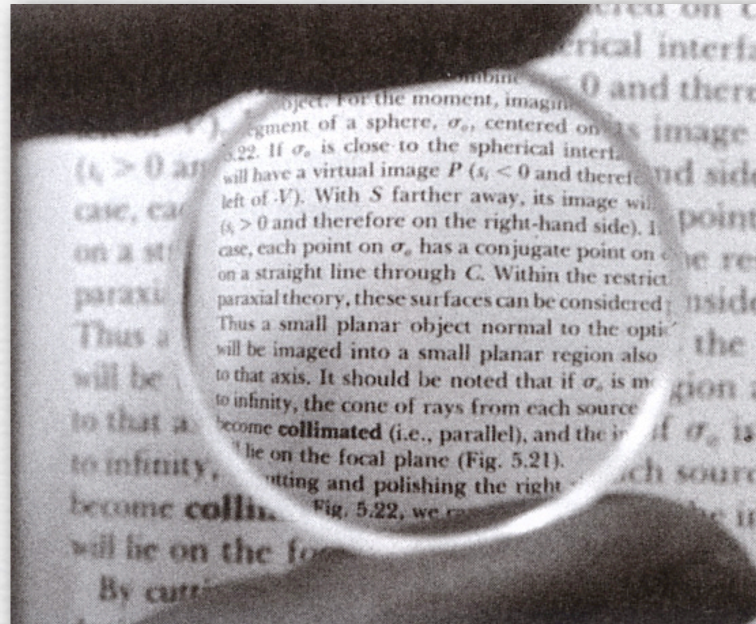


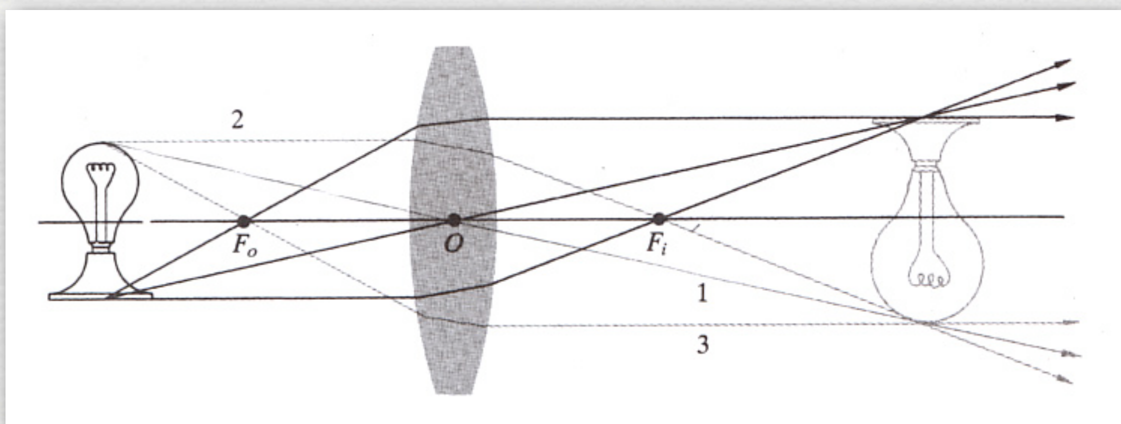
...producing a real image

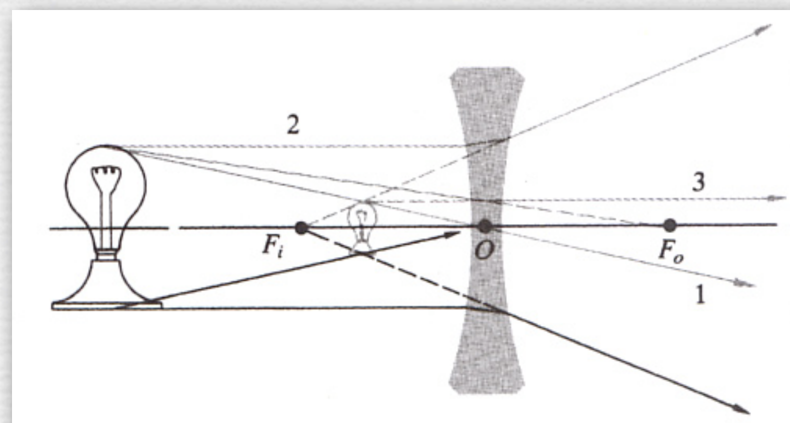
...producing a virtual image

Convex versus concave lenses





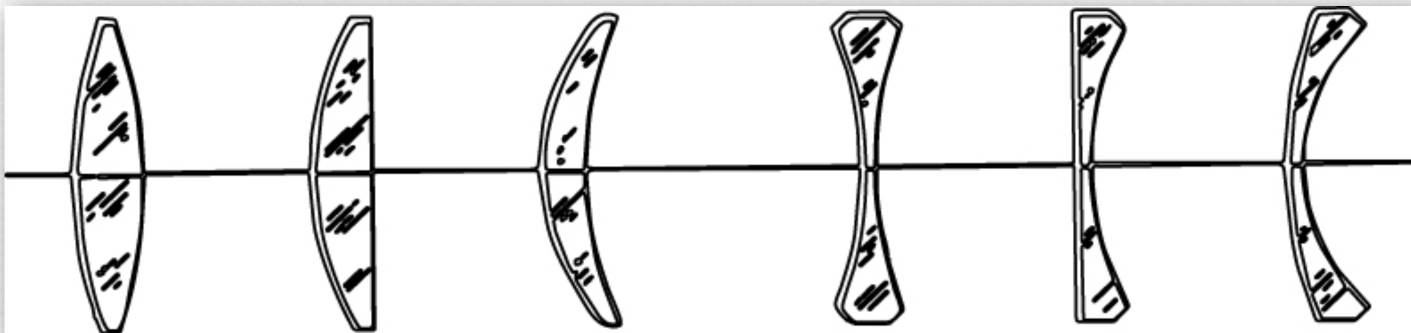




...producing a real image

...producing a virtual image

A menagerie of lenses



(Smith)

biconvex | plano-convex | pos. meniscus | biconcave | plano concave | neg. meniscus

Q. Given the lensmaker's formula, how do you tell if parallel rays entering a lens will converge or diverge?

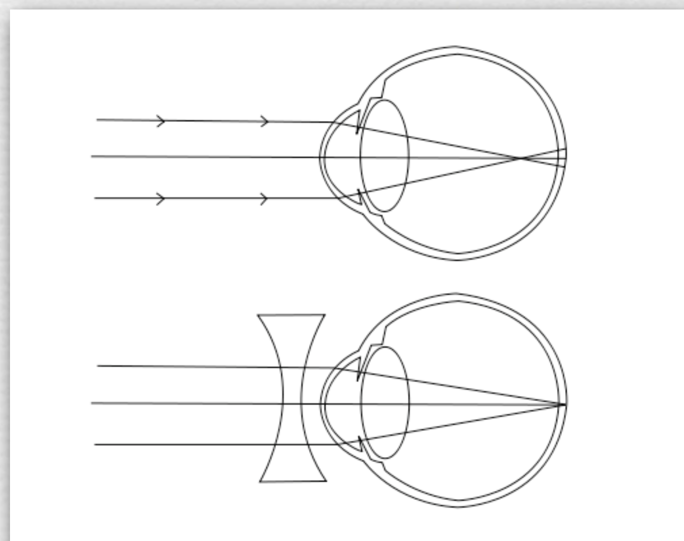
$$\frac{1}{s_o} + \frac{1}{s_i} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$

The power of a lens

$$P = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$

- units are meters^{-1}
- a.k.a. diopters

(wikipedia)

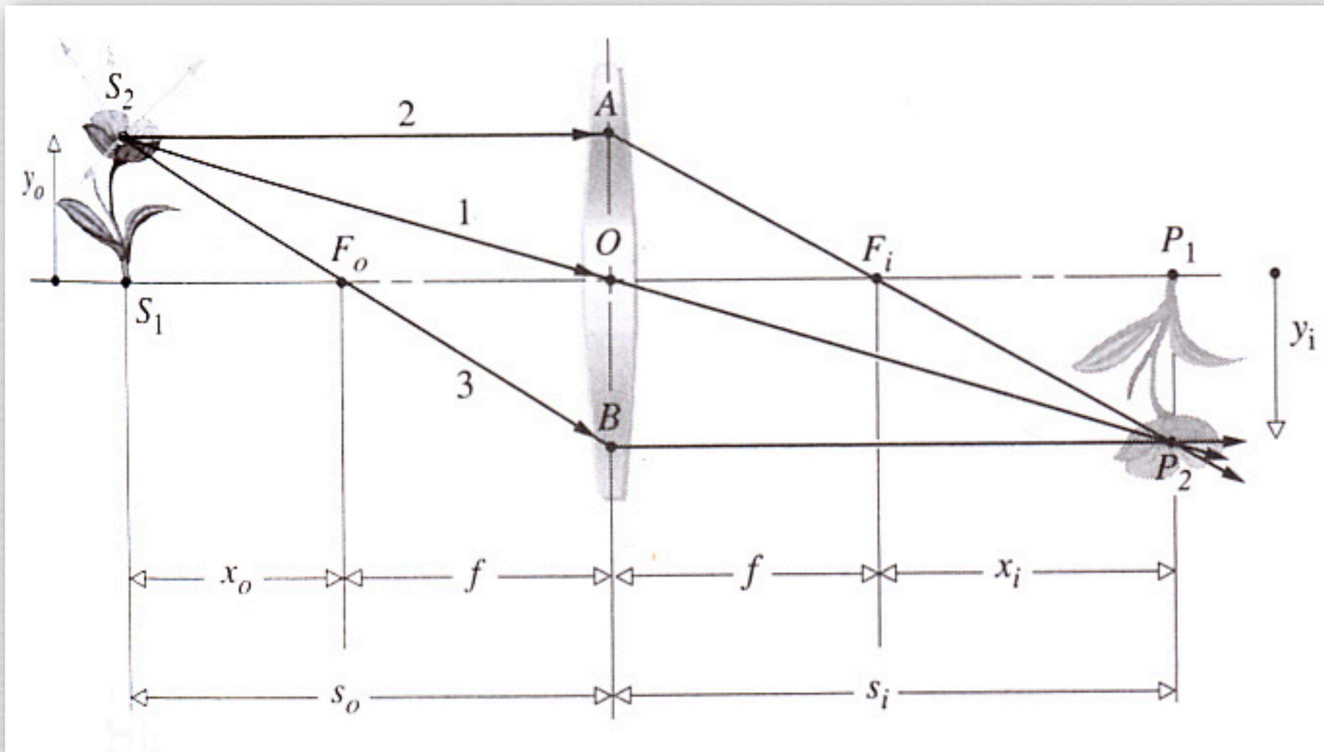


- my eyeglasses have the prescription
  - right eye: -0.75 diopters
  - left eye: -1.00 diopters

Q. What's wrong with me?
A. Myopia

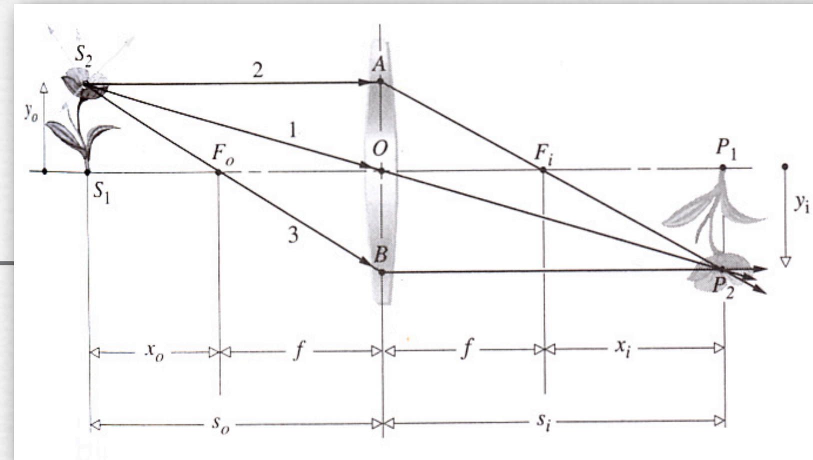
Newtonian form of the lens equation

(Hecht)



$$\frac{|y_i|}{y_o} = \frac{s_i}{s_o} \quad \text{and} \quad \frac{|y_i|}{y_o} = \frac{s_i - f}{f} \quad \text{imply} \quad x_o\, x_i = f^2$$ (Hecht, eqn 5.23)

Magnification



$$x_o \, x_i = f^2$$

- lateral magnification

$$M_T \triangleq \frac{y_i}{y_o} = -\frac{s_i}{s_o} \qquad \text{(Hecht, eqn 5.24)}$$

  - negative for a convex lens, because it inverts the image

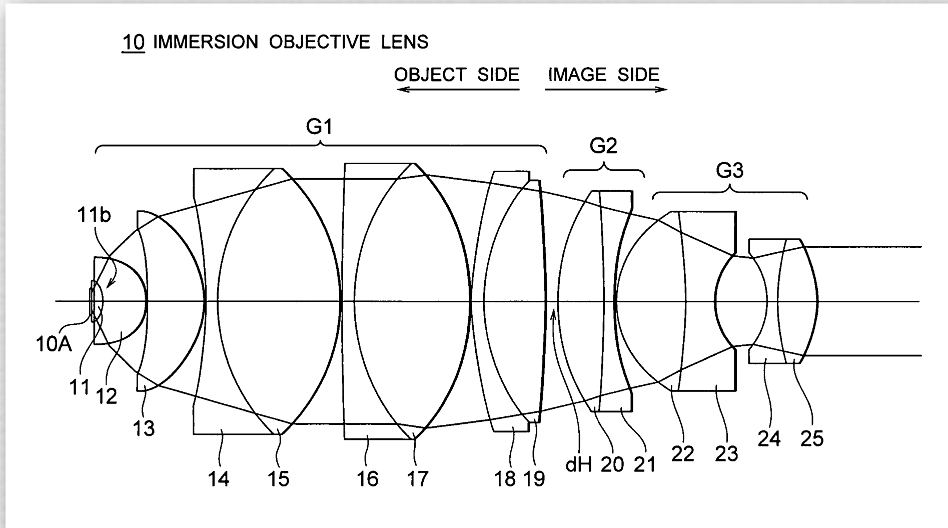
- longitudinal magnification

$$M_L = \frac{dx_i}{dx_o} = \frac{-f^2}{x_o^2} = -M_T^2 \qquad \text{(Hecht, eqn 5.25)}$$

  - equal to the (negative) square of lateral magnification

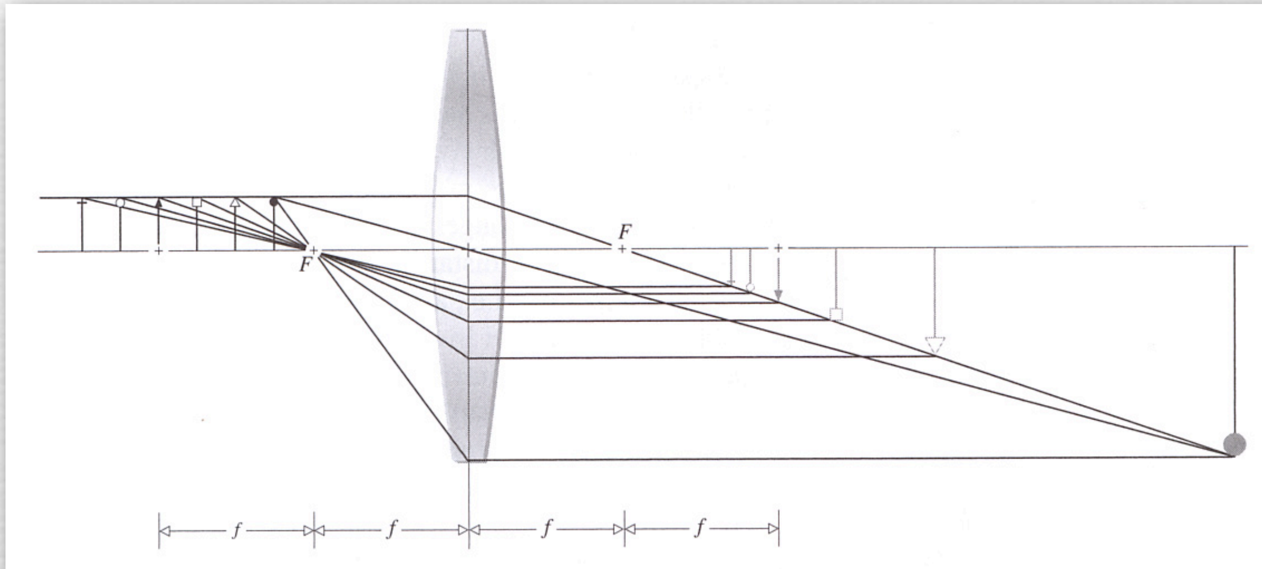
In my lecture I showed this as dxo/dxi. That is flipped. The correct formula is shown above.

Example: 100× microscope objective



- 1 micron laterally on specimen becomes 100 microns at a microscope's camera sensor (about 15 pixels)
- 1 micron axially on specimen becomes 10,000 microns (10mm) at the sensor - well beyond the depth of focus
- depth of field of a 100× objective is less than 1 micron

Lenses perform a 3D perspective transform



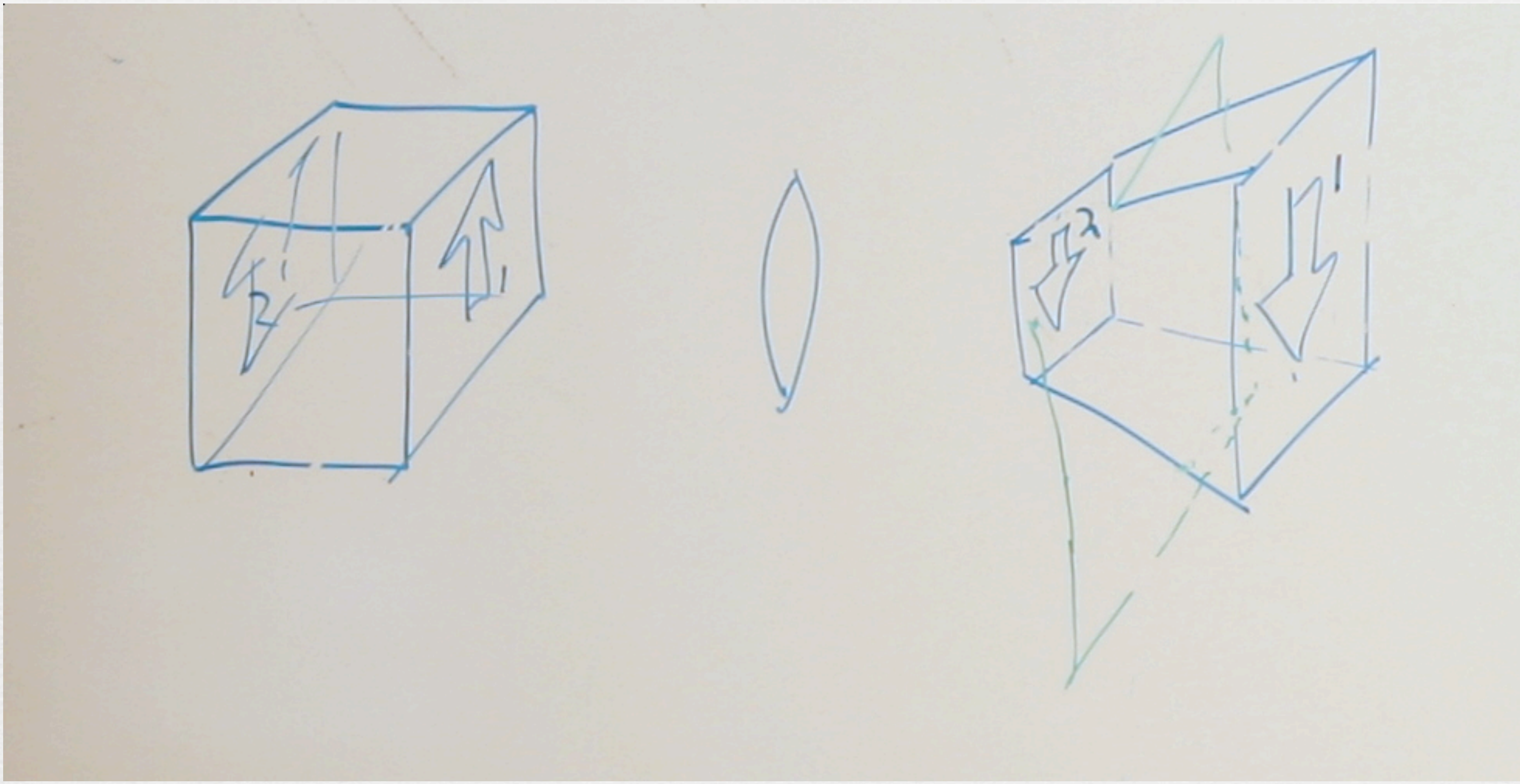
(Hecht)

(FLASH DEMO)

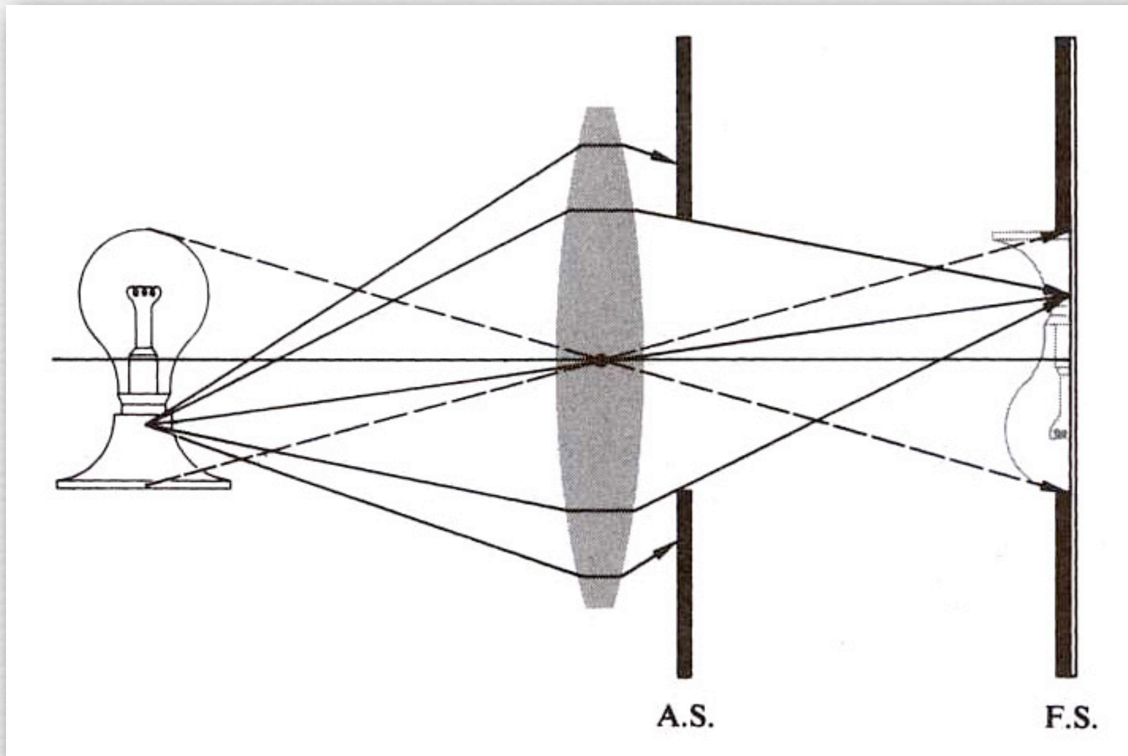
http://graphics.stanford.edu/courses/cs178-09/applets/thinlens.swf

- lenses transform a 3D object to a 3D image;
  the sensor extracts a 2D slice from that image
- as an object moves linearly toward the camera,
  its image moves non-proportionately
- as you move a sensor (or lens) linearly,
  the in-focus object plane moves non-proportionately
- as you refocus a camera, the image changes size !

Lenses perform a 3D perspective transform



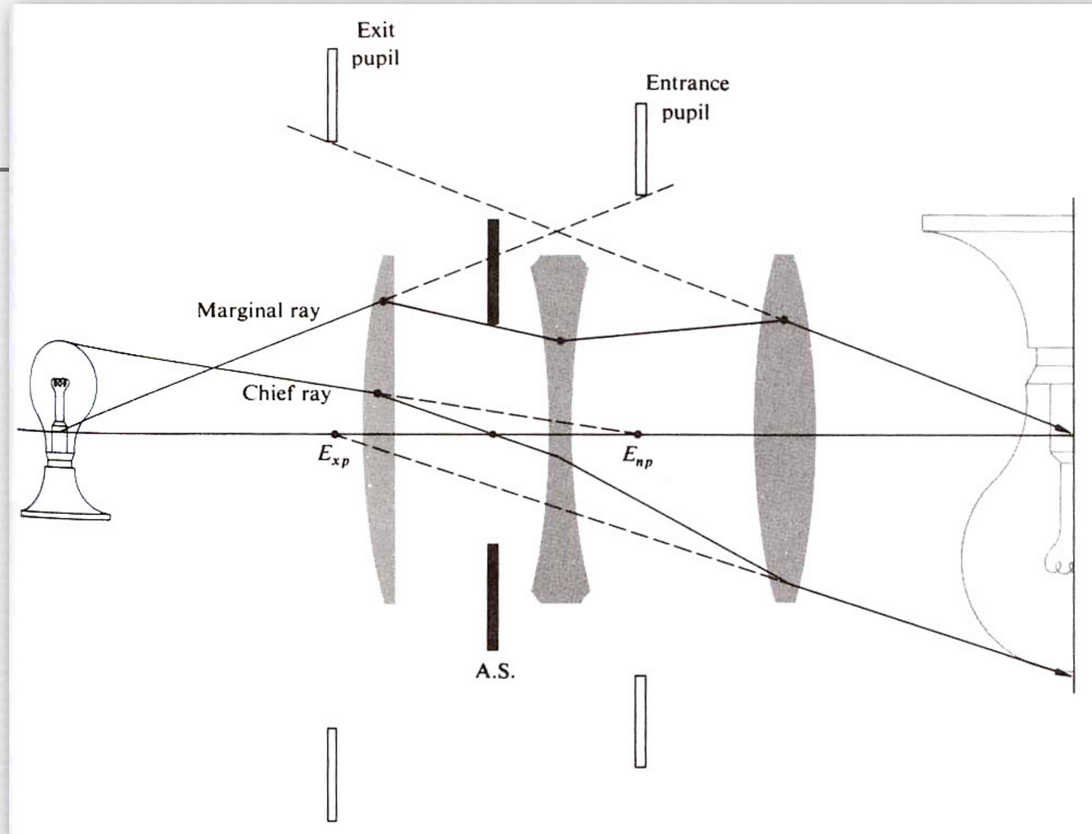
Stops



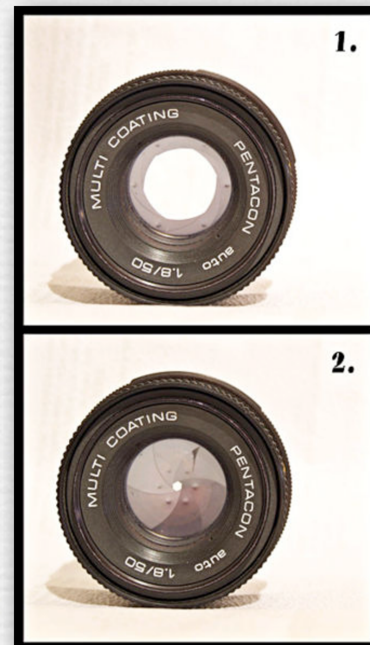
(Hecht)

- in photographic lenses, the *aperture stop* (A.S.) is typically in the middle of the lens system
- in a digital camera, the *field stop* (F.S.) is the edge of the sensor; no physical stop is needed

Pupils



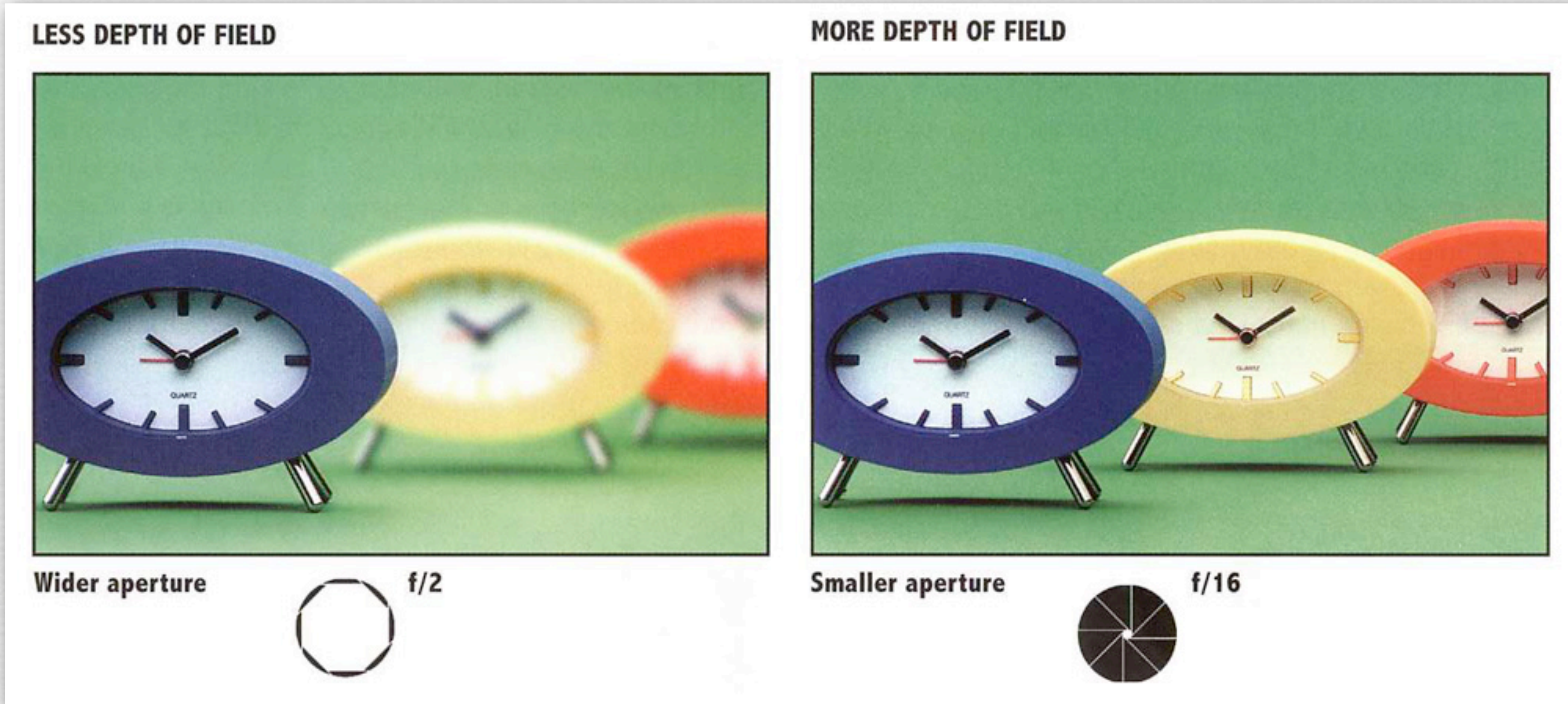
(Hecht)



(wikipedia)

- the *entrance pupil* is the image of the aperture stop as seen from an axial point on the object
- the *exit pupil* is the image of the aperture stop as seen from an axial point on the image plane
- the center of the entrance pupil is the center of perspective
- you can find this point by following two lines of sight

Depth of field

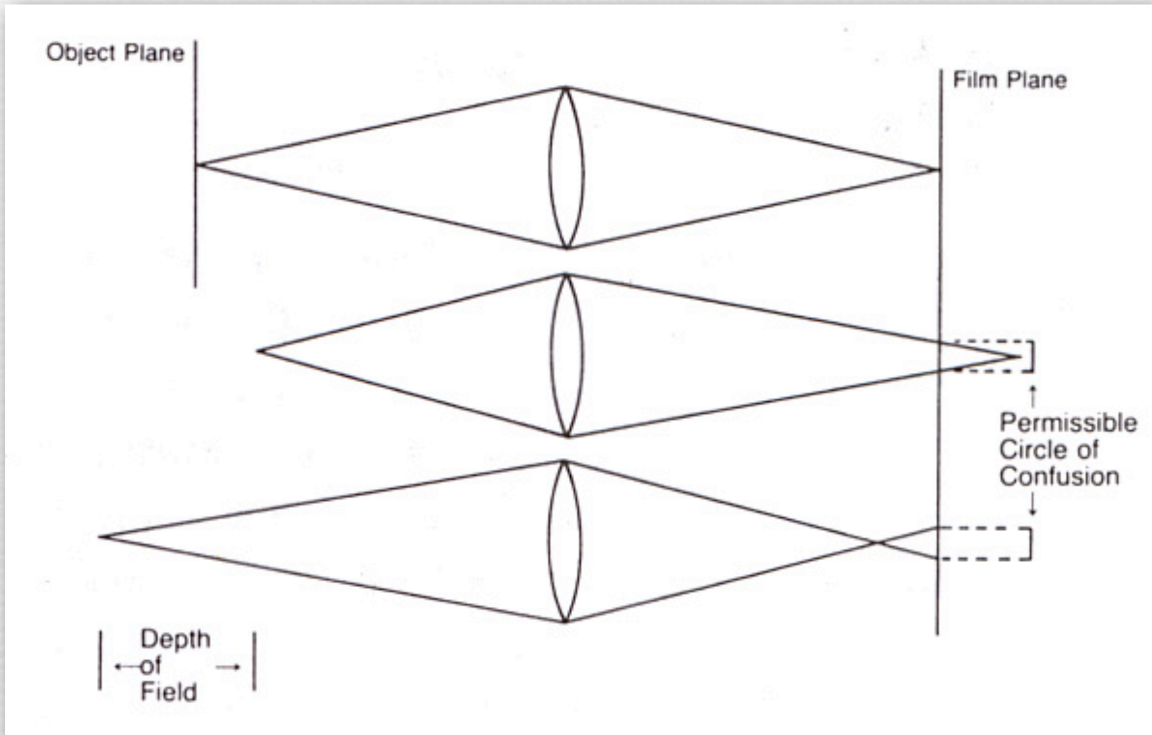


(London)

$$N = \frac{f}{A}$$

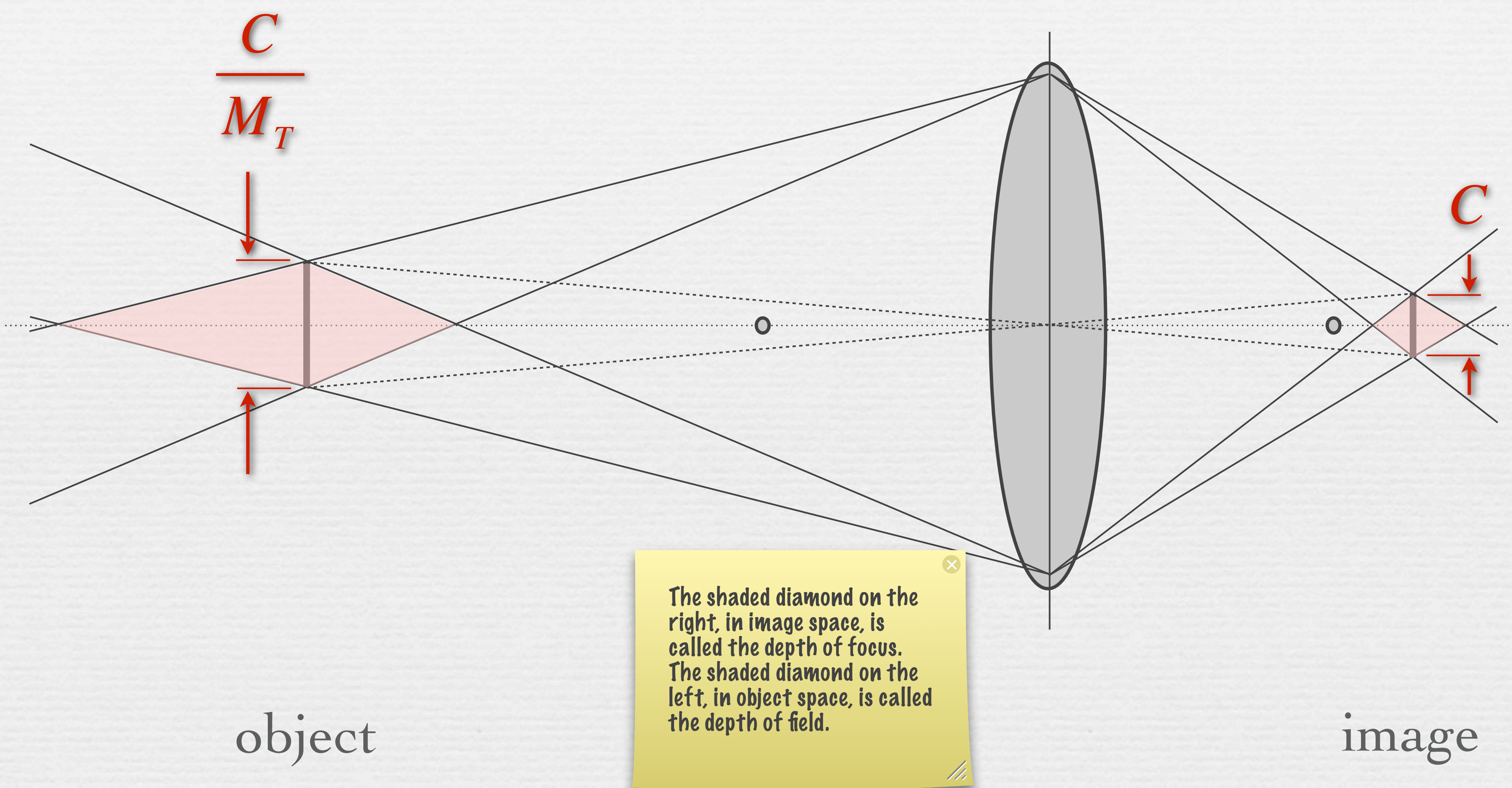
- lower N means a wider aperture and less depth of field

Circle of confusion (C)



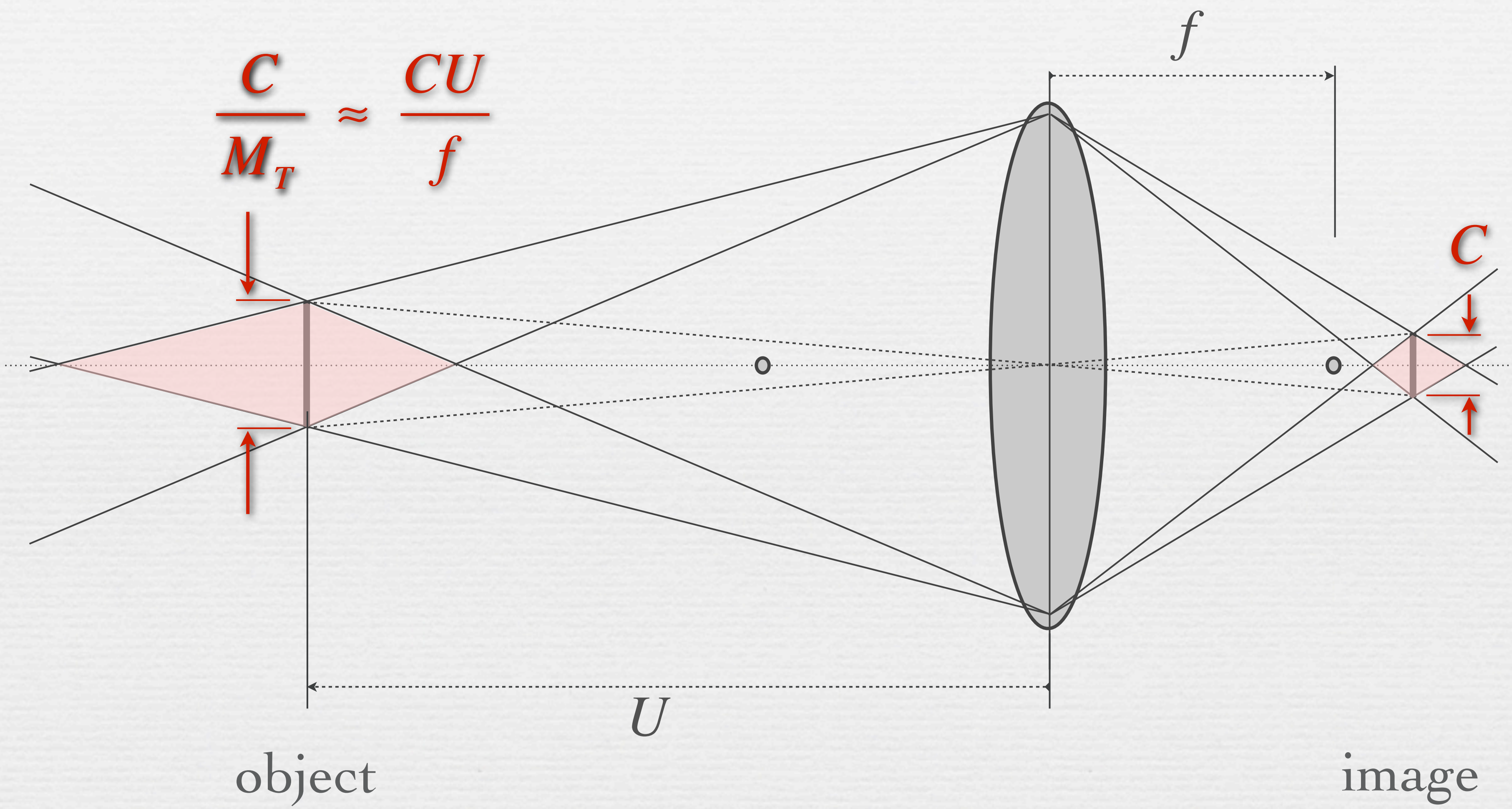
- C depends on sensing medium, reproduction medium, viewing distance, human vision,...
    - for print from 35mm film, 0.02mm is typical
    - for high-end SLR, 6μ is typical (1 pixel)
    - less if downsizing for web, or lens is poor

Depth of field formula



- DoF is asymmetrical around the in-focus object plane
- conjugate in object space is typically bigger than C

Depth of field formula



- DoF is asymmetrical around the in-focus object plane
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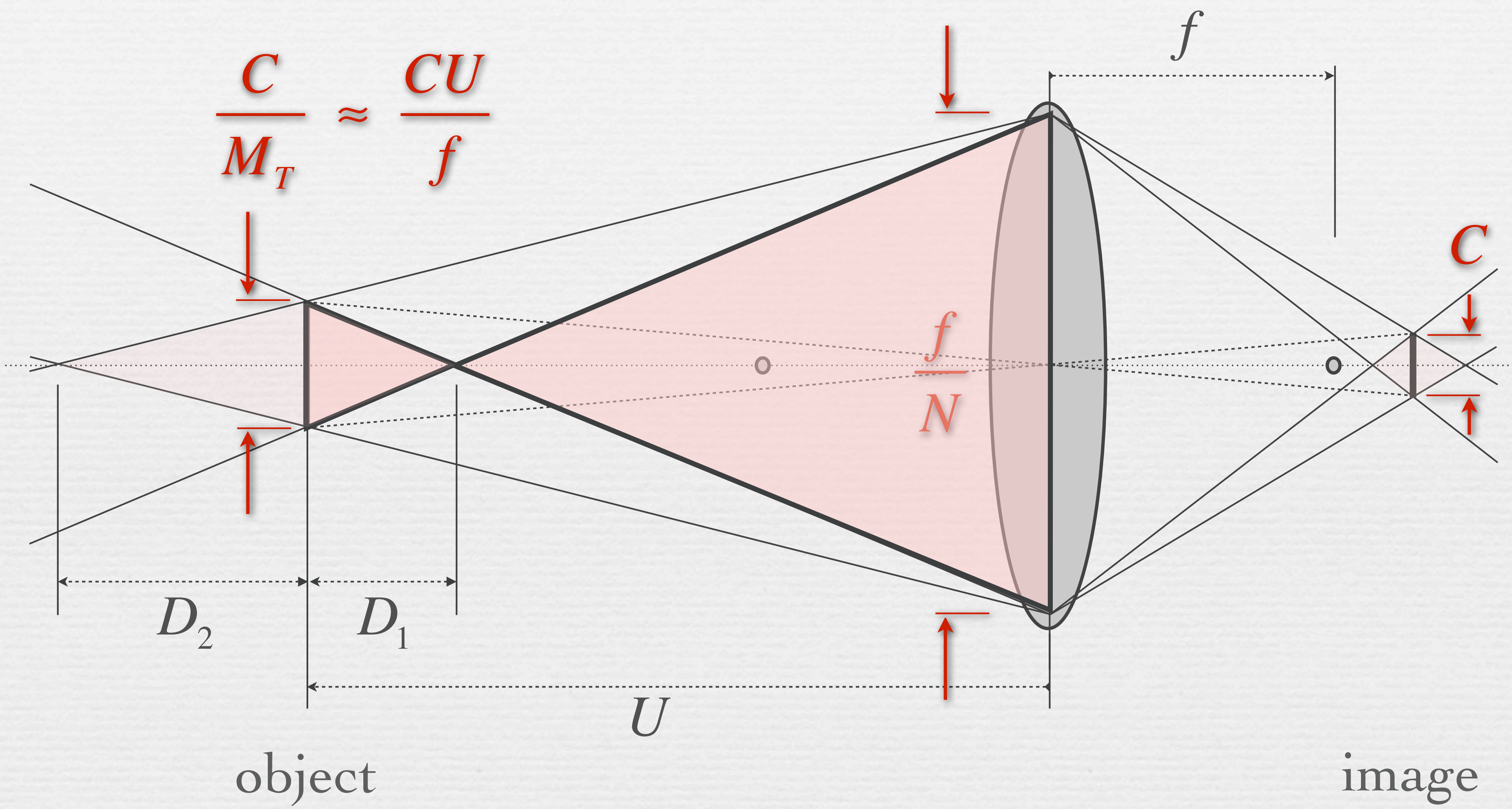
Depth of field formula



$$\frac{D_1 f}{CU} = \frac{U - D_1}{f / N} \quad \ldots.. \quad D_1 = \frac{NCU^2}{f^2 + NCU} \qquad D_2 = \frac{NCU^2}{f^2 - NCU}$$

Depth of field formula

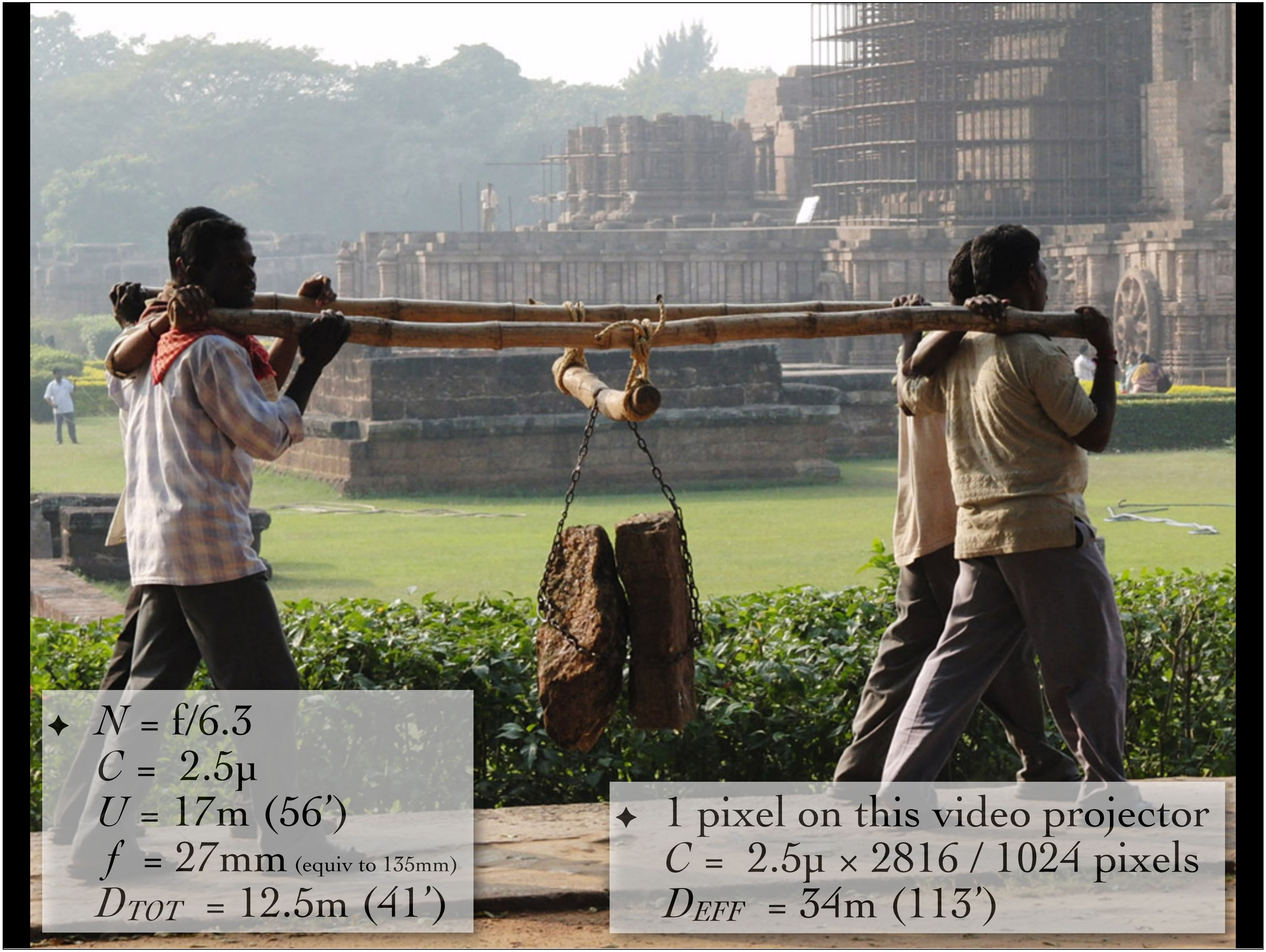
$$D_{TOT} = D_1 + D_2 = \frac{2NCU^2 f^2}{f^4 - N^2C^2U^2}$$

- $N^2C^2D^2$ can be ignored when conjugate of circle of confusion is small relative to the aperture

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

- where
  - N is F-number of lens
  - C is circle of confusion (on image)
  - U is distance to in-focus plane (in object space)
  - f is focal length of lens


$D_{TOT} \approx \dfrac{2NCU^2}{f^2}$
N = f/4.1
C = 2.5µ
U = 5.9m (19’)
f = 73mm (equiv to 362mm)
D_{TOT} = 132mm
1 pixel on this video projector
C = 2.5µ × 2816 / 1024 pixels
D_{EFF} = 363mm



✦ N = f/6.3
C = 2.5µ
U = 17m (56')
f = 27mm (equiv to 135mm)
DTOT = 12.5m (41')
✦ 1 pixel on this video projector
C = 2.5µ × 2816 / 1024 pixels
DEFF = 34m (113')


✦ N = f/5.6
C = 6.4μ
U = 0.7m
f = 105mm
D_{TOT} = 1.6mm
✦ 1 pixel on this video projector
C = 6.4μ × 5616 / 1024 pixels
D_{EFF} = 8.7mm

Canon MP-E
65mm 5:1 macro

These numbers were replaced on 6/4/09, after a student pointed out that they didn't work out. I'm still not confident in them.

- N = f/2.8
  C = 6.4µ
  U = 31mm
  f = 65mm

(use $N' = (1+M_T)N$ at short conjugates (M_T=5 here)) = f/16

D_{TOT} = 0.048mm! (48µ)

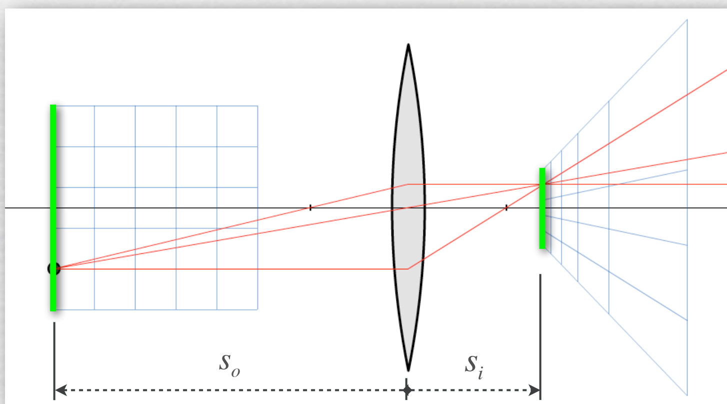
(Mikhail Shlemov)

Sidelight: macro lenses

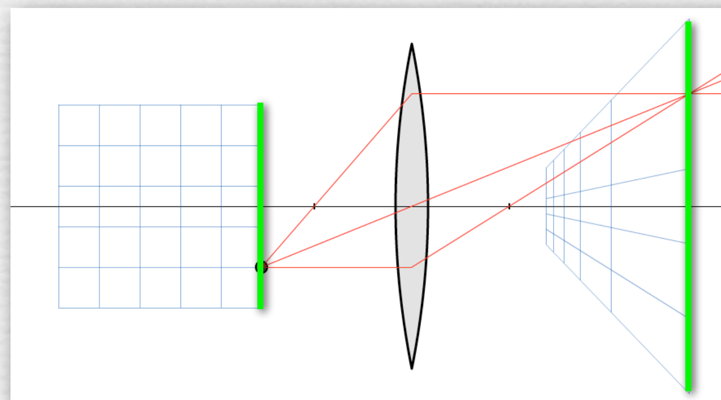
This and next slide added 4/14/09

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Q. How can the Casio EX-F1 at 73mm and the Canon MP-E 65mm macro, which have similar f's, have such different focusing distances?



normal


macro

- A. Because they are built to allow different s_i
  - this changes s_o, which changes magnification $M_T \triangleq -s_i / s_o$
  - macro lenses are well corrected for aberrations at short s_o

Extension tube: converts a normal lens to a macro lens

- toilet paper tube, black construction paper, masking tape
- camera hack by Katie Dektar (CS 178, 2009)

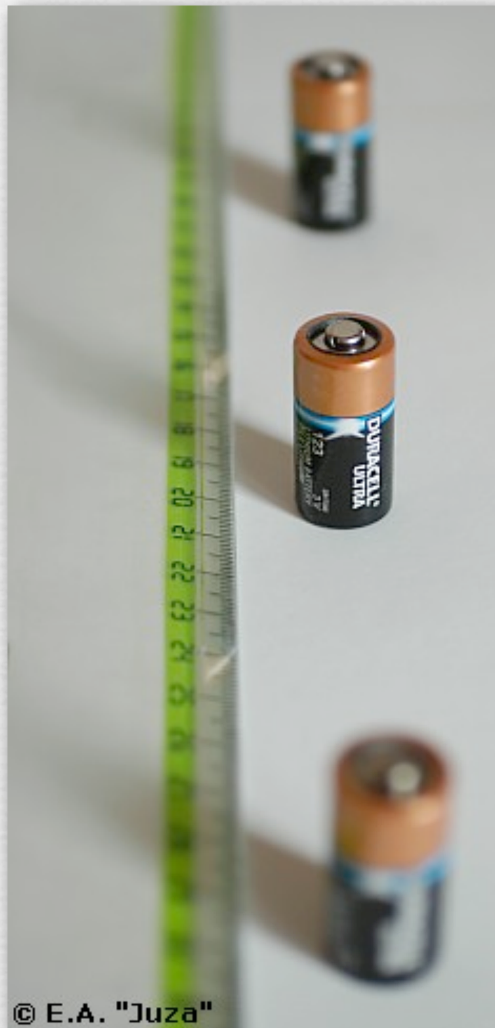
DoF is linear with aperture

(juzaphoto.com)

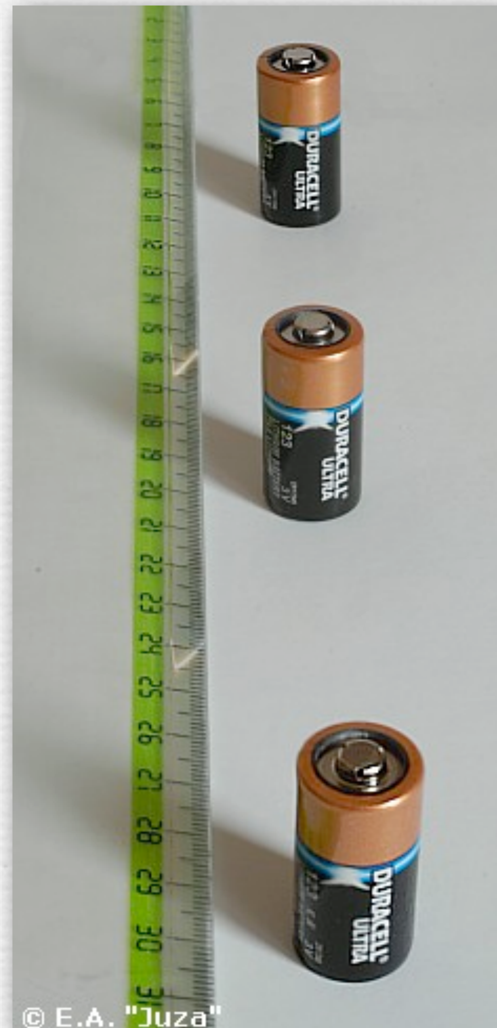
$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178-09/applets/dof.swf



f/2.8



f/32

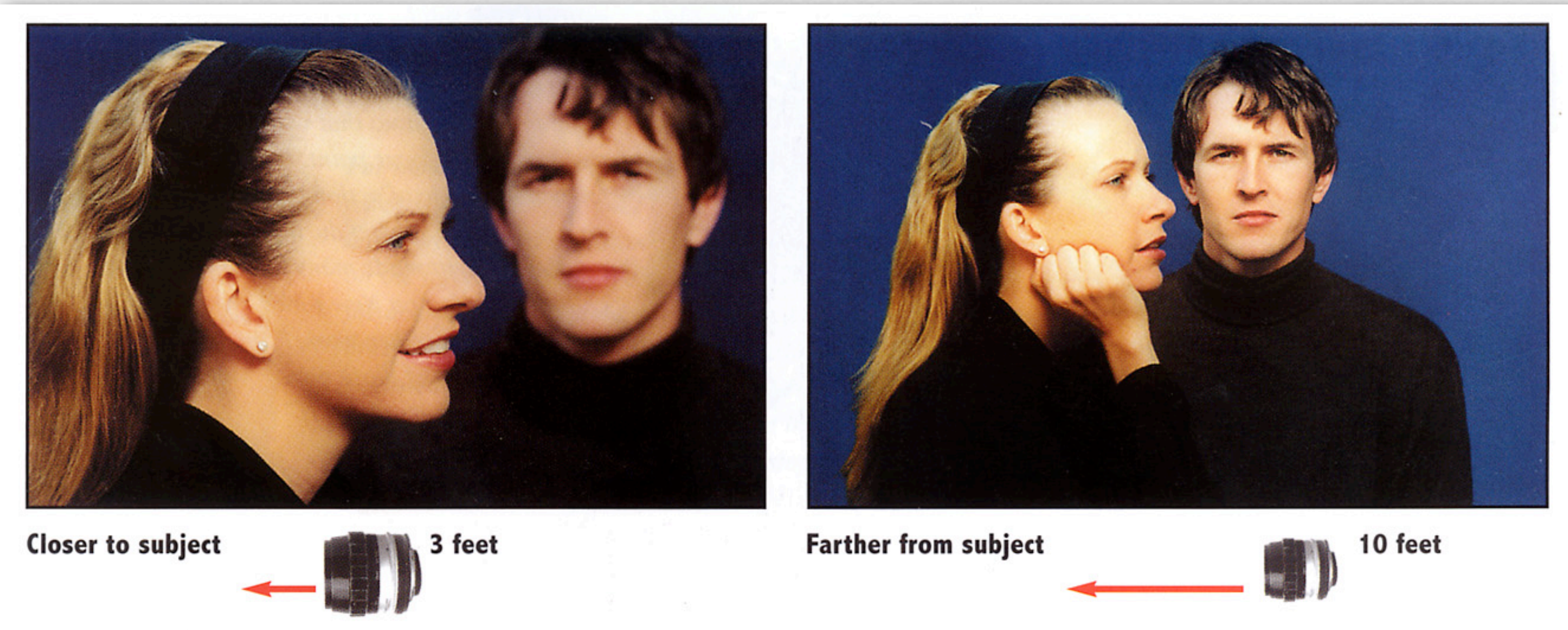
DoF is quadratic with focusing distance

(we already know this, because M_T scales with U, and M_L goes as the square of M_T)

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178-09/applets/dof.swf



(London)

Hyperfocal distance

- the back depth of field

$$D_2 = \frac{NCU^2}{f^2 - NCU}$$

- becomes infinite if

$$U \geq \frac{f^2}{NC} \triangleq H$$

- In that case, the front depth of field becomes

$$D_1 = \frac{H}{2}$$

- so if I had focused at 32m, everything from 16m to infinity would be in focus on an HD projector, including the men

I incorrectly stated in class that when you plug U = f^2/NC into the formula for D2 you get 0/0. You get H/0, which is infinity. And when you plug it into the formula for D1 you get H/2, as shown on the slide.

Last slide covered on Tuesday. The remaining slides were covered on Thursday.

- N = f/6.3
  C = 2.5μ × 2816 / 1024 pixels
  U = 17m (56')
  f = 27mm (equiv to 135mm)
  D_{TOT} = 34m on video projector
  H = 32m (106')

This calculation earlier (and incorrectly) assumed an HD projector. The numbers actually work out for a normal projector, as shown. Note added 5/4/09.

(FLASH

http://graphics.stanford.edu/courses/cs178-09/applets/dof.swf

DoF is inverse quadratic with focal length

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

On a point and shoot camera, depth of field is always large. Why? After all, the N's are comparable. But the focal lengths f's are typically smaller, reflecting the physically smaller camera. Thus, depth of field is much larger.

(FLASH DEMO)

(London)

DoF and the dolly-zoom

- if we zoom in (change f)
  and stand further back (change U) by the same factor

$$D_{TOT} \approx \frac{2NCU^2}{f^2}$$

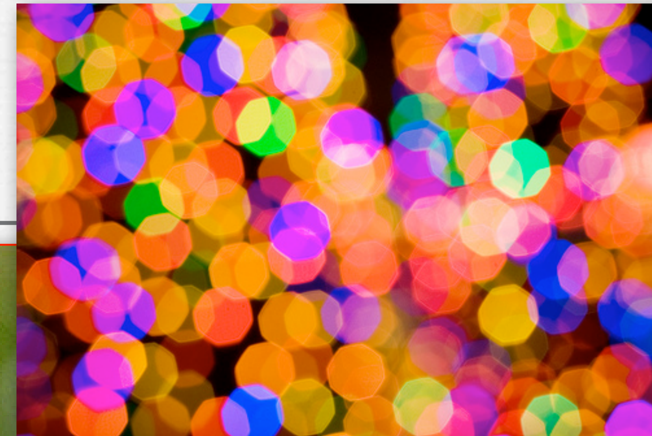
- the depth of field stays the same!
  - useful for macro when you can't get close enough

50mm f/4.8

(juzaphoto.com)

200mm f/4.8,
moved back 4× from subject

Parting thoughts on DoF: the zen of *bokeh*



Canon 85mm
prime f/1.8 lens

Bokeh is pronounced bow'–keh, with the emphasis on the 1st syllable. It's from Japanese. I pronounced it incorrectly in class.

- the appearance of sharp out-of-focus features in a photograph with shallow depth of field
  - determined by the shape of the aperture
  - people get religious about it
  - but not every picture with shallow DoF has evident bokeh...

Natasha Gelfand (Canon 100mm f/2.8 prime macro lens)

Parting thoughts on DoF: seeing through occlusions

(Fredo Durand)

For slide credits, see end of second optics talk.

- depth of field is not a convolution of the image
  - i.e. not the same as blurring in Photoshop
  - DoF lets you eliminate occlusions, like a chain-link fence

Seeing through occlusions