

# Sampling and pixels

CS 178, Spring 2009

Begun on Tuesday, 4/14/09, finished on  
Thursday, 4/16. Additional slide (#24)  
added on 4/21.



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# Announcements (from whiteboard)

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Reading for this week:

- London, 4 - exposure & sensors
- Hamamatsu online articles - see course web page
- Möller, Real-Time Rendering 5.6.1 - sampling & reconstruction

Outline:

- optics redux
- sampling & pixels (Tue)
- photons & sensors (Thu)

BRING CAMERAS  
to SECTION !!



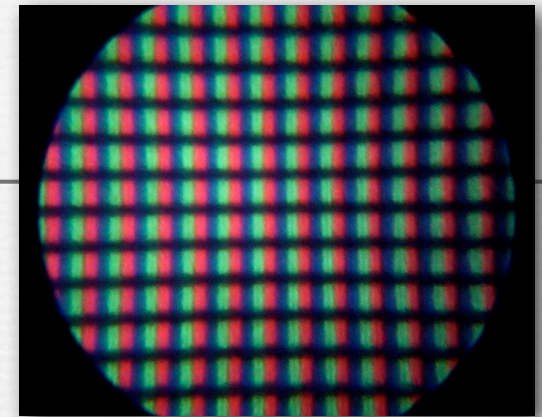
# Outline

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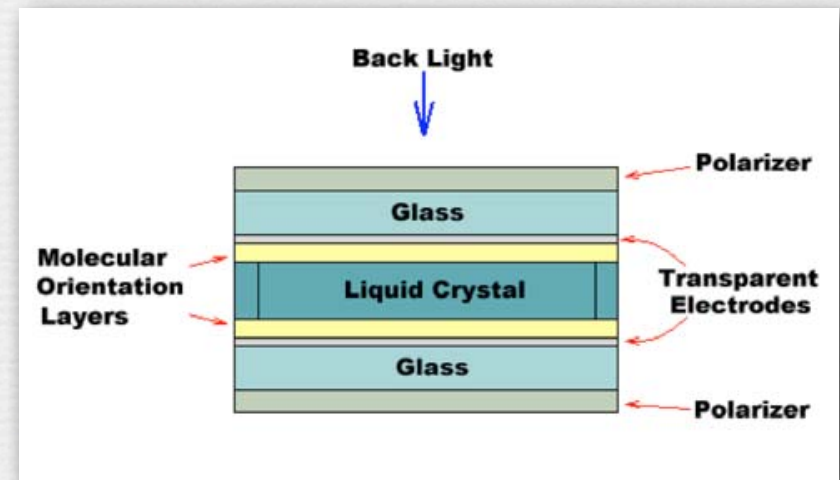
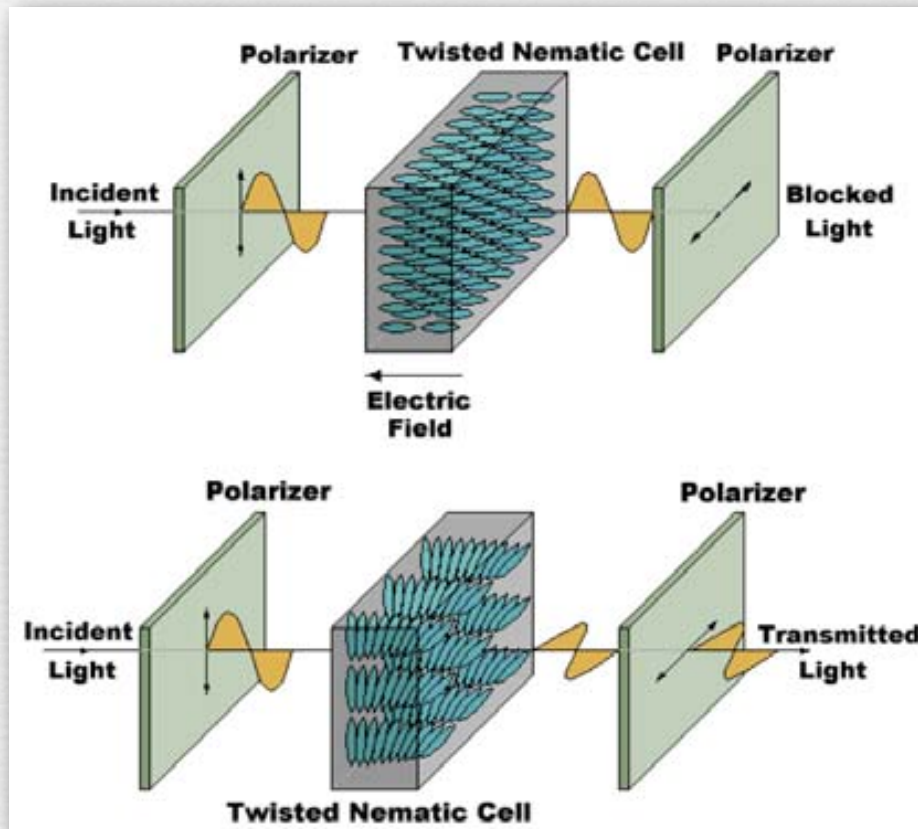
- ◆ image output technologies
  - displays
  - printing
- ◆ human perception
  - human spatial sensitivity
    - How many pixels?
  - human intensity discrimination
    - How many bits per pixel?
- ◆ sampling and reconstruction
  - including aliasing and antialiasing

I skipped human intensity discrimination. I'll cover it when I talk about exposure metering, noise, and ISO.

# Liquid crystal displays



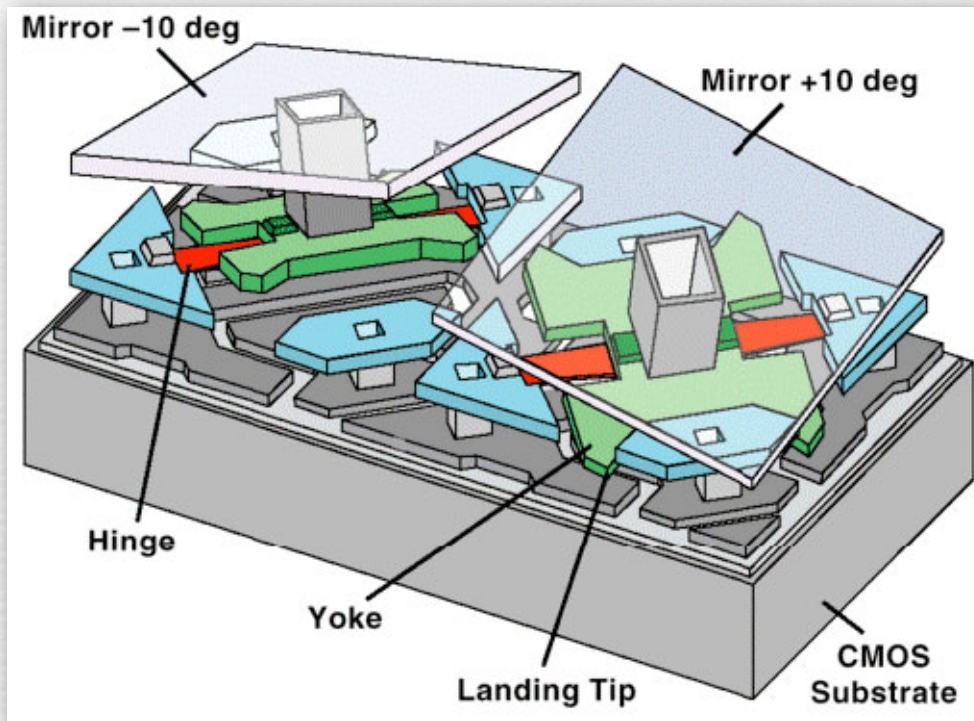
(Katie Dektar)



- ◆ cell on (top): LC aligned, light blocked → black
- ◆ cell off (bottom): LC twists light  $90^\circ$ , light transmitted → white
- ◆ color produced using a superimposed array of R,G,B filters



# Dynamic micro-mirror device (DMD)



## Other acronyms:

DLP = digital light processing

SLM = spatial light modulator

- ◆ each mirror is 7-16 $\mu$  on a side
  - created using MEMS technology
- ◆ mirrors vibrate at KHz rates
  - fraction of time spent facing optics determines gray level
- ◆ color produced using spinning color wheel or LEDs



# Organic LEDs (OLEDs)

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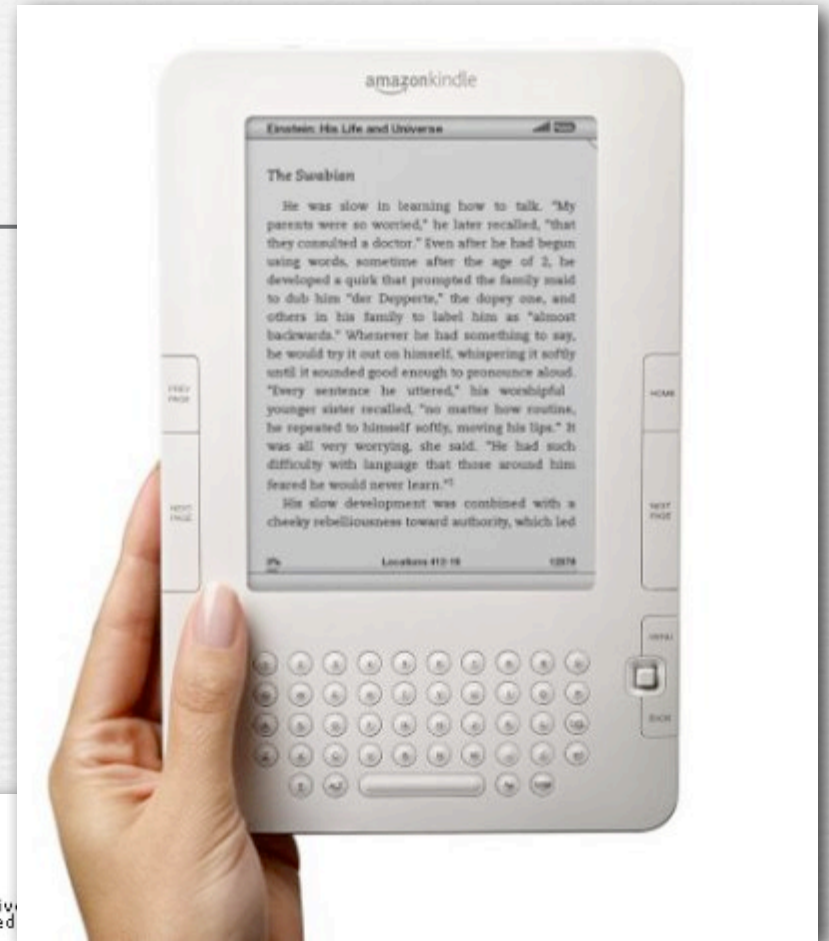
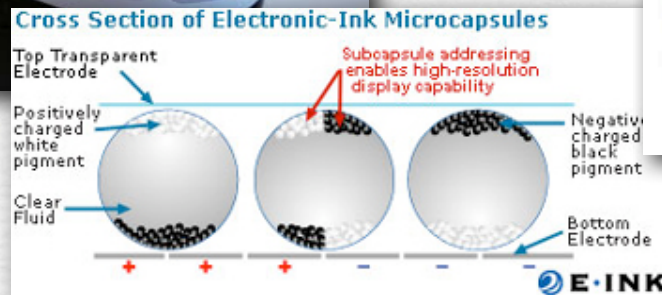


(<http://www.artlebedev.com/everything/optimus>)

- ◆ requires no backlight
- ◆ low power, thin, flexible (bendable)
- ◆ lousy color quality



# Electronic ink



Amazon's Kindle 2

- ◆ black (-) & white (+) particles in a clear fluid
  - position in fluid controlled by a transparent grid of electrodes
- ◆ flexible, requires no power except when changing images
- ◆ changing images is slow

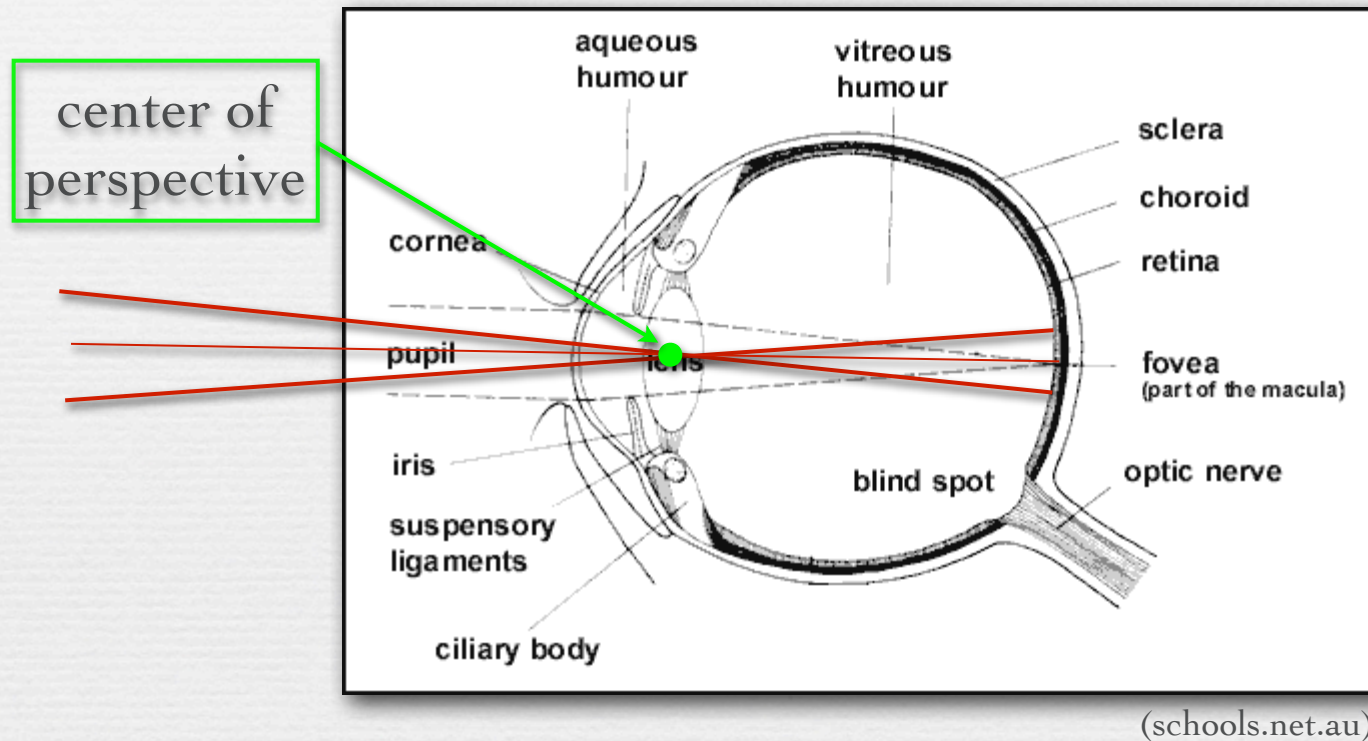
# Other image output technologies

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- ◆ display
  - CRT = cathode ray tube (obsolete)
  - LCOS = liquid crystal on silicon - like LCD but reflective
  - plasma panel - emissive
- ◆ printing
  - dye sublimation
  - inkjet
  - offset



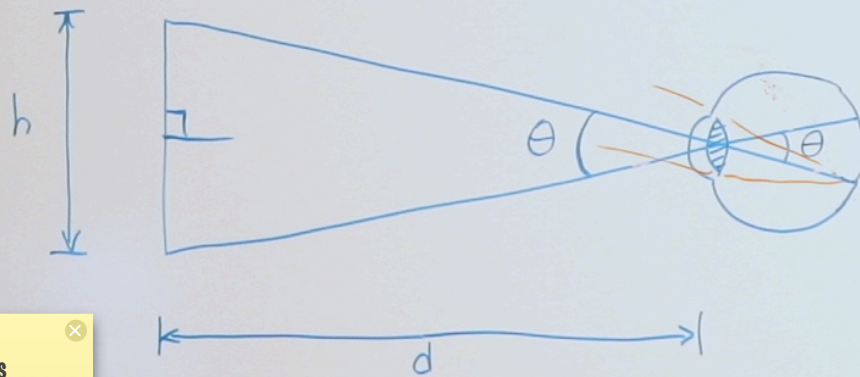
# Human field of view



- ◆ center of ray bundle is nominal line of sight
- ◆ a range of such lines constitute a field of view
  - the proper measure (from “natural perspective”) is retinal arc
  - maximum human FOV is  $180^\circ$ , with  $140^\circ$  of binocular overlap
  - human fovea (area of highest acuity) is  $1^\circ$

# Human retinal arc

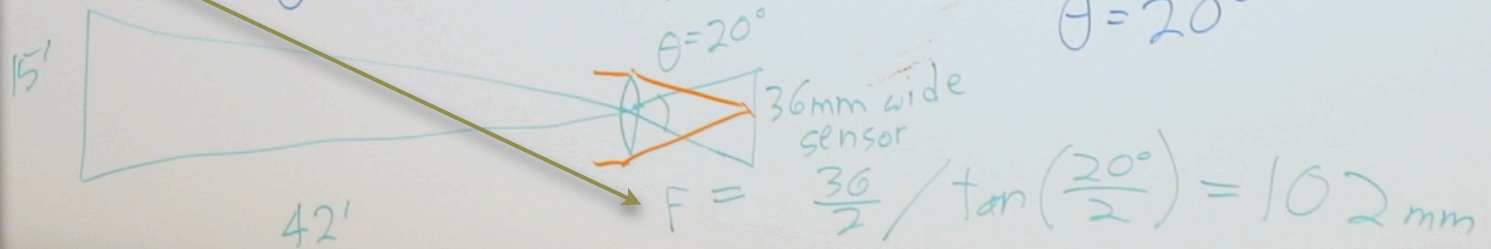
Human retinal arc:



Ex: laptop  
 15" diagonal 16:9  
 (8" height)  
 viewed from 18"  
 $\theta = 25^\circ$

whiteboards  
 15' wide  
 viewed from 42'  
 $\theta = 20^\circ$

$$\theta = 2 \arctan \left( \frac{h}{2d} \right)$$



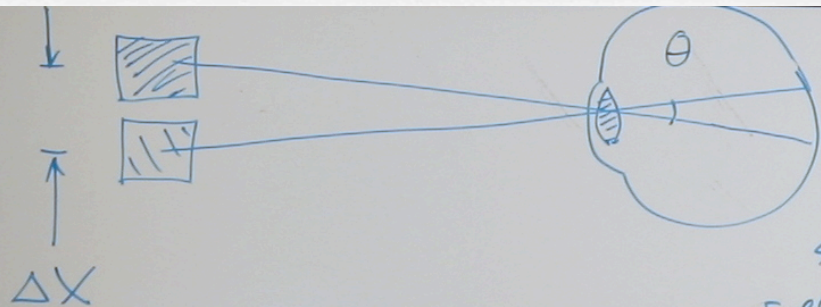
I made an error here. This calculation gives us  $s_i$ , not  $f$ . However, at long conjugates (meaning the object is far away),  $f$  is nearly equal to  $s_i$ .

To be sure, let's compute it correctly. Applying the formula  $1/s_o + 1/s_i = 1/f$ , where  $s_o$  is 42' and  $s_i$  is 102mm, we obtain  $f = 101$ mm.

Andrew reported (from the back of the room) 100mm, which is very close. Indeed, he was deliberately set a bit wide to make sure he didn't clip the whiteboards.



# Spatial frequency of a medium



Spatial Freq. of  
the medium

Laptop: 900 pixels high on 8" height  
 $\Delta x = 8" / 900 = 0.0089" / \text{pixel}$   
 $\frac{1}{\Delta x} = 112 \text{ dpi}$  (1 pixel = 1 dot)

Kindle: 800 pixels high  
4.8" high  
 $\frac{1}{\Delta x} = 167 \text{ dpi}$

R

# Spatial frequency on the retina

Spatial frequency in retinal arc

Laptop:  
 $\theta = 2 \arctan\left(\frac{p}{2d}\right) = 0.057^\circ$   
 $\frac{1}{\theta} = 17.6 \text{ cycles/degree}$   
 cutoff of human visual system  
 (foveal, high contrast)  
 = 50 cyc/deg

gigapixel.org  
 40k x 20k pix  
 72" x 36"  
 view at 48"  
 $\frac{1}{\theta} = 56 \text{ cyc/deg}$

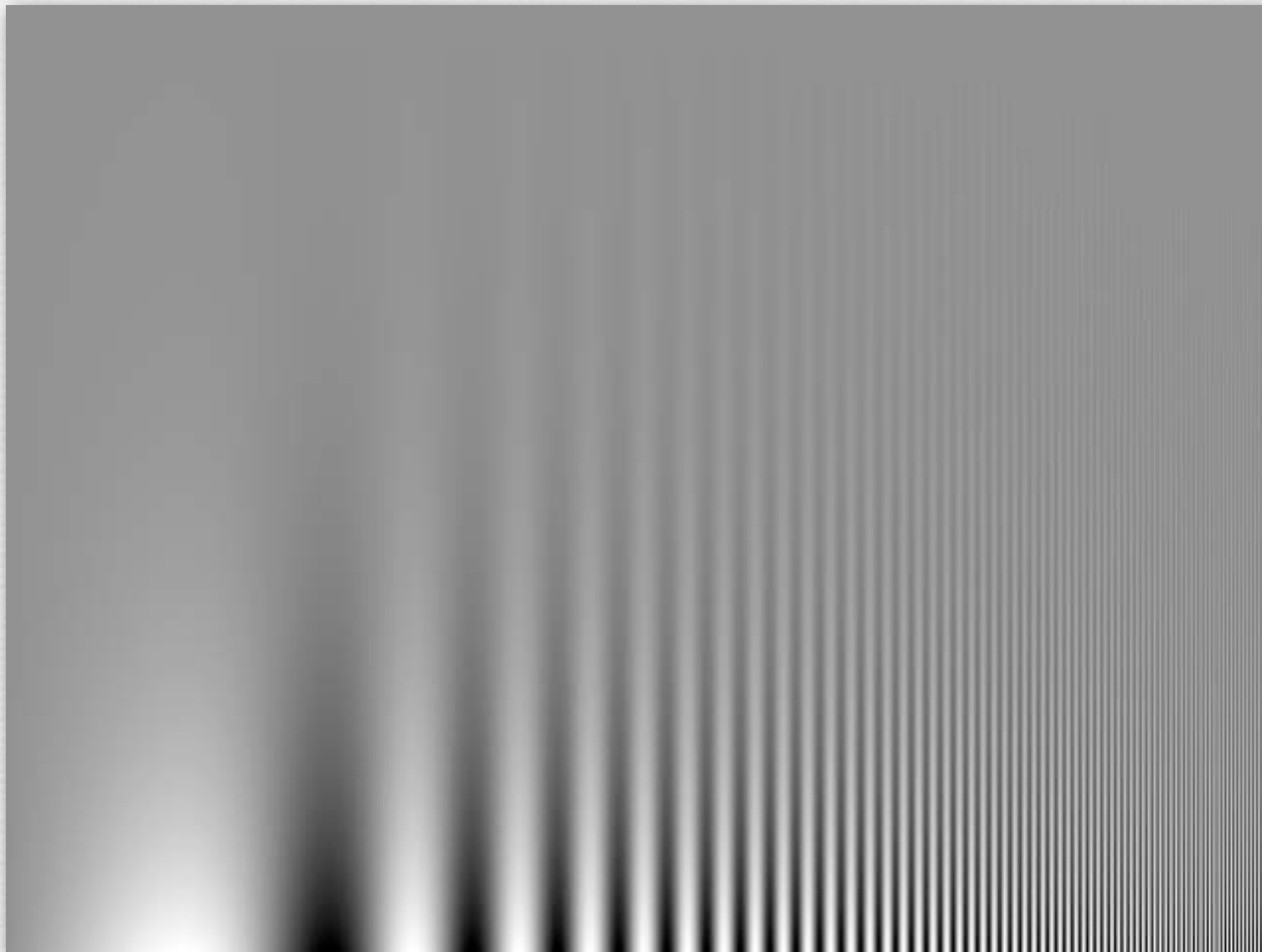
Epson intject  
 1400 dpi

I made an arithmetic error here. The answer should be 232 cycles per degrees (under our assumption that each cycle requires 2 pixels). This is much finer than the human cutoff, which is about 50 cycles per degree. To just barely match human acuity, you would want to blow up his photograph to 72" x (232/50) = 27 feet wide! (assuming you still view it from 4' away)



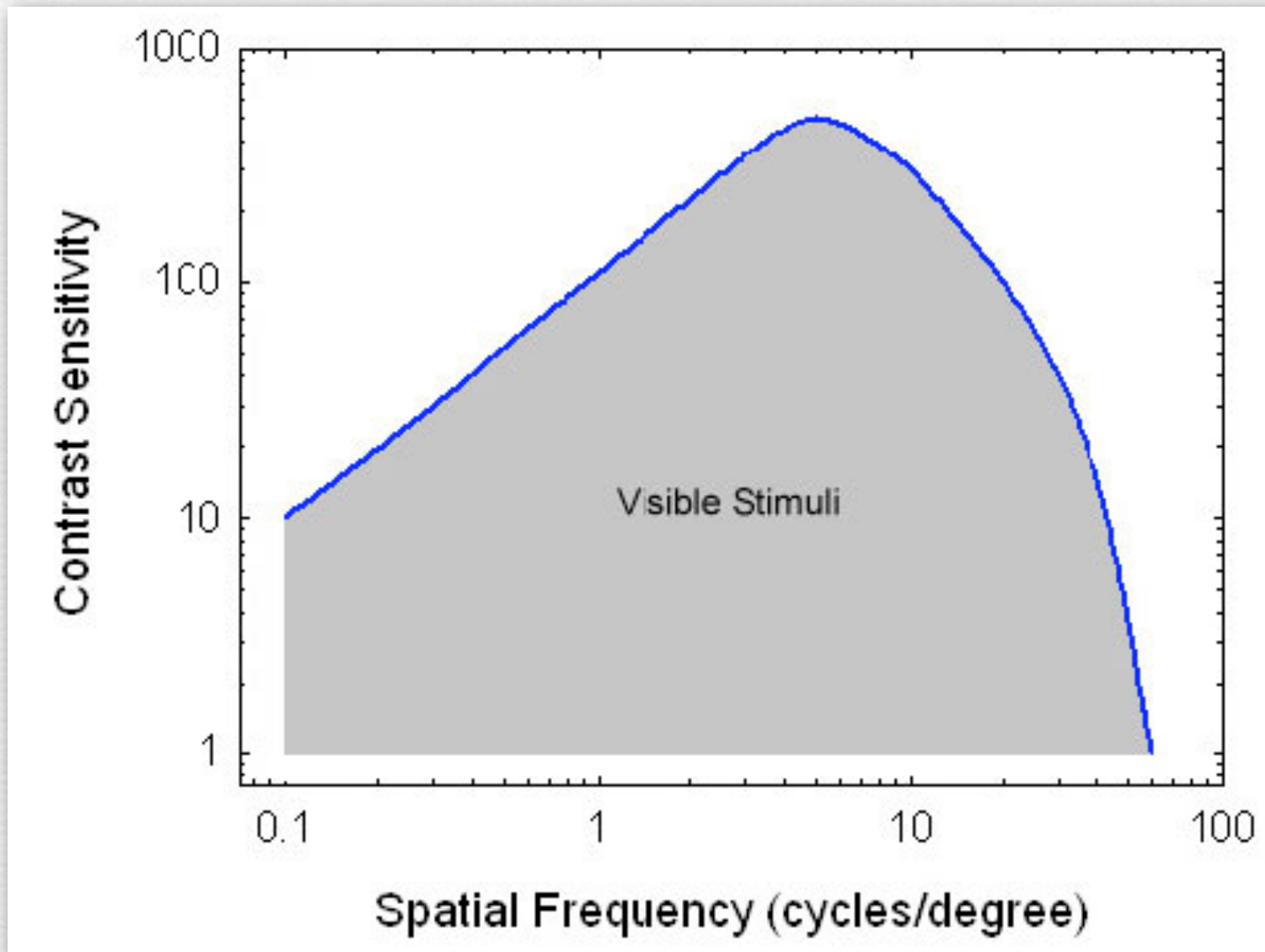
# Human spatial sensitivity

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# Human spatial sensitivity

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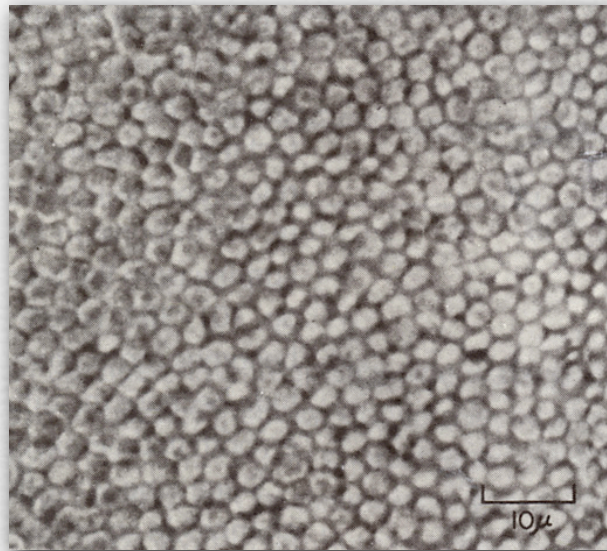
(Graham Flint)

## Balboa Park, San Diego

(original is 40K × 20K pixels, Gates Hall print is 72" × 36")

# The human retina

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(Cornsweet)

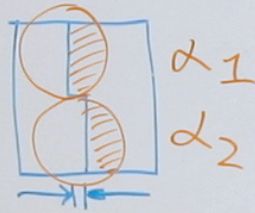
- ◆ retina contains 100M rods and 5M cones (3 types: L,M,S)
  - size of cones is 1 $\mu$  (foveal), 10 $\mu$  (peripheral)
  - expressed as retinal arc: 30 arcsecs (L,M), 600 arcsecs (S)
- ◆ eyes are diffraction limited
  - pupil is 4mm
  - circle of confusion on retina is 6 $\mu$  (bigger than a foveal cone)



# Vernier acuity

# Vernier acuity

Vernier acuity :



Humans can detect 5 seconds of arc

Saccades

$$|\alpha_1 - \alpha_2| > \epsilon$$

Beware of jaggies

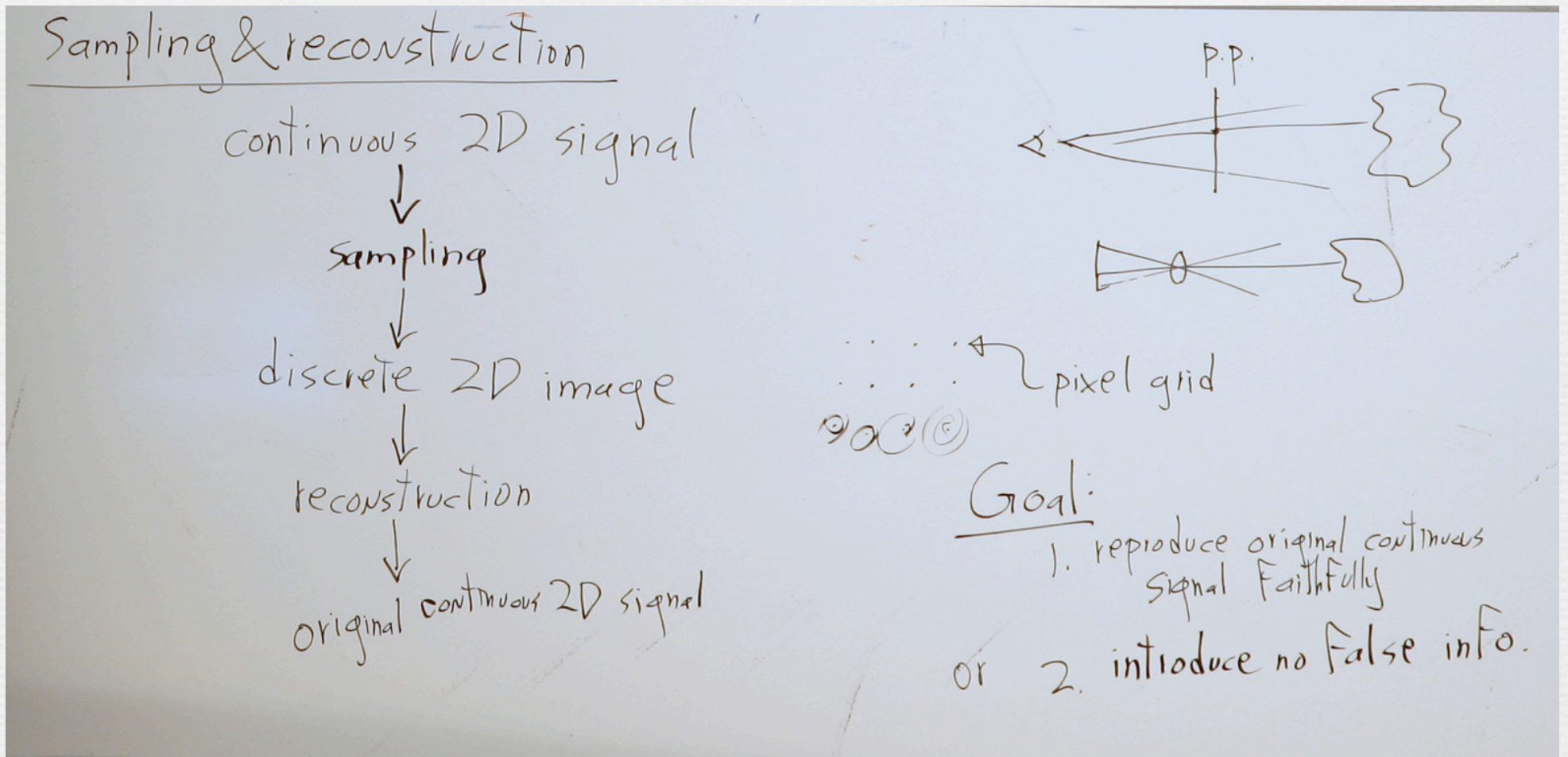
my laptop > 18,000 x 30,000





# Sampling and reconstruction

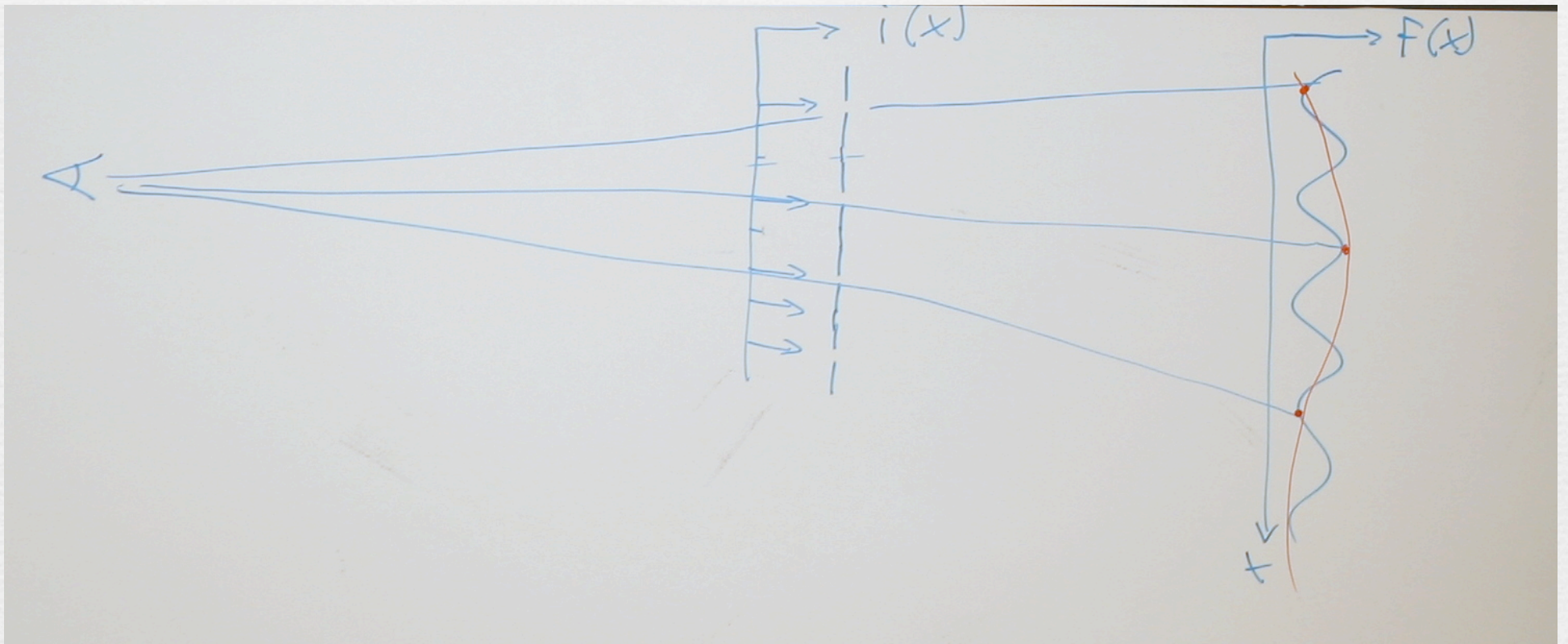
## ◆ the pipeline



# Sampling and reconstruction

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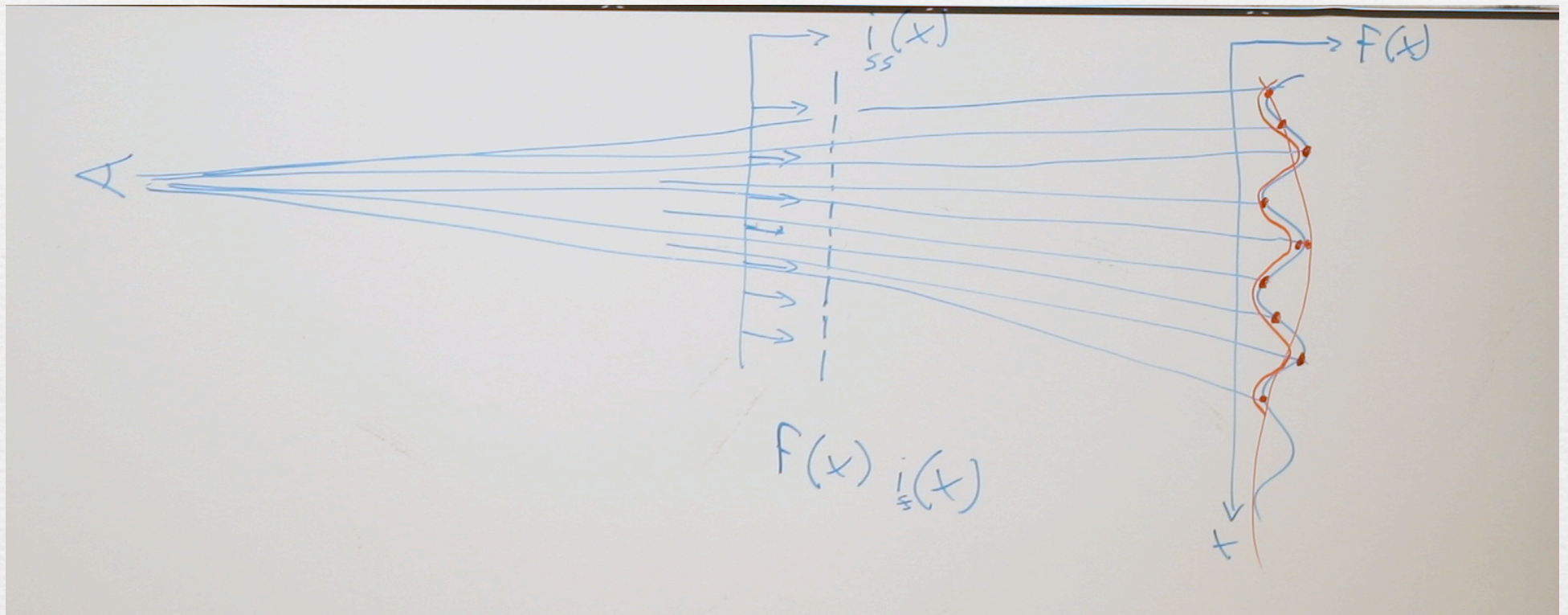
- ♦ insufficiently fine sampling, producing *aliasing* (high frequencies masquerading as low frequencies)





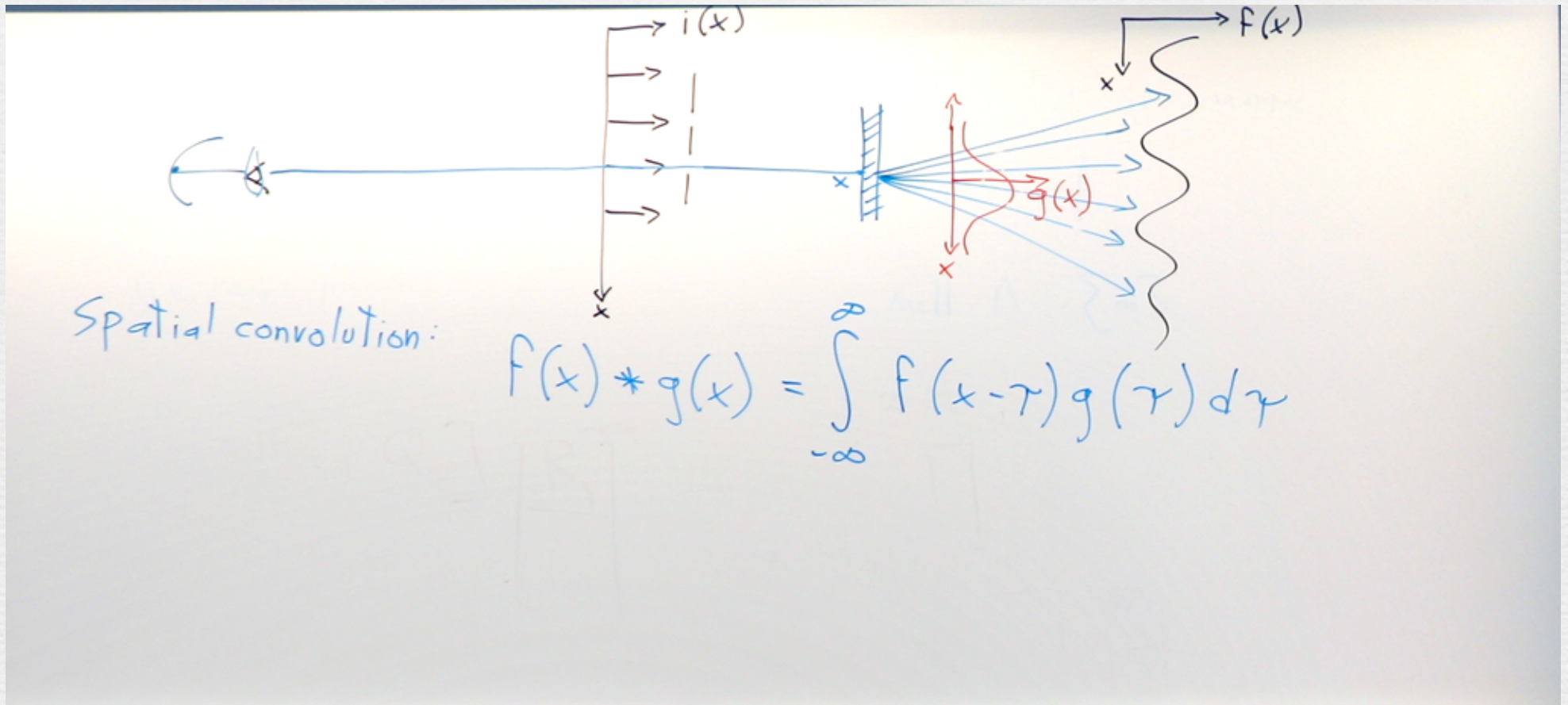
# Sampling and reconstruction

- ♦ solution #1: raise the sampling rate



# Sampling and reconstruction

- ♦ solution #2: filter the signal before sampling, a.k.a. *pre-filtering*, performed using a spatial convolution

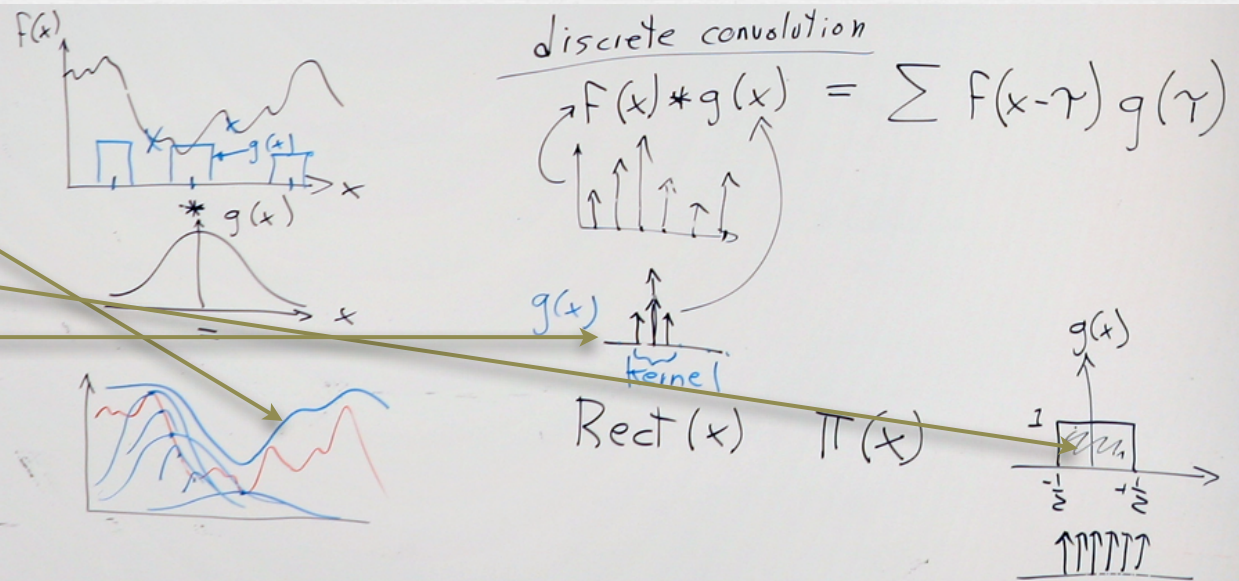




# A graphical construction for how convolution works

- ◆ on the left is continuous convolution (an integral);  
on the right is discrete convolution (a sum)
  - a blur filter in Photoshop performs a discrete convolution
- ◆ at bottom right is the Rect function - a.k.a. box filter
  - a square pixel in a digital camera performs a (continuous) convolution of the focused image with a (2D) Rect function, thereby pre-filtering it to avoid aliasing when converted to pixels

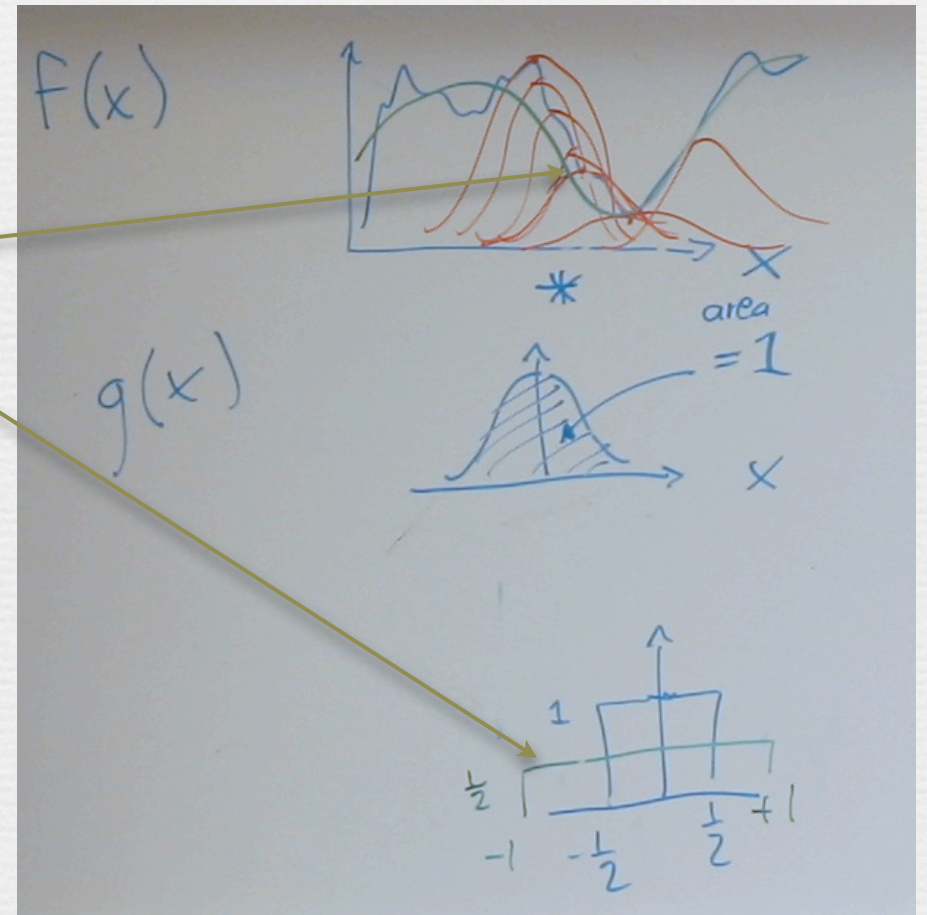
In this graphical construction for convolution, I should have been more careful when drawing the smoothed result to draw it passing \*through\* the original signal, not above it. For continuous convolution, this is accomplished by dividing the integral by the area under the filter function  $g(x)$  (or  $r(x)$ ), which by design should be 1.0. For discrete convolution, this means dividing by the sum of the non-zero taps in the filter, which similarly should sum to 1.0 by design. This division is called normalization. When convolution is used for reconstruction, i.e. for interpolation between samples, this normalization ensures that the black (or white) area between the samples being interpolated doesn't make the result look too dark (or washed out). See also next slide, added 4/21/09.



# Normalization during convolution

- ♦ with appropriate normalization, the smoothed result of convolution passes more or less through the original function values

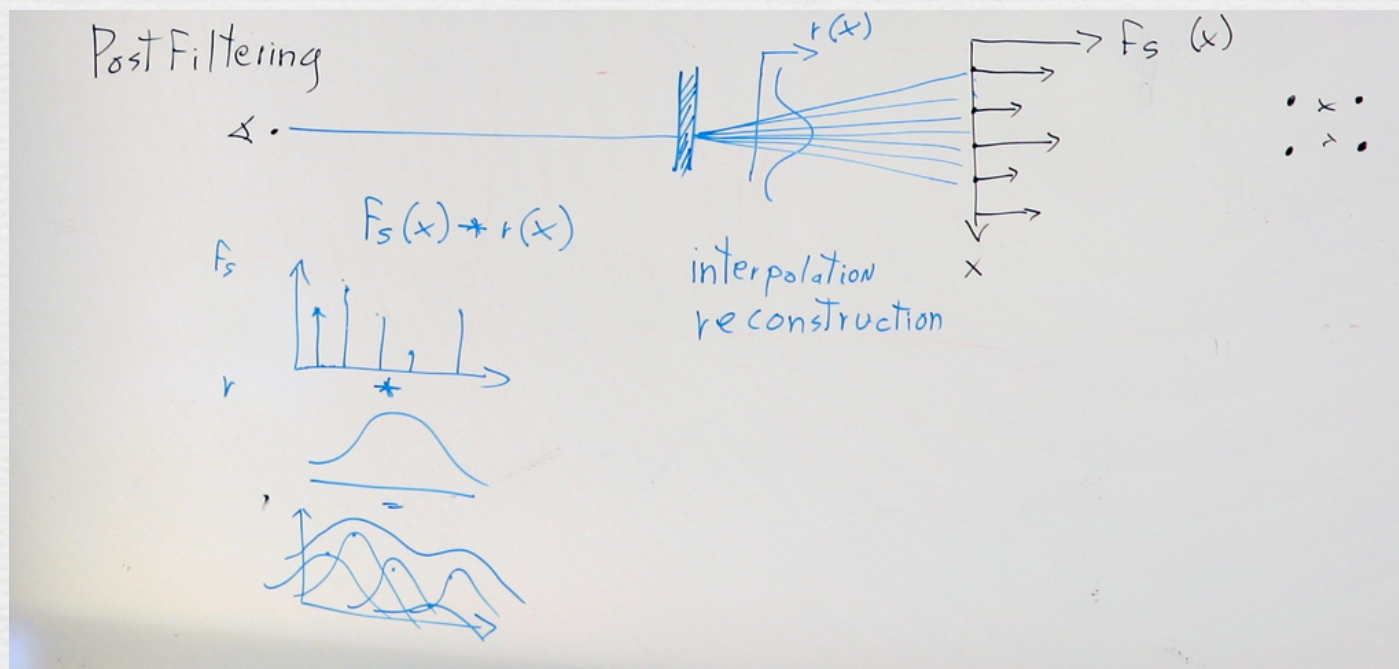
As noted in class on 4/21/09, in order to make the area of  $g(x)$  equal to 1, it may be necessary to make its maximum value less than 1. An example, for the Rect function, is shown in green at lower right. In this case, the vertically scaled copies (red curves) will not reach as high as the original function values. However, if the area of  $g(x)$  is 1, then the result of the convolution integral is guaranteed to pass more or less through the original function values.





# Sampling and reconstruction (cont.)

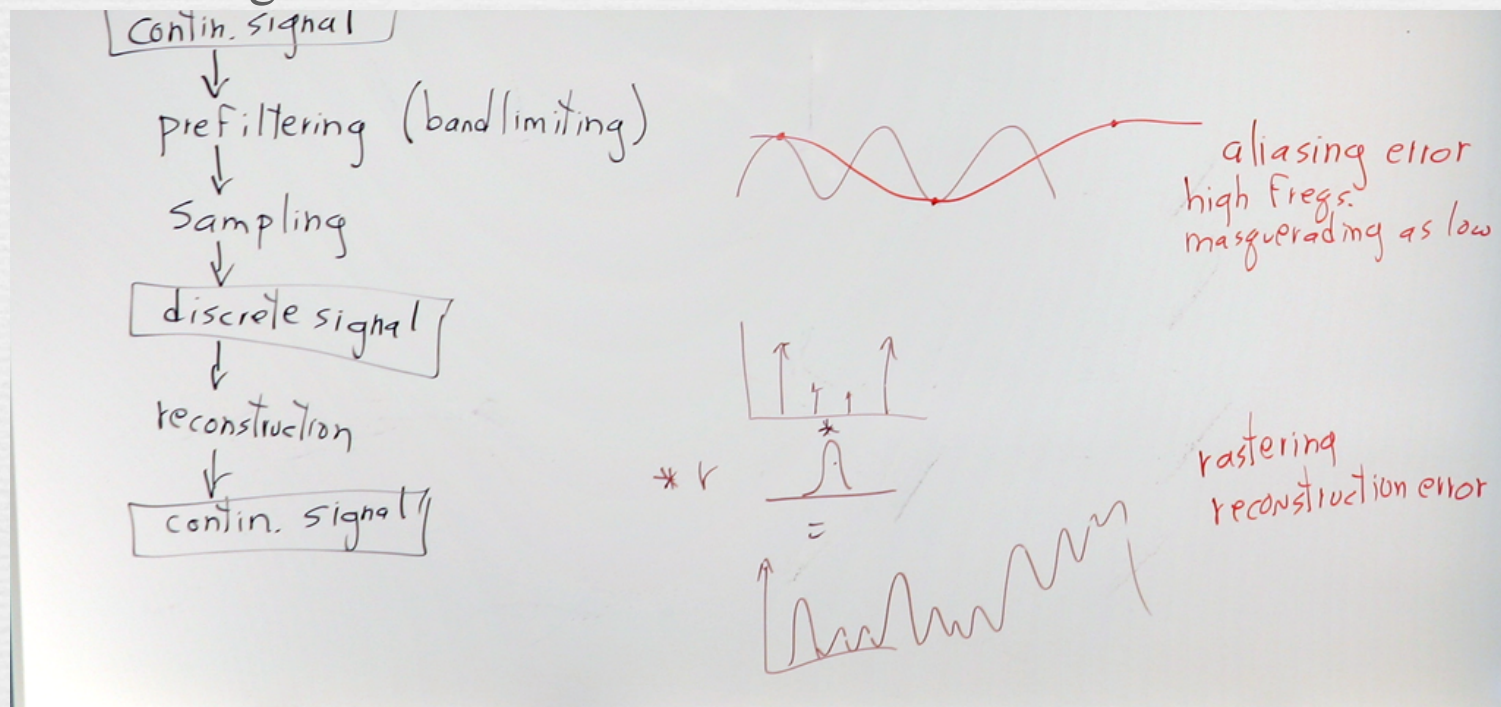
- ◆ reconstruction (a.k.a. *post-filtering*) by performing another spatial convolution, typically with a filter of a different size
  - its task is to interpolate between the discrete samples
- ◆ in computer graphics and digital photography, we don't worry about reconstruction; we hope the display or printer does it
  - blurring in the human eye helps with the reconstruction task



# Sampling and reconstruction (cont.)

## ♦ summary of the pipeline

- aliasing error is high frequencies masquerading as low frequencies due to insufficient sampling or prefiltering
- reconstruction error is a residual of the sampling pattern due to insufficient postfiltering, a.k.a. rastering in the case of an image
- no amount of postfiltering can remove aliasing error; once moire is in the image, it's too late to remove it

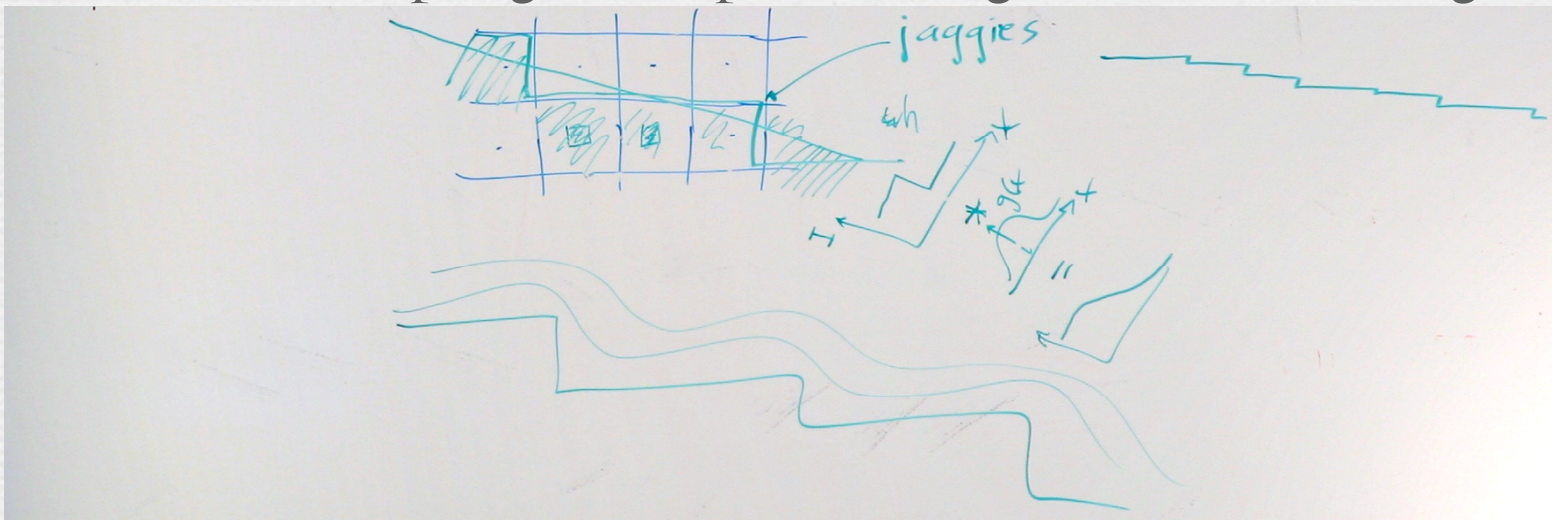




# Example of spatial aliasing: jaggies in computer graphics

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- ◆ an edge (e.g. of a colored triangle) contains (infinitely) high frequencies; merely sampling it produces jaggies
  - the rhythm of these jaggies is aliasing - a false low frequency
- ◆ to remove the jaggies, blur the edge (in software), i.e. draw it with pixels of intermediate colors
  - feathering a Selection in Photoshop will do this for you
- ◆ no amount of blurring a jaggy image will remove the jaggies
  - that's attempting to use postfiltering to remove aliasing







# Slide credits

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◆ Pat Hanrahan

◆ Cornsweet, T.N., *Visual Perception*, Kluwer Academic Press, 1970.