Color I: trichromatic theory

CS 178, Spring 2012

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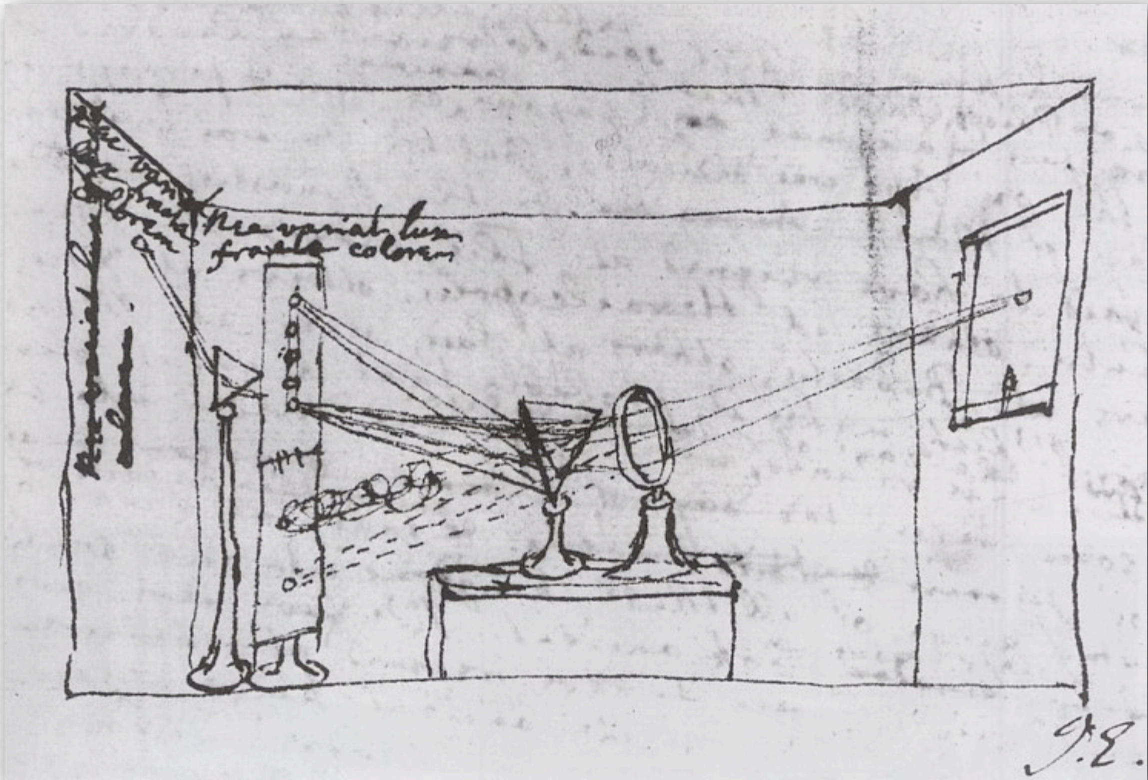
Outline

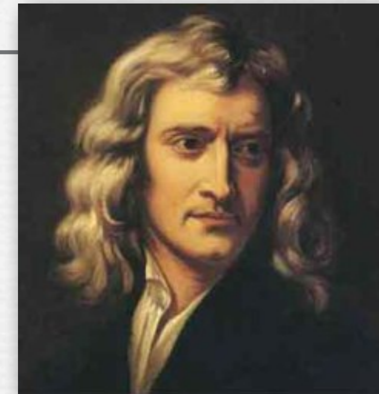
- spectral power distributions
- color response in animals and humans
- 3D colorspace of the human visual system
  - and color filter arrays in cameras
- reproducing colors using three primaries
- additive versus subtractive color mixing

- cylindrical color systems used by artists (and Photoshop)
- chromaticity diagrams
  - color temperature and white balancing
  - standardized color spaces and gamut mapping

Newton's Experimentum Crucis



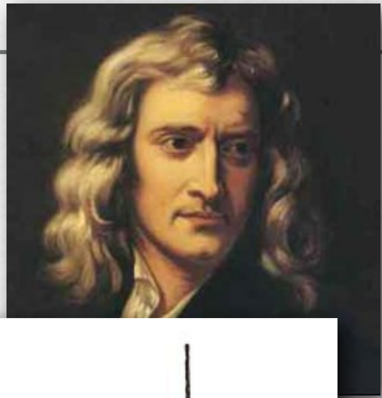
(Robin)



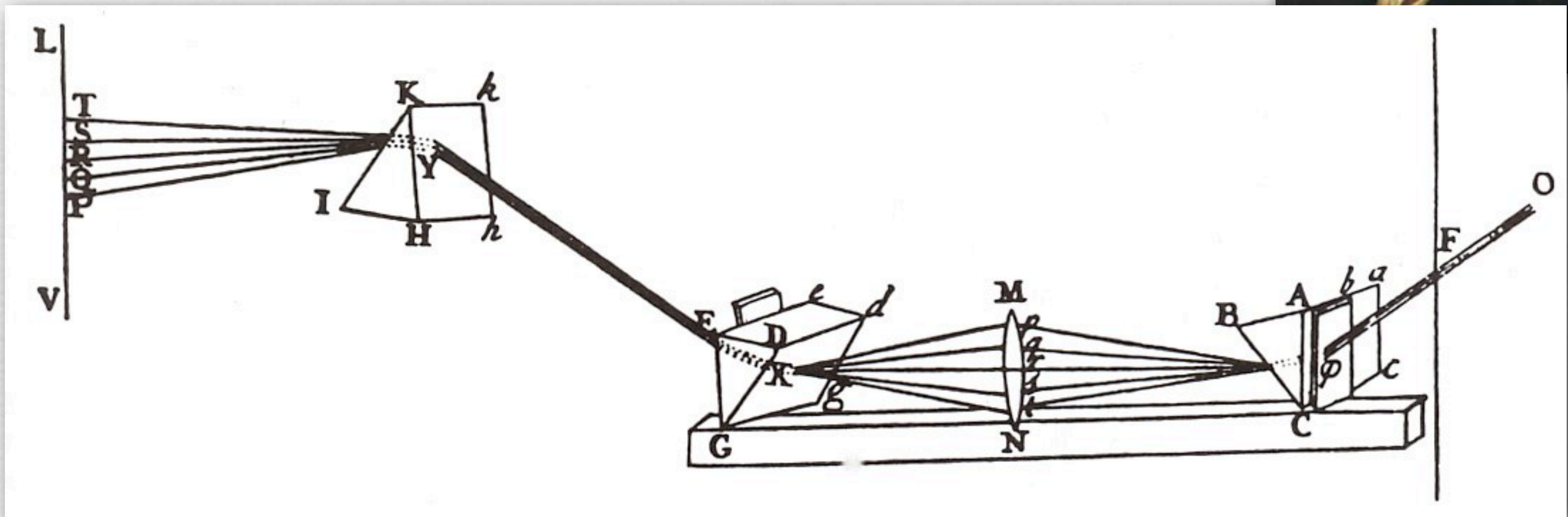
Isaac Newton
(1643-1727)

- sunlight can be divided into colors using a prism
- these colors cannot be further divided using a 2nd prism
- experiment performed 1665, drawing made in 1672

Newton's Experimentum Crucis

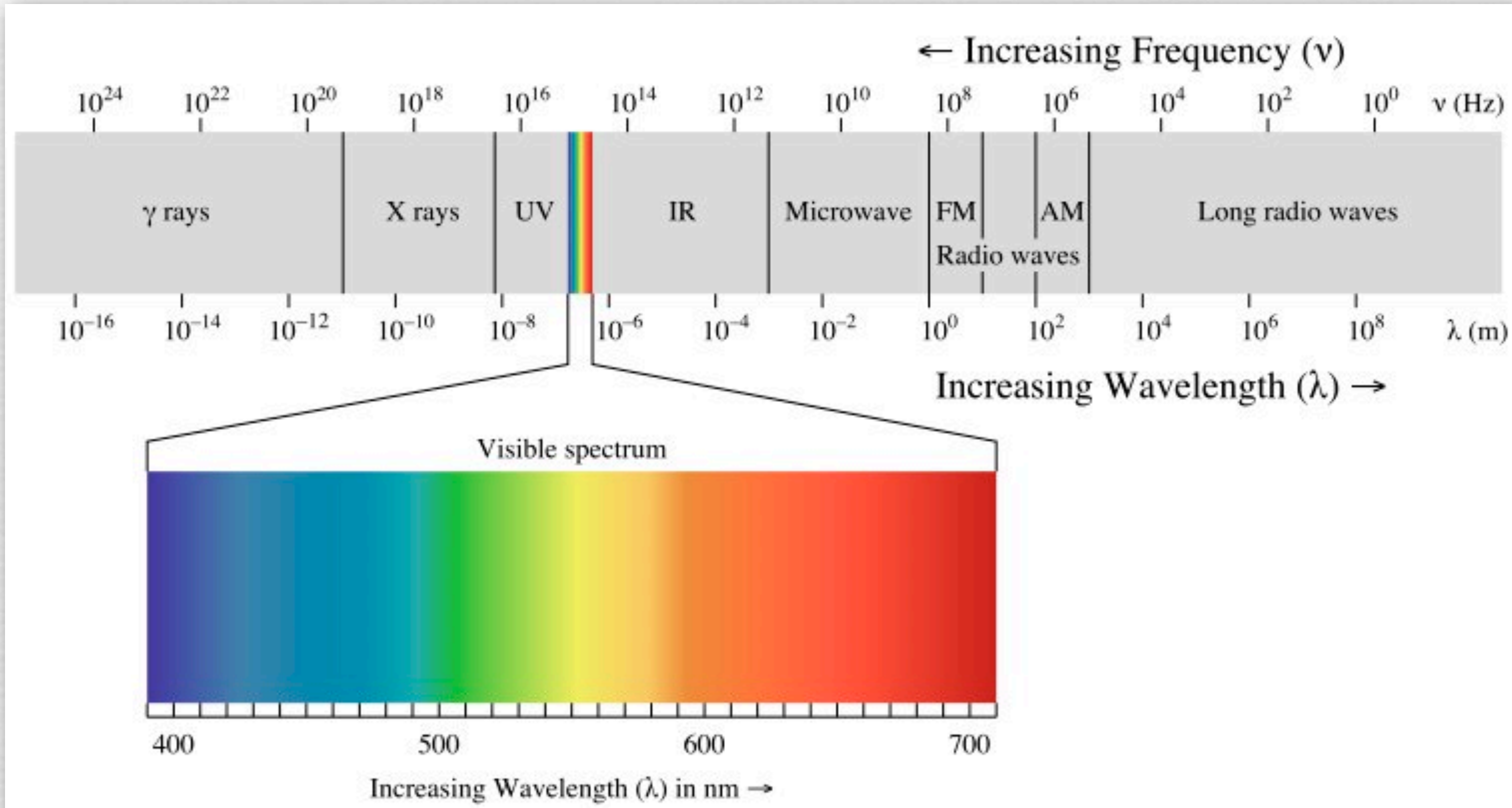


(Robin)



- alternatively, the divided colors can be recombined using a lens and 2nd prism into a new beam that has exactly the same properties as the original

The visible light spectrum



(wikipedia)

- wavelengths between 400nm and 700 nm (0.4μ - 0.7μ)
- exactly the colors in a rainbow

The visible light spectrum

(Dan Bush)

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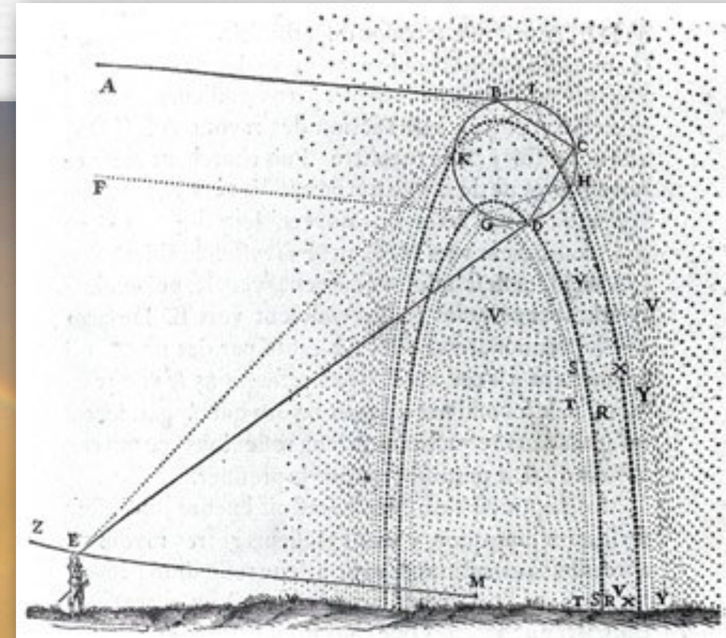
- exactly the colors in a rainbow

The visible light spectrum

Rene Descartes,
Formation of a Rainbow
(1637)



- wavelengths between 400nm and 700 nm (0.4μ - 0.7μ)
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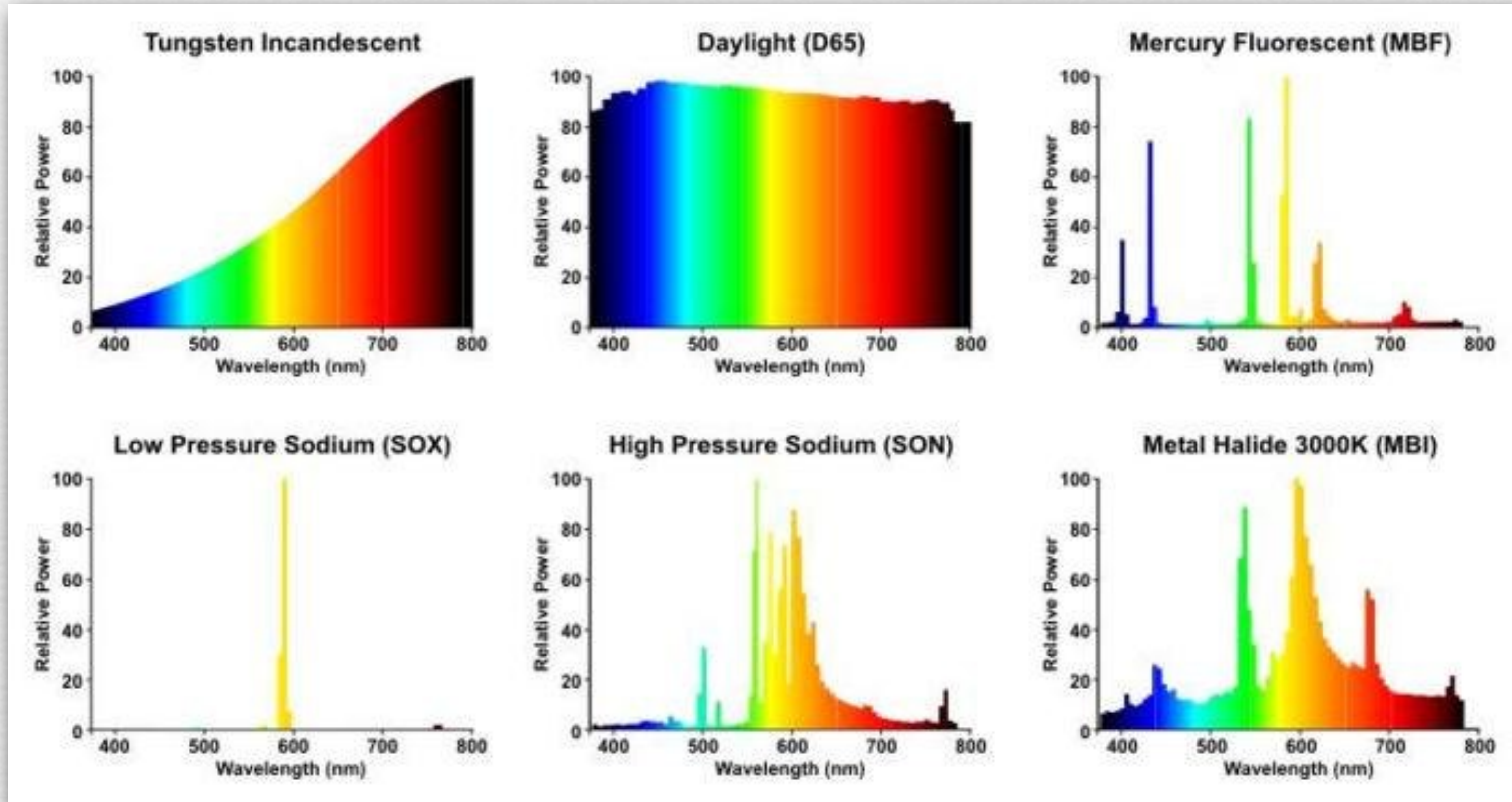
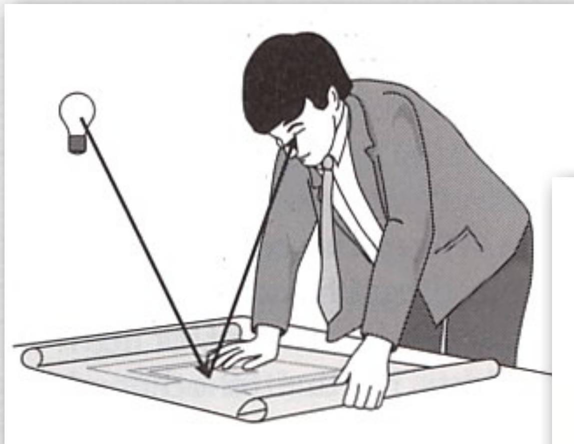
Spectral power distribution (SPD)



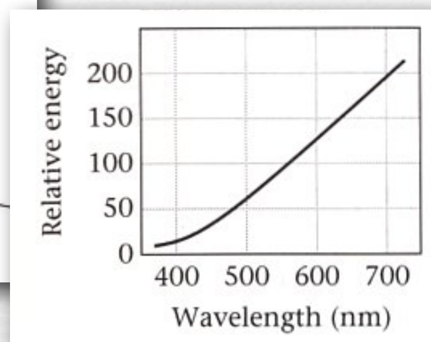
(LampTech)

- units of power are watts (joules per second)
- shown here are spectra of common illumination sources
- plots above are relative amounts (%) of each wavelength

Interaction of light with matter

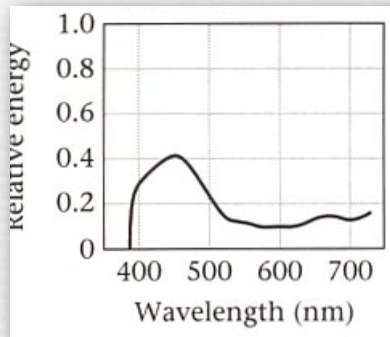


light is reflected by an object



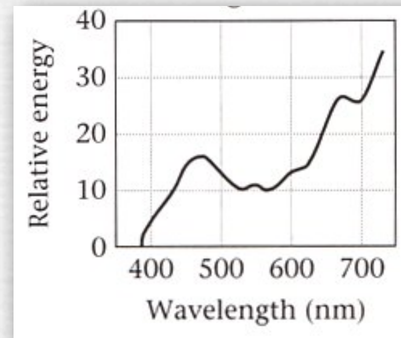
illumination

×



reflectance

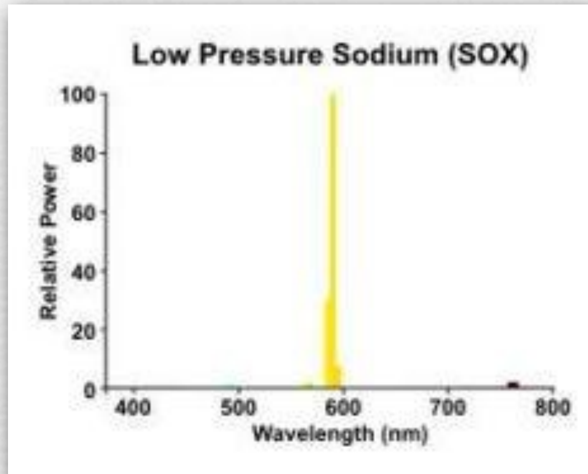
=



stimulus that enters your eye

- spectrum of illumination is multiplied wavelength-by-wavelength by reflectance spectrum of object
  - cause is absorption by the material
  - so the spectrum you see depends on the illumination
- transmittance operates the same way

Example

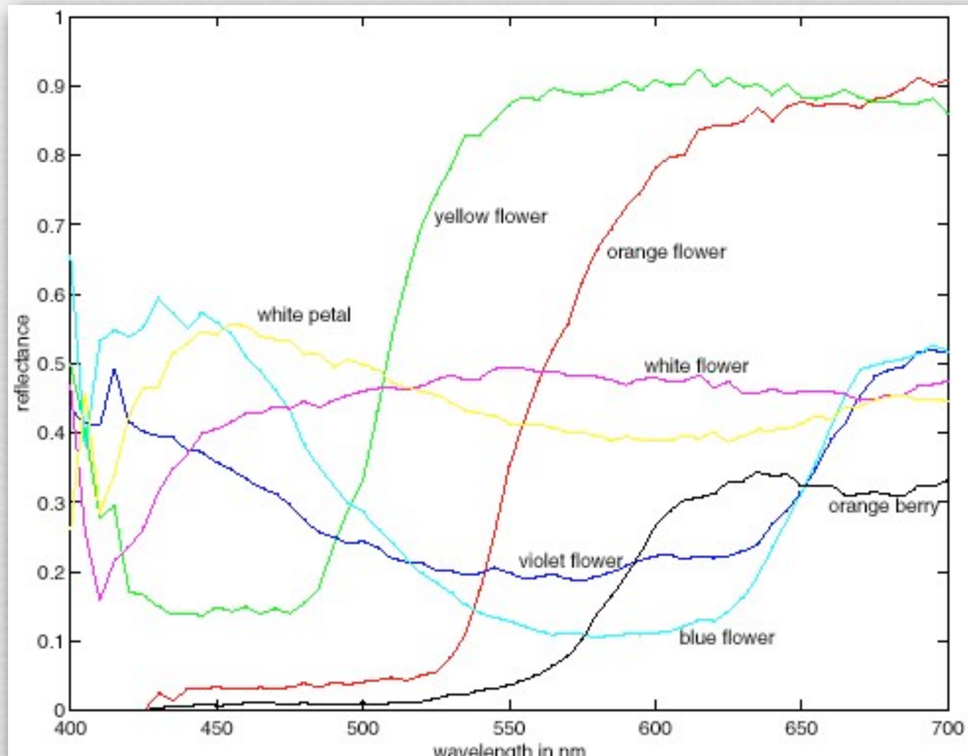


×

= black

my old van

Examples of reflectance spectra



- two reflectance spectra that match (i.e. are metamers) under one illuminant may not match under another
- clothes that match in the store may not match outdoors

Questions?

- ✦ two different spectra may appear alike to us
  - white petal and white flower (above left)
  - these are called *metamers*
- ✦ Newton observed this, but could not explain it

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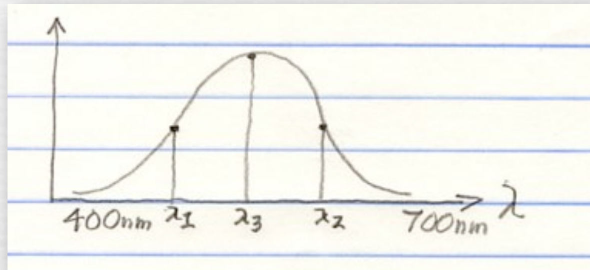
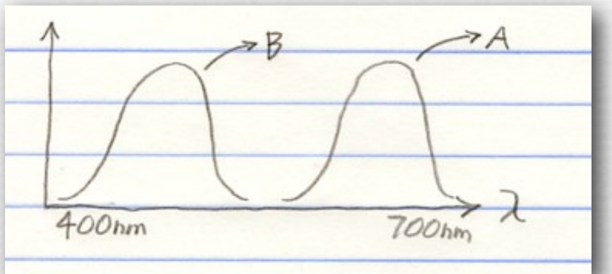
Monochromats (contents of whiteboard)



1

1. organisms having only one kind of retinal receptor cannot distinguish changes in intensity from changes in wavelength, hence they have no *color discrimination*
   - for example a unit amount of λ_1 versus λ_2 above
   - or a unit amount of λ_1 versus half as much of λ_3 (assuming the sensitivity to λ_3 is twice the response to λ_1)
   - example: horseshoe crab

Dichromats
(contents of whiteboard)



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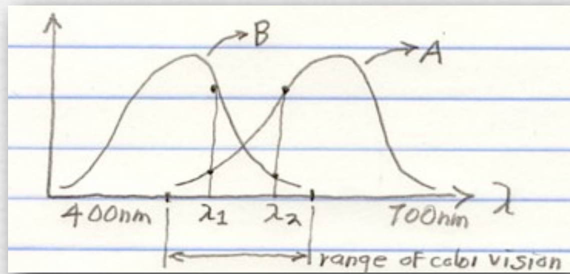

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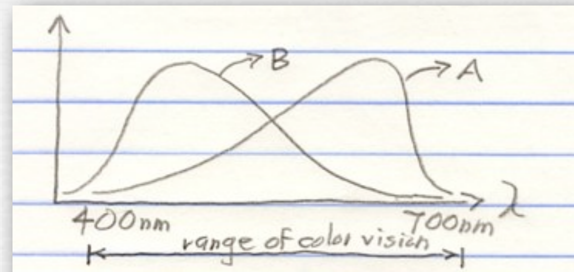


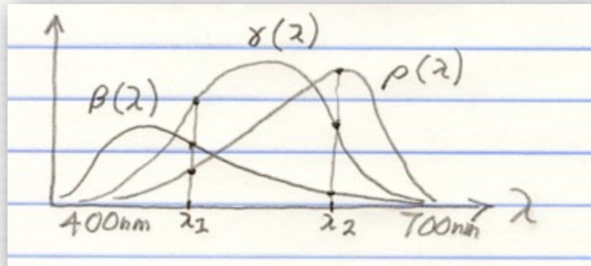
4

2. this organism can discrimate a response in the range wavelengths covered by A versus B, but cannot discriminate within those ranges

3. this organism has color discrimination over the range of wavelengths shown
   - for each wavelength within this range, the ratio of responses of receptors A and B is unique; hence the organism can identify which wavelength (e.g. λ_1 or λ_2) it's looking at

4. this organism has a larger range of color vision
   - example: dog, horse

Trichromats (contents of whiteboard)



5

5. humans can discrimate wavelengths from 400nm to 700nm
   - we can also discriminate mixtures of wavelengths that dichromats cannot; this will become clearer later

- at the retinal level, our response to light is linear
   a. if the response to a unit stimulus at λ_1 of is $(\rho_1, \gamma_1, \beta_1)$, and to a unit stimulus at λ_2 is $(\rho_2, \gamma_2, \beta_2)$, then the response to a superposition of stimuli λ_1 and λ_2 is $(\rho_1 + \rho_2, \gamma_1 + \gamma_2, \beta_1 + \beta_2)$
   b. the response to n units of a stimulus at λ_1 is $(n\,\rho_1, n\,\gamma_1, n\,\beta_1)$
   c. a system that obeys *superposition* (a) and *scaling* (b) is *linear*

Human response to an arbitrary stimulus

(Berns)

spectrum of stimulus arriving in one small area on retina

×

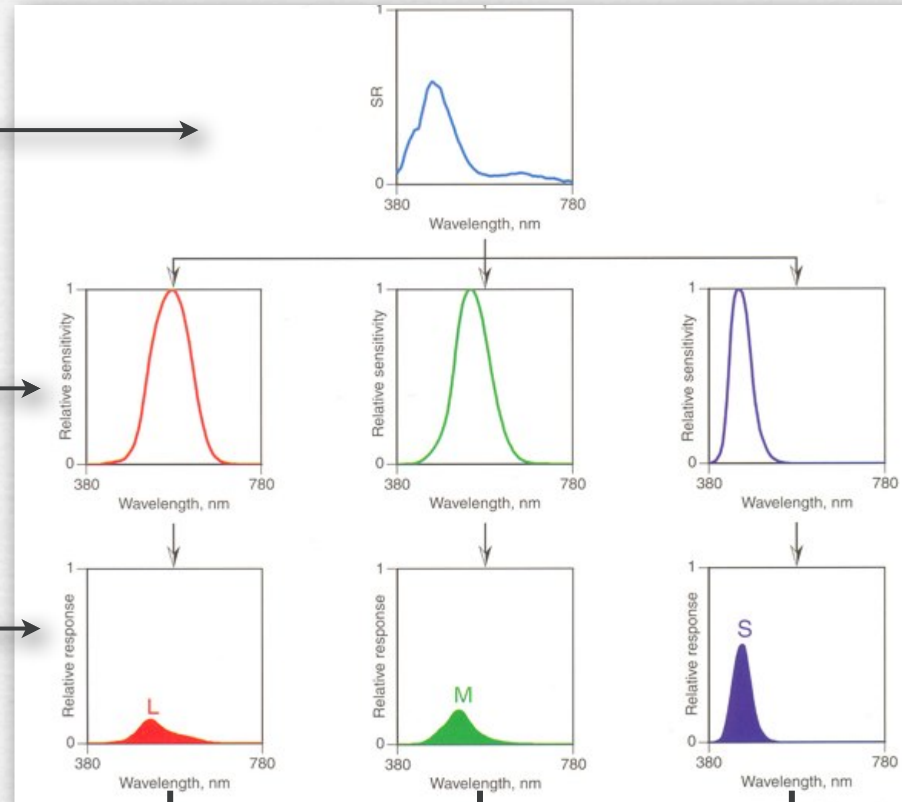
spectral sensitivity of each type of cone (L,M,S)

=

multiply wavelength-by-wavelength to get response spectra

$\int$

integrate over wavelengths to get total response for that type of cone



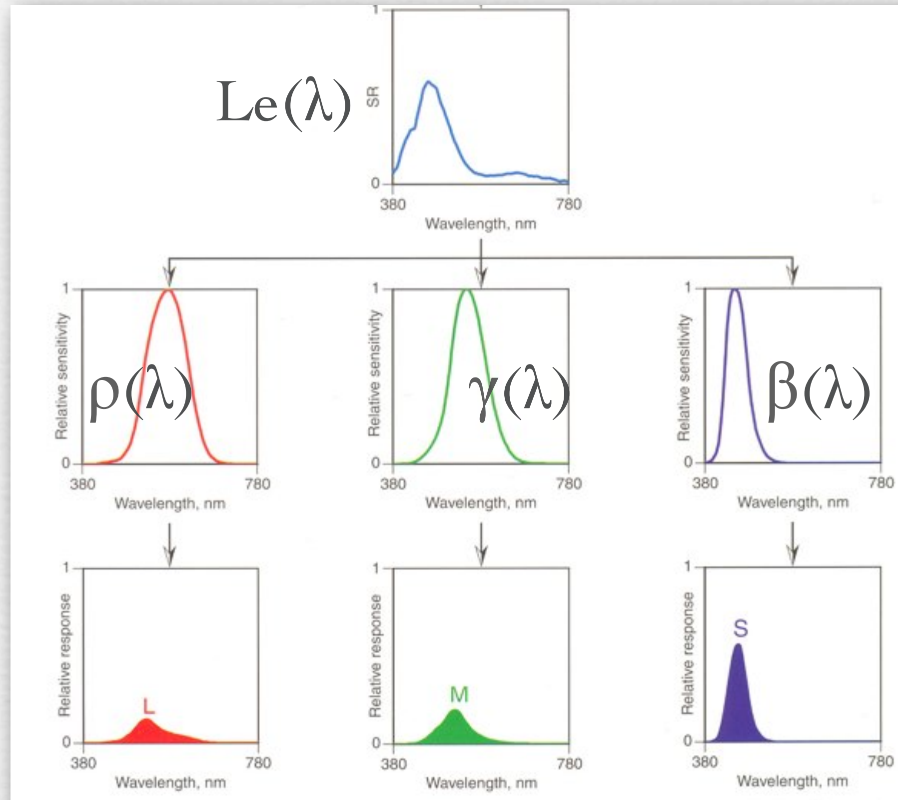
ρ γ β

- output is three numbers (ρ, γ, β) per area on retina

Human response to an arbitrary stimulus

(Berns)

- stated algebraically, given a stimulus spectrum $L_e(\lambda)$, the human response to it (ρ, γ, β) are the integrals over all visible wavelengths of our responses
  $L_e(\lambda)\,\rho(\lambda)$,
  $L_e(\lambda)\,\gamma(\lambda)$,
  $L_e(\lambda)\,\beta(\lambda)$
  to each constituent wavelength λ, i.e.



$$(\rho,\gamma,\beta) = \left(\int_{400\,nm}^{700\,nm} L_e(\lambda)\,\rho(\lambda)\,d\lambda,\ \int_{400\,nm}^{700\,nm} L_e(\lambda)\,\gamma(\lambda)\,d\lambda,\ \int_{400\,nm}^{700\,nm} L_e(\lambda)\,\beta(\lambda)\,d\lambda \right)$$

Questions?

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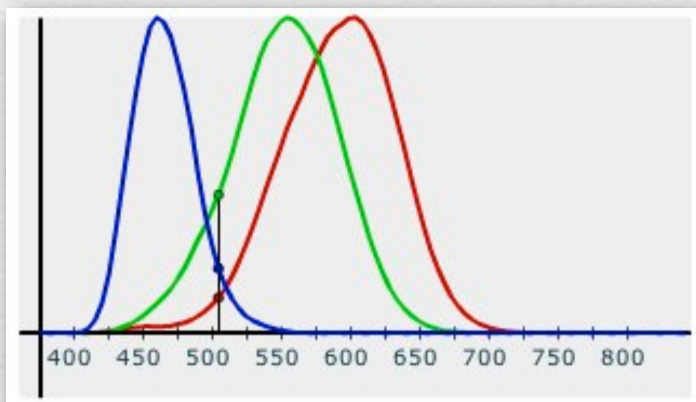
Human 3D colorspace

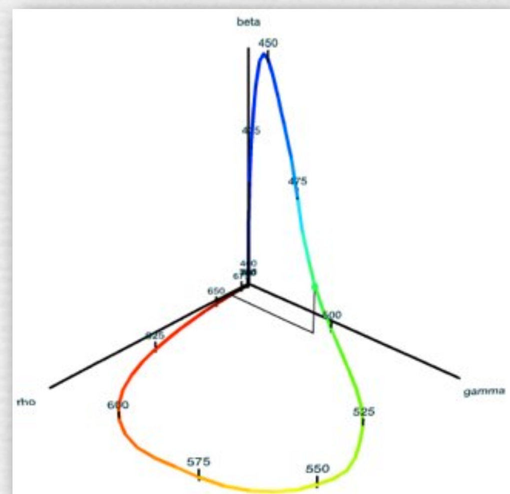
- the three types of cones in our retina (Long, Medium, Short wavelength) define the axes of a three-dimensional space
- our response to any stimulus spectrum can be summarized by three numbers (ρ, γ, β) and plotted as a point in this space
- our responses to all visible single-wavelength spectra (a.k.a. pure wavelengths λ, i.e. positions along the rainbow), if connected together, form a curve in this space, called the *locus of spectral colors*; the sequence of (ρ, γ, β) numbers form the *tristimulus sensitivity functions* ρ(λ), γ(λ), and β(λ)



sensitivity functions

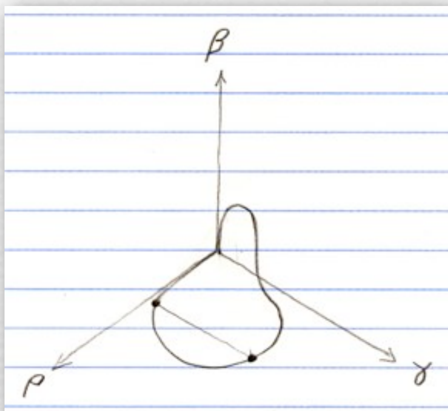
(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/locus.html

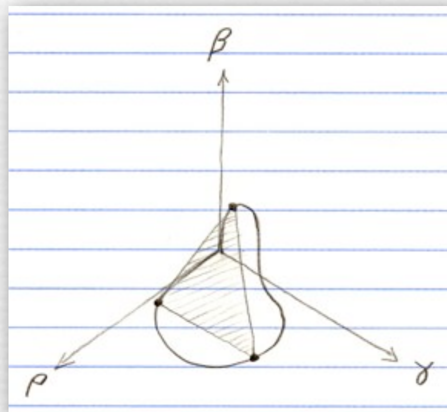


spectral locus

Properties of human 3D colorspace (1 of 2) (contents of whiteboard)



1



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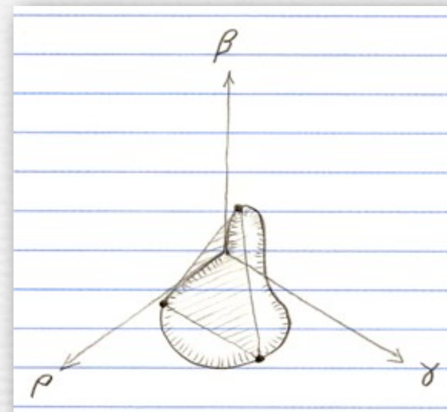


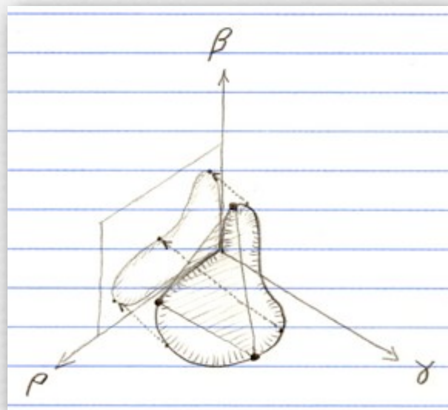
3

1. our response to any mixture ($\sum = 1$) of two pure wavelengths falls on a line connecting the responses to each wavelength

2. our response to any mixture ($\sum = 1$) of three pure wavelengths falls on a triangle connecting the responses to each wavelength; our response to any mixture or scaling ($\sum \leq 1$) of three pure wavelengths falls in a tetrahedron defined by this triangle and the origin

3. our responses to all possible mixtures or scalings ($\sum \leq 1$) of all visible wavelengths forms an irregular volume called the *gamut of perceivable colors*, equal to the convex hull of the spectral locus

Properties of human 3D colorspace (2 of 2) (contents of whiteboard)

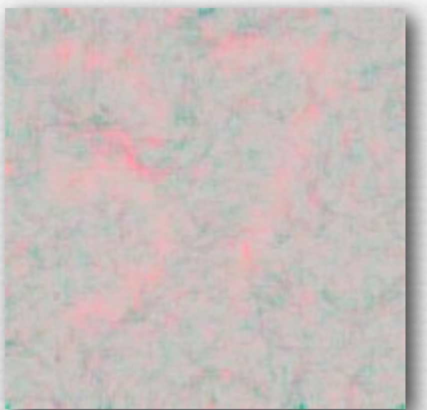


4

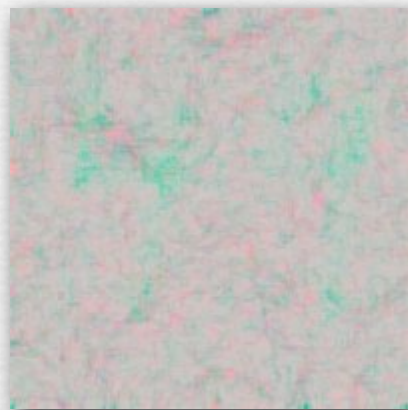
4. to a deuteranope - a color-blind person who is missing their medium-wavelength receptor, i.e. their gamma receptor - this diagram is squashed into the rectangle shown above on the rho-beta plane
    - as a result, spectra whose (ρ, γ, β) responses lie along the dotted lines cannot be distinguished; they will appear as the same color, i.e. as metamers
    - by a similar argument, many spectra distinguishable to pentachromats (e.g. Mallard ducks) are indistinguishable to trichromats (humans)

Color blindness

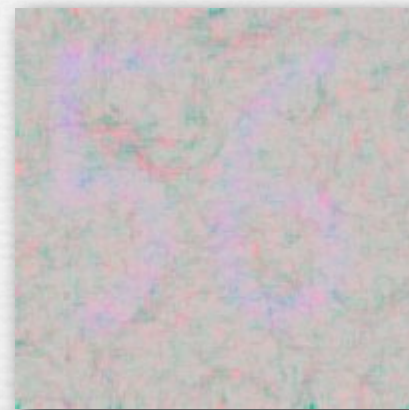
37?



49?



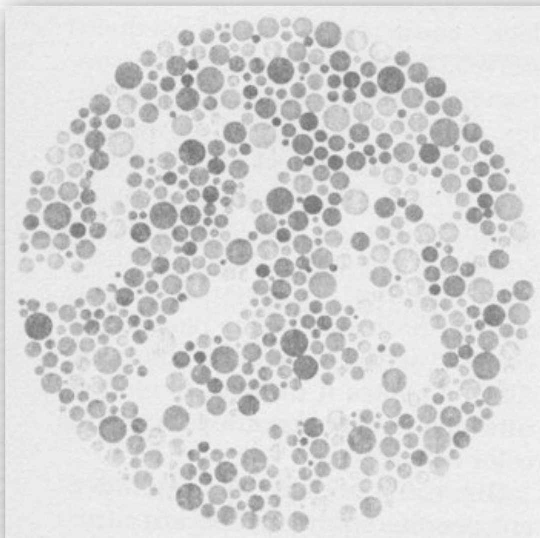
56?

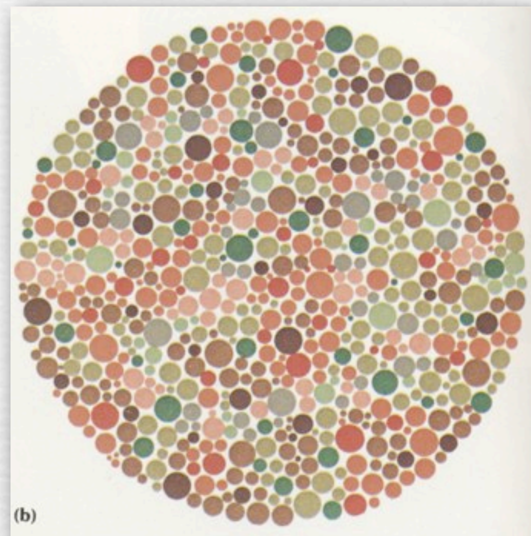


(wikipedia)

- protanopia (1% of males)
- deuteranopia (1% of males)
- tritanopia (< 1% of both genders)
- protanomaly (1% of males)
- deuteranomaly (6% of males)
- tritanomaly (< 1% of both genders)

The advantage of being color blind

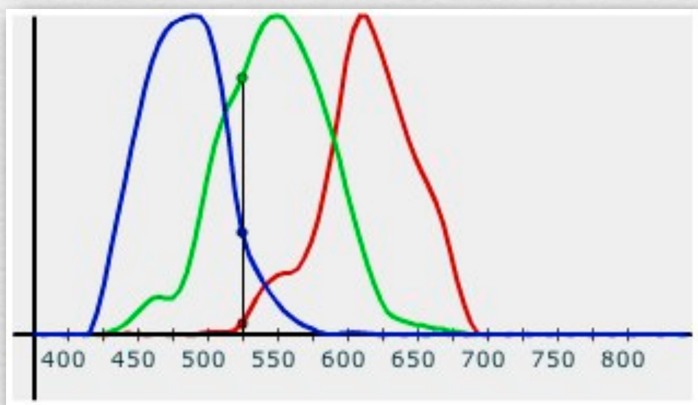




- the maze (at left) is recreated (at right) using subtle intensity differences, but overridden by stronger red-green color differences
- only a deuteranope can see the maze at right

Canon 30D color filters

- you want the camera's R, G, and B color filters to have the same spectral sensitivities as our L, M, and S cones
  - you don't want objects in the real world to be metamers to one system and not the other
  - otherwise, colored patterns the camera sees might be invisible to a person (bad), or patterns you see might be invisible to a camera (also bad)

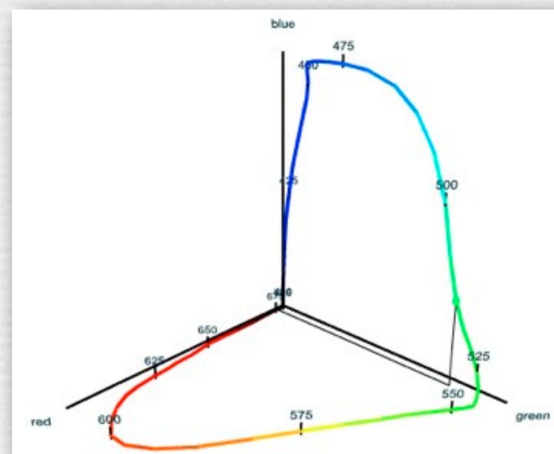


filter transmissivity

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/locus.html

Questions?



spectral locus

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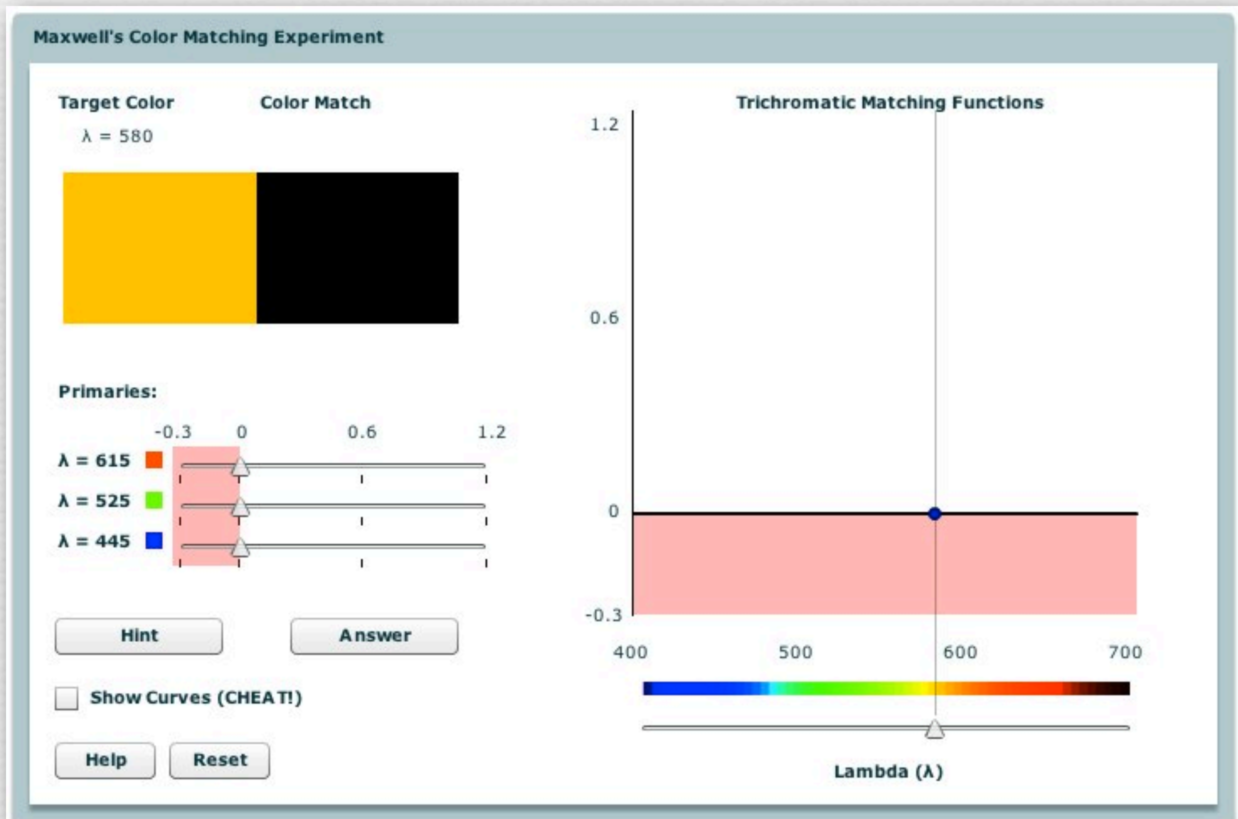
Maxwell's color matching experiment

- Maxwell actually used a slightly different procedure
  - see http://www.handprint.com/HP/WCL/color6.html for details
  - the procedure below is used in modern versions of the experiment



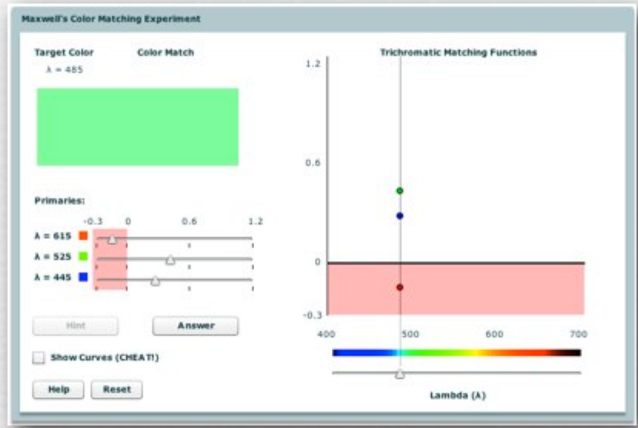
(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/colormatching.html

Maxwell's color matching experiment

(summary of live demo)

1



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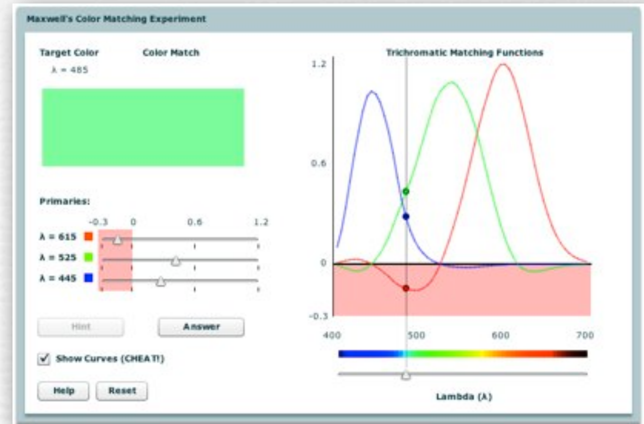


3

1. given a stimulus wavelength, the amount of each primary required to match it is given by three numbers $(\bar{r}, \bar{g}, \bar{b})$
2. some stimuli cannot be matched unless first desaturated by adding a primary to it before matching; the amount added is denoted by negative values of $\bar{r}$, $\bar{g}$, or $\bar{b}$
3. the sequence of $(\bar{r}, \bar{g}, \bar{b})$ values, some negative, required to match the locus of spectral colors across all λ, form the *trichromatic matching functions* $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$ for a particular set of 3 primaries

Human response to an arbitrary stimulus (contents of whiteboard)

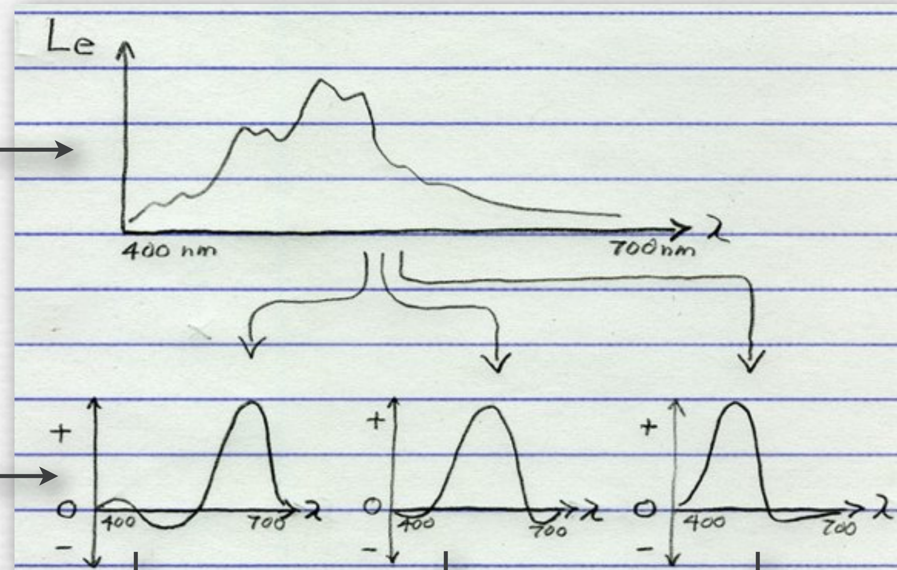
spectrum of stimulus

×

multiply wavelength-by-wavelength
by the matching functions
$\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$
for a particular set of 3 primaries

$\int$

then integrate over wavelengths to
get the amount of that primary
required to reproduce that spectrum



R G B

Young-Helmholtz trichromatic theory

Thomas Young
(1773-1829)

James Clerk Maxwell
(c. 1860)

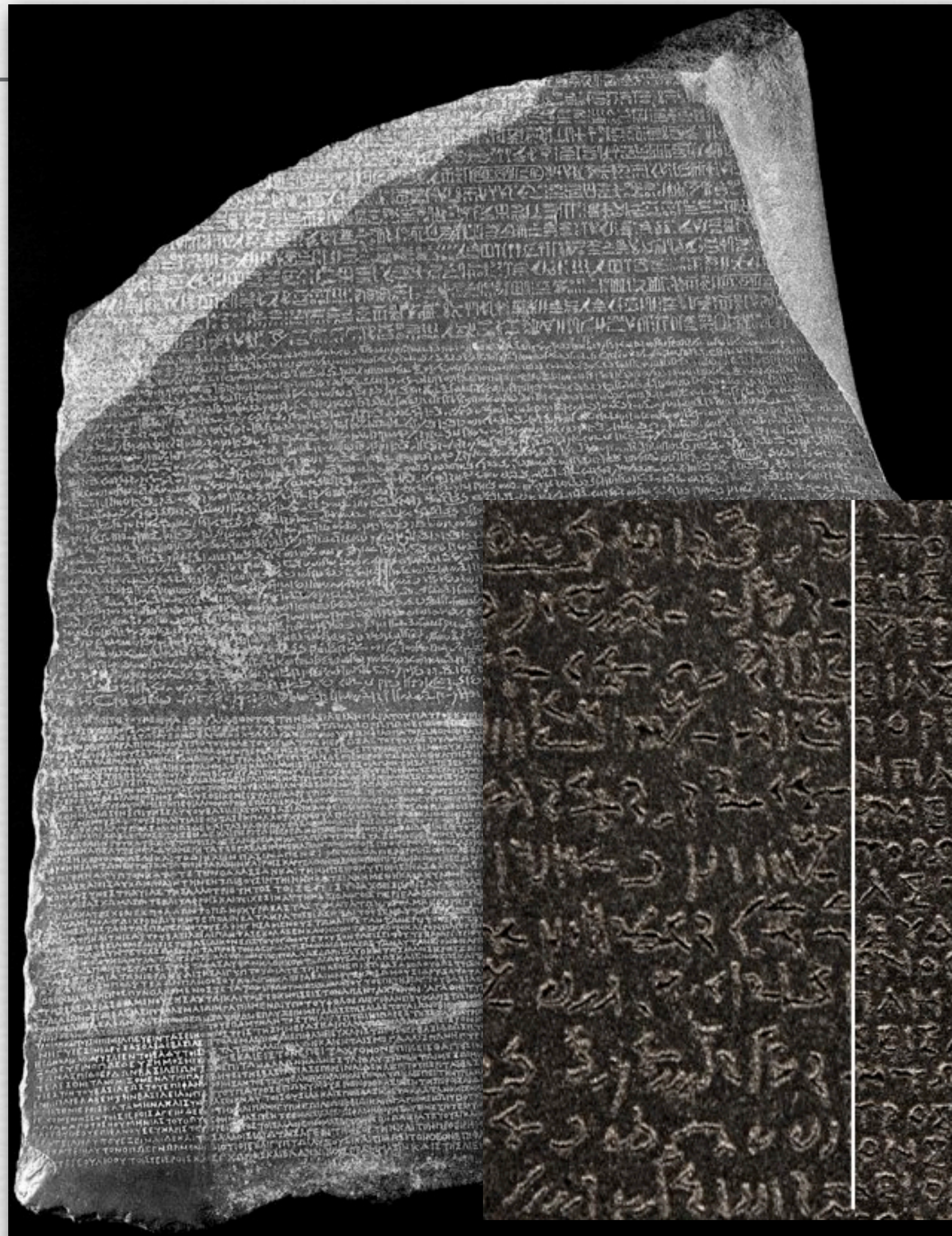
Hermann von Helmholtz
(1821-1894)

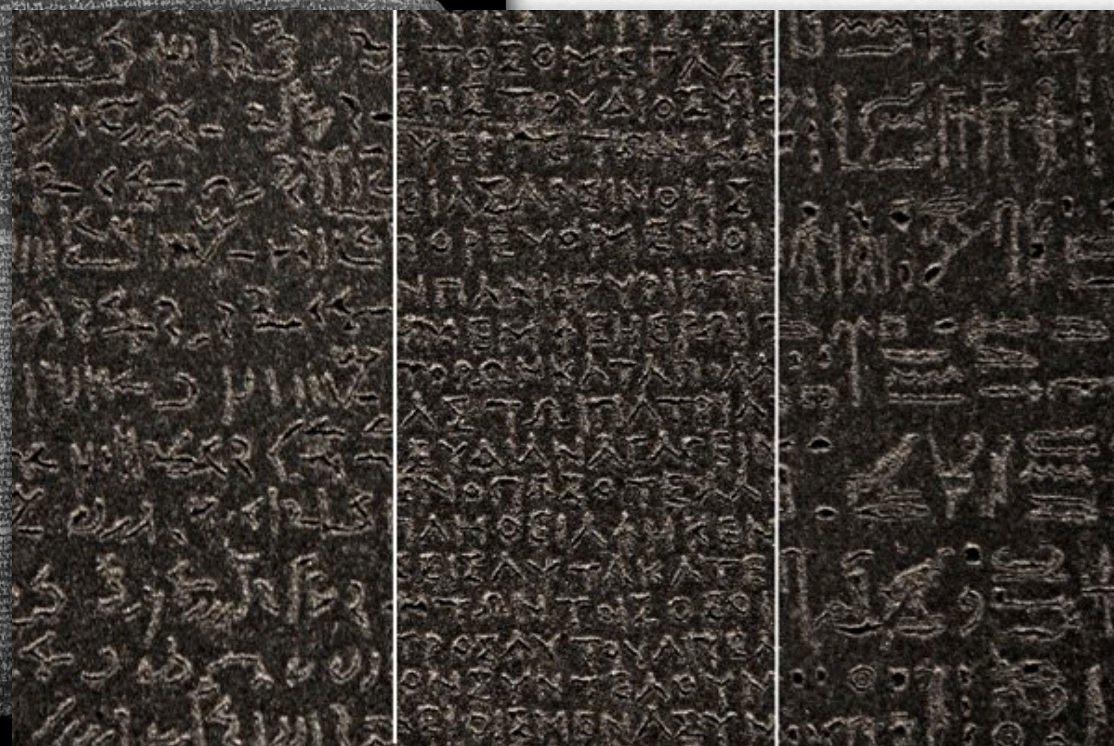
- spectra can be visually matched using mixtures of *primary colors*; such matches are called *metamers*
- due to the linearity of human retinal response, given a stimulus spectrum $L_e(\lambda)$, the amounts of each primary R, G, B required to match it, for any particular choice of 3 primaries, are the integrals over all visible wavelengths of the amounts $\bar{r}(\lambda)$, $\bar{g}(\lambda)$, and $\bar{b}(\lambda)$ required to match each constituent wavelength λ, *i.e.*

$$(R,G,B) = \left(\int_{400nm}^{700nm} L_e(\lambda)\,\bar{r}(\lambda)\,d\lambda,\ \int_{400nm}^{700nm} L_e(\lambda)\,\bar{g}(\lambda)\,d\lambda,\ \int_{400nm}^{700nm} L_e(\lambda)\,\bar{b}(\lambda)\,d\lambda \right)$$

Young-Helmholtz trichromatic theory

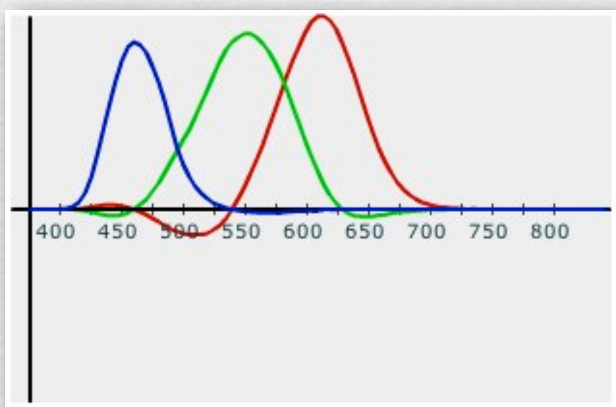
Thomas Young
(1773-1829)





3D interpretation of color matching

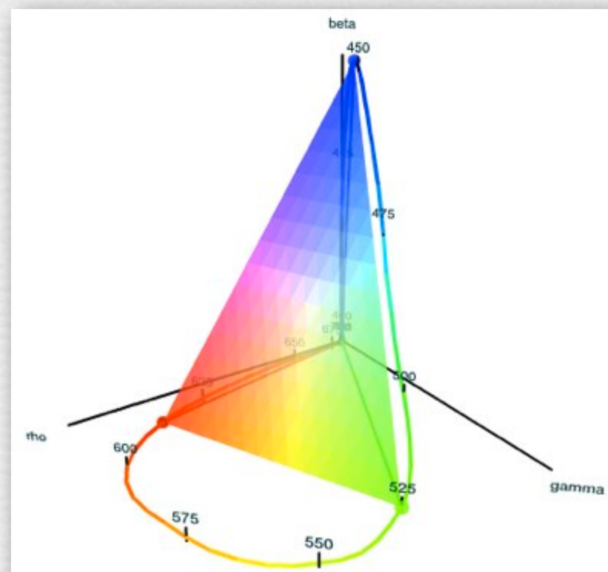
- our response to varying amounts of a primary forms a vector in (ρ, γ, β) space, rooted at the origin
- to provide a normal range of color vision, three primaries are required, and their vectors must not lie on a plane
- our responses to all possible mixtures and scales ($\sum \leq 1$) of three primaries form a tetrahedron called the *gamut of reproducible colors* for these primaries



RGB matching functions

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/locus.html

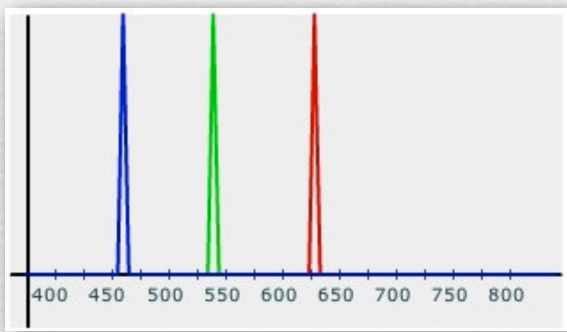


gamut of reproducible colors

3D interpretation of color matching

- the spectrum of each of the three primaries can be a pure wavelength (1) or a mixture of wavelengths (2)
- impure primaries have a smaller gamut in (ρ, γ, β) space
- additional primaries can be added to increase the gamut

1



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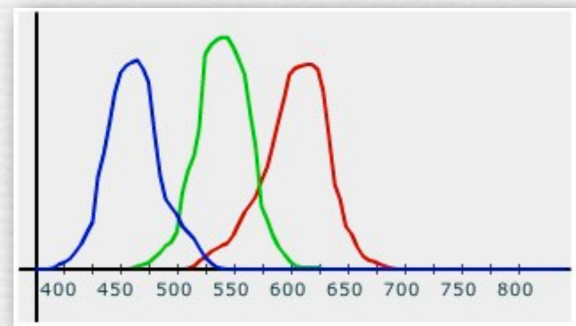

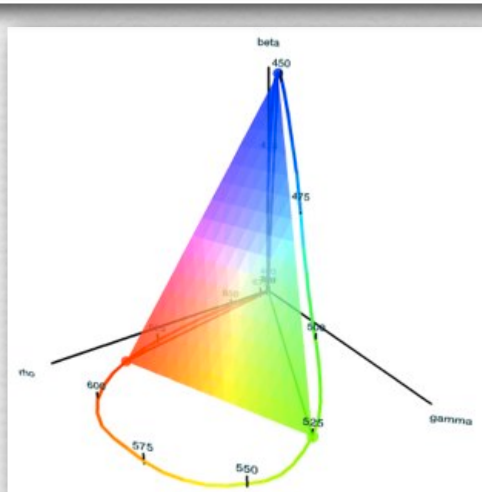

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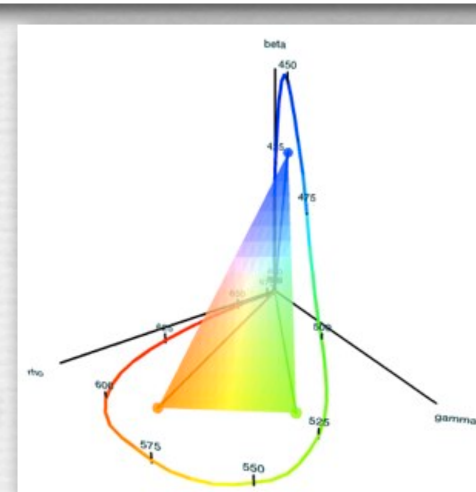
http://graphics.stanford.edu/courses/cs178/applets/locus.html

Questions?



Outline

- spectral power distributions
- color response in animals and humans
- 3D colorspace of the human visual system
  - and color filter arrays in cameras
- reproducing colors using three primaries
- ☞ additive versus subtractive color mixing
- cylindrical color systems used by artists (and Photoshop)
- chromaticity diagrams
  - color temperature and white balancing
  - standardized color spaces and gamut mapping

Additive versus subtractive mixing

- demo using color guns and filters

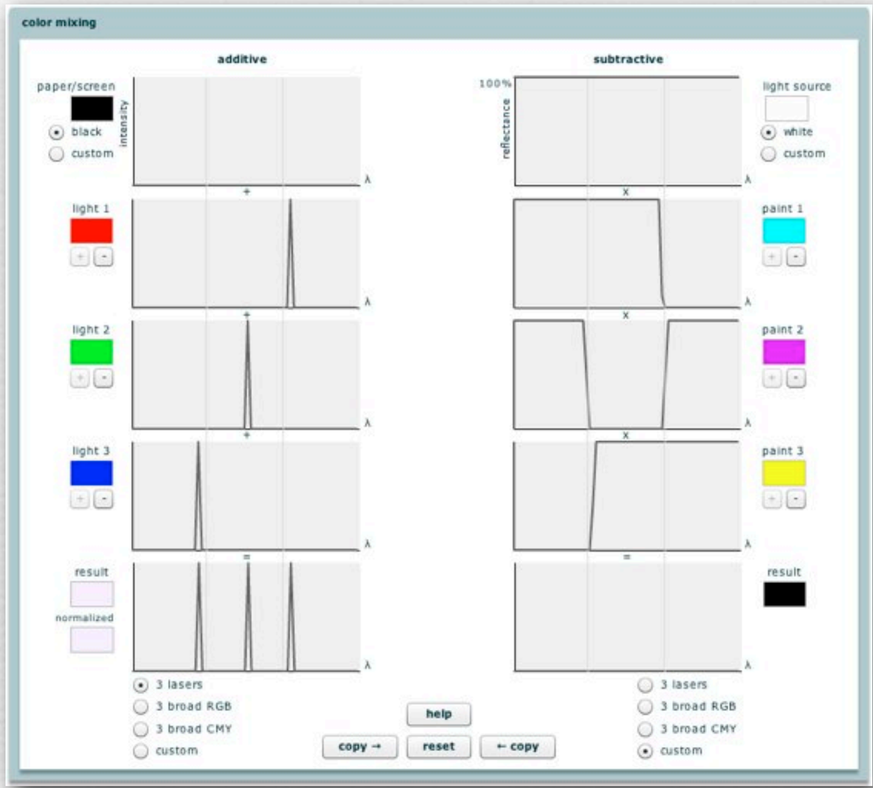
Additive versus subtractive mixing

(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/ColorMixing-narrowCMY.swf

- superimposed colored lights or small adjacent dots combine *additively* - by adding their spectra wavelength-by-wavelength
- layered dyes or sequenced color filters combine *subtractively* - by multiplying their transmittance spectra wavelength-by-wavelength

Additive versus subtractive mixing

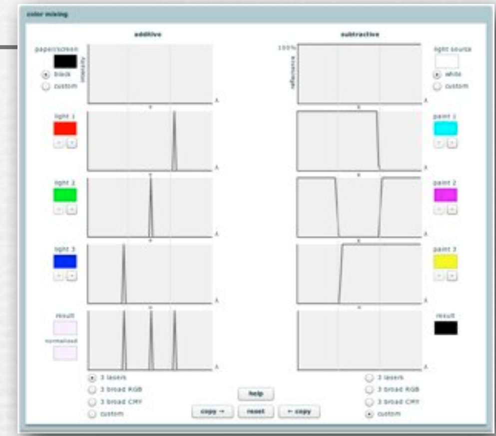


(FLASH DEMO)

http://graphics.stanford.edu/courses/cs178/applets/colormixing.html

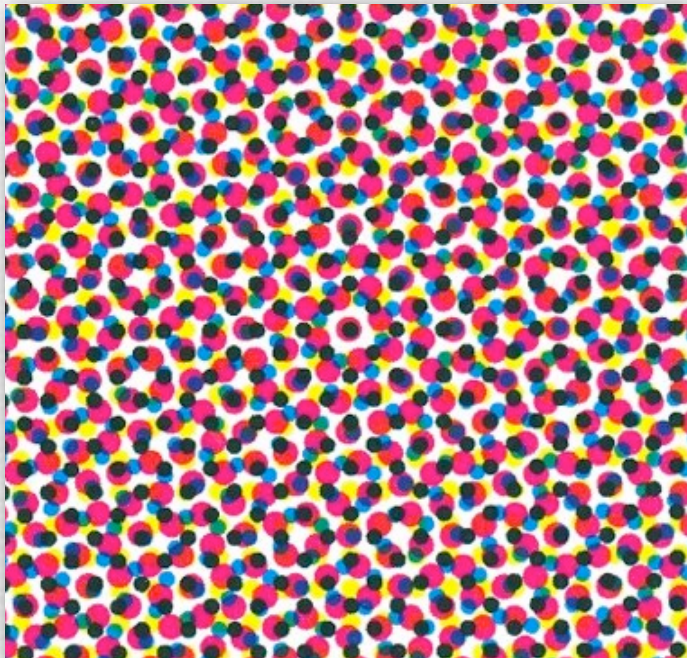
- superimposed colored lights or small adjacent dots combine *additively* - by adding their spectra wavelength-by-wavelength
- layered dyes or sequenced color filters combine *subtractively* - by multiplying their transmittance spectra wavelength-by-wavelength

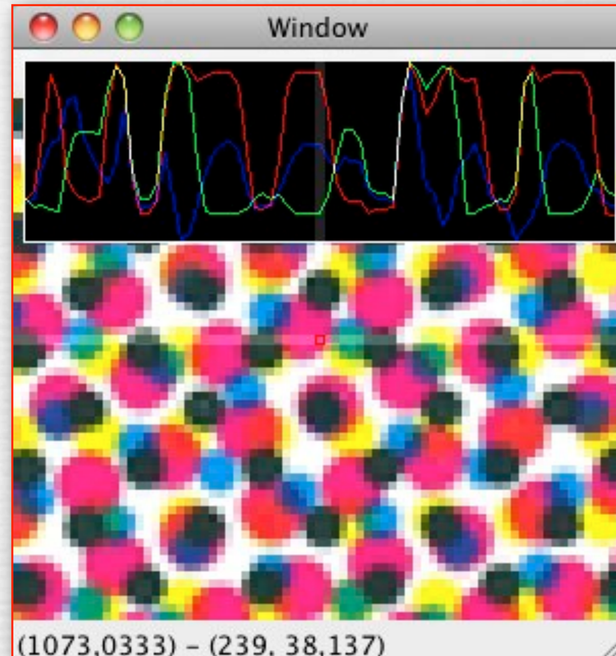
Additive versus subtractive mixing



- narrow spectra, widely spaced in wavelength, are best for primaries to be combined additively
- wide spectra that overlap are best for primaries to be combined subtractively, but product of all three must be black
- the particular spectra chosen are flexible; additive primaries need not be R,G,B, nor subtractive primaries C,M,Y
- additional primaries may be added to either system, resulting in a larger gamut of reproducible colors; adding black to a subtractive system (called CMYK) ensures a deep black
- note: additive mixing can be interpreted as interpolation between points in rho-gamma-beta space, but subtractive mixing cannot, because the two spectra must be multiplied together, not added

Color printing





- patches of the 3 subtractive primaries (C,M,Y) overlap partially on the page, making patches of 8 meta-primaries (Wh,C,M,Y,MY,CY,CM,CMY), which combine additively in the eye when viewed from a distance
  - M×Y=R, C×Y=G, C×M=B
  - these effects are modeled by the *Neugebauer equations*