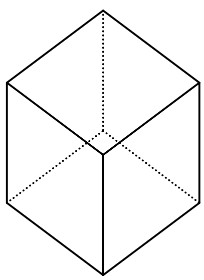
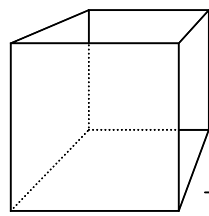
Solutions

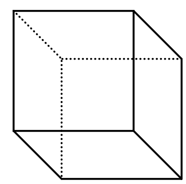
1. (12 points) Indicate which type of projection was used for each cube rendering. Lengths that appear equal in the renderings should be taken as being equal.
    a) Oblique parallel projection
    b) Orthographic projection
    c) Perspective projection



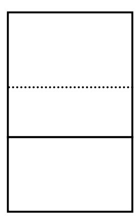
B



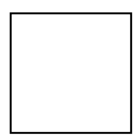
C



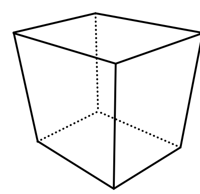
A



A



B



C

Grading: 2 points for each correct answer, 0 points for other answers.

Explanations, left to right, top to bottom:

a) B, orthographic: recognizable because parallel lines are still parallel yet the front face is tilted.

b) C, perspective: you can see the back face is smaller than the front one, and the horizontal edges along the sides are all receding towards a vanishing point.

c) A, oblique parallel: the front face is viewed as a square, without viewing the cube straight on (which, as in (e), is the only case where you'll get a true square from an orthographic projection). The X and Y axes are in the image plane, with the Z axis tilted up and left (in the –X, +Y direction).

d) A, oblique parallel: again, the front face is viewed as a square but other parts of the cube are still visible. The X and Y axes are in the image plane, with the Z axis tilted down (-Y direction). You could get *almost* this view with an appropriately rotated orthographic projection, but neither visible face would be a square as it is here.

e) B, Orthographic: with no rotations applied at all, the X and Y axes are in the normal directions in the image plane and the Z axis points into the screen. It can't be an oblique parallel projection because it's not oblique (no shear in Z); a perspective projection with no rotation would produce the same *visible* surfaces but the back face would project to a smaller square and would have shown up as hidden (dotted) lines in the rendering above.

f) C, perspective: easily recognized by the fact that the opposite edges of any face are not parallel, but converge on the vanishing points.

(Graded by Matt.)

2. (8 points) When rendering a scene that contains both opaque and semi-transparent polygons, what is the *weakest* constraint on drawing order that guarantees that the scene will be rendered correctly? Circle the best answer. Assume OpenGL rendering using depth buffering and, for the semi-transparent polygons, using alpha blending. Also assume non-interpenetrating polygons.

   a) all polygons (must be drawn) in back-to-front order
   b) opaque polygons in any order, then transparent polygons in any order
   c) opaque polygons in any order, then transparent polygons in back-to-front order
   d) transparent polygons in back-to-front order, then opaque polygons in any order
   e) transparent polygons in back-to-front order, opaque polygons interspersed in any order
   f) all polygons in any order

   Answer: The correct answer is c. Digital compositing is not commutative. Once the colors of two polygons have been combined using a compositing operator (such as OVER), there is no way to insert a new polygon between them and still produce the correct color. Thus, polygons for which compositing is relevant - semi-transparent polygons in this problem - must be drawn in order. Moreover, they must be drawn in order with respect to *all* polygons in the scene, whether opaque or semi-transparent. This reduces the choices to a or c.

   Some people said a. This answer will produce a correct image, so we gave 2 points for it, but it is not the *weakest* possible constraint. If the scene is divided into opaque and semi-transparent polygons, and the opaque polygons are drawn first, then they may be drawn in any order; Z-buffering is sufficient to resolve their intervisibility. Then, semi-transparent polygons may be drawn, and assuming they are drawn in back-to-front order, if one falls in front of the frontmost polygon at a pixel, as determined by the Z-buffer, then it can be blended OVER it, and the correct image will be produced. This answer, (c), is less constraining than (a), and thus it is the correct answer.

3. (8 points) Match the surface material to the goniometric diagram by writing the corresponding letter in a diagram's box. Not all surface materials will have a corresponding goniometric diagram. The goniometric diagrams are drawn for a single incoming light direction, $\vec{L}$, and all reflection directions in the plane of $\vec{L}$ and $\vec{N}$ (the surface normal). The diagrams plot radiance.

a) ideal mirror (such as a standard mirror)

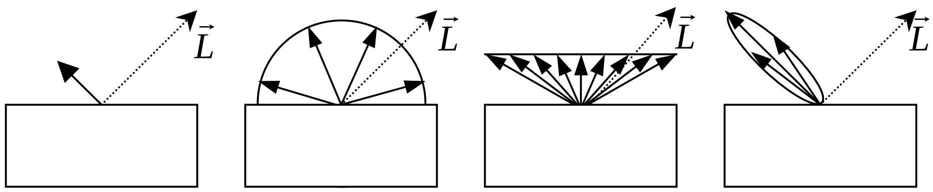
b) dark glossy surface (such as black plastic)

c) dark mirror (such as a half-silvered mirror on a black velvet background)

d) ideal diffuse / Lambertian (such as flat wall paint)

e) isotropic scatterer (such as a dusty surface)

f) retroreflective surface (outgoing direction = incoming, like a highway reflector)



Answer: From left to right: c, d, e, b.

In the first diagram, the reflected light is exactly in the mirror direction, without any light going in any other direction. This indicates a mirror surface of some kind. The amplitude of reflected light (as indicated by the length of the outgoing vector) is smaller than the amplitude of incoming light, so this suggests that the surface is not reflecting all of the light. Therefore, it cannot be the ideal mirror, so it must be the dark mirror.

In the second diagram, the light is reflected diffusely. The outgoing light is uniformly distributed in all directions. This is characteristic of Lambertian reflection: no matter what angle you view it at, it looks the same.

In the third diagram, the reflected light has greater amplitude at grazing angles. This is characteristic of an isotropic scatterer, where radiant intensity is constant for all view angles (and therefore radiance is greater at grazing angles).

In the fourth diagram, the reflected light is centered around the mirror direction but is not perfectly reflected in only that direction. Therefore, the highlight would be strongest at the mirror direction but would also be visible at angles close to the mirror direction. This is exactly what a glossy surface looks like.

Two points were awarded for each correct match.

(Graded by Yu.)

4. (8 points) Match the BRDF to the goniometric diagram by writing the corresponding letter in a diagram's box. Not all BRDFs will have a corresponding goniometric diagram. These goniometric diagrams are identical to those in the previous problem. $\vec{R}$ is the normalized mirror reflection direction, $\vec{V}$ is the normalized viewing direction, the k's are arbitrary constants, and $\delta(\vec{R})$ denotes a unit-amplitude reflection in direction $\vec{R}$.

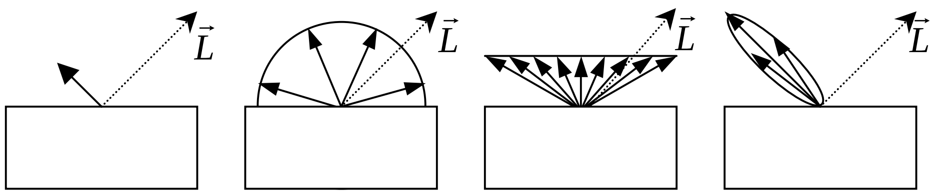
   a) $f_r = k_a\ (\vec{R} \bullet \vec{V})$

   b) $f_r = k_b\ (\vec{R} \bullet \vec{V})^4$

   c) $f_r = k_c\ /\ (\vec{N} \bullet \vec{V})$

   d) $f_r = k_d\ \delta(\vec{R})$

   e) $f_r = k_e$



Answer: From left to right: d, e, c, b.

As a reminder, the BRDF is the ratio of reflected radiance to incoming irradiance. In terms of these diagrams, the correct equation is simply the one that mathematically matches the shape in the diagram.

In the first diagram, the reflection is exactly in the mirror reflection direction, but not necessarily the same amplitude as the incoming light, so the BRDF will be some constant multiplied by δ. This corresponds to answer d.

In the second diagram, the reflected light is equal in all directions. Therefore, the BRDF must be constant in all directions. The BRDF in e is a constant.

In the third diagram, the reflected light is greater when the surface is viewed at grazing angles. That is, when $\vec{N} \bullet \vec{V}$ is small (almost at right angles), the amount of reflected light is large. Therefore, the BRDF must depend on 1 / ($\vec{N} \bullet \vec{V}$), i.e. choice c.

In the fourth diagram, the amount of reflected light is greatest at the mirror direction but falls off quickly when the viewing angle is not equal to the reflected angle. This means that there is some dependence on a power of ($\vec{R} \bullet \vec{V}$) (the Phong approximation for specular reflection). In order to distinguish between choices a and b, you should realize that if the answer were a, the shape of the reflection would be a circle. The greater the exponent, the "thinner" the shape of the reflection. Therefore, the answer must be b.

(9 points) As every scout knows, you can start a fire on a sunny day by holding a magnifying glass between the sun and a piece of paper placed on the ground.

f) Is the radiance of the sun as seen from the focal point of the lens *more*, *less*, or the *same* as the radiance as seen from the same point in the absence of the magnifying glass? Circle the correct choice.

g) Is the irradiance due to the sun at the focal point *more*, *less*, or the *same* as the irradiance at the same point in the absence of the magnifying glass? Circle the correct choice.

h) Is the radiant flux due to the sun across the piece of paper *more*, *less*, or the *same* as the radiant flux at the same point in the absence of the magnifying glass? Circle the correct choice. Assume the piece of paper is larger than the magnifying glass

Answer: same, more, same. Three points were given for each correct answer.

Explanation: Radiance measures flux per unit solid angle per unit projected area. Placing a magnifying glass between the paper and the sun does not increase the apparent brightness (radiance) of the sun as seen from the paper at the focal point. It does, however, increase the apparent size of the sun, which leads to more deposition of energy (irradiance) on the paper.

More specifically, radiance measures flux per unit solid angle per unit projected area. Inserting the magnifying glass does not increase flux per unit solid angle; it merely increases the solid angle. If you don't believe this, try looking at a not-too-bright light source (a diffused fluorescent light works well) through a magnifying glass. Note that the light source appears larger. This reflects the increase in solid angle. Now place a small aperture between your eye and the magnifying glass (make a barrel with your fist). Think of this as a ``unit solid angle''. Remove the magnifying glass. Does the brightness coming through the aperture change? It doesn't. This shows the constancy of radiance.

For part (c), it is useful to think of the flux traveling from the sun to the piece of paper as a bundle of (near-) parallel rays. Assuming that the magnifying glass does not extend beyond the boundaries of the piece of paper (relative to the direction of these lines), then the presence of the magnifying glass neither increases nor decreases the number of lines in this bundle; it merely gathers a subset of them together and focuses them to a point on the paper. This causes some parts of the paper to receive more irradiance and others less, but the total number of lines (hence the total flux) is the same.

Warning: Never look at the sun directly, never hold a magnifying glass between your eye and the sun, and never look directly at its focused spot on a reflective surface.