Reflection Models

Today

- Types of reflection models
- The BRDF
- The reflection equation
- Ideal reflection and refraction
- Ideal diffuse

Thursday

- Glossy and specular reflection models
- Rough surfaces and microfacets
- Self-shadowing
- Anisotropic reflection models

Questions

1. How is light measured? *Radiometry*
2. How is the spatial distribution of light energy described? *Radiance*
3. How is reflection from a surface characterized?
4. What are the conditions for equilibrium flow of light in an environment?

Reflection Models

Definition: Reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident side without change in frequency.

Properties

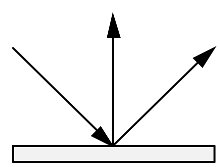
- Spectra and Color
- Polarization
- Directional distribution

Theories

- Phenomenological
- Physical

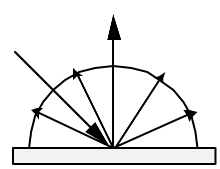
Types of Reflection Functions

Ideal Specular



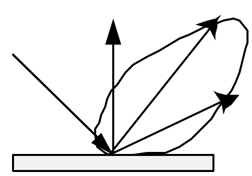
- Reflection Law
- Mirror

Ideal Diffuse



- Lambert's Law
- Matte

Specular



- Glossy
- Directional diffuse

Materials

Plastic

Rough Metal

Matte

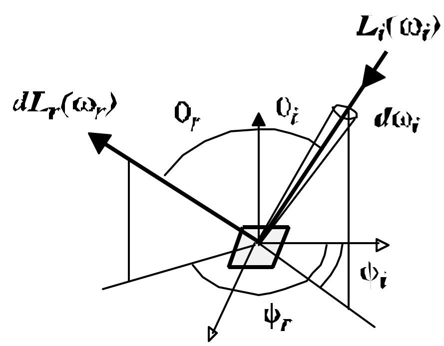
From Apodaca and Gritz, *Advanced RenderMan*

The BRDF

Bidirectional Reflectance-Distribution Function (BRDF)

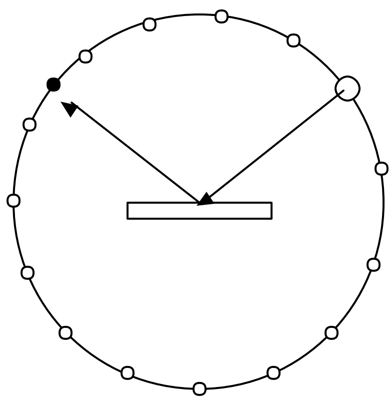
$$d\Phi_i = L_i(w_i)\cos q_i \, dw_i \, dA$$

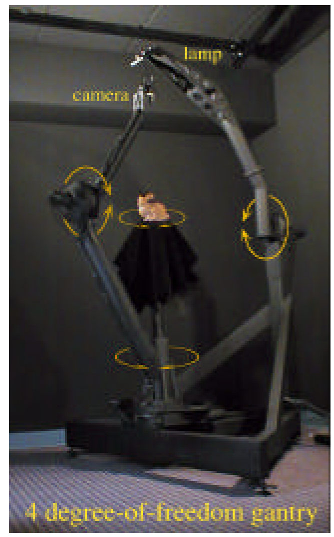
$$d^2\Phi_r = dL_r(w_r)\cos q_r \, dw_r \, dA$$



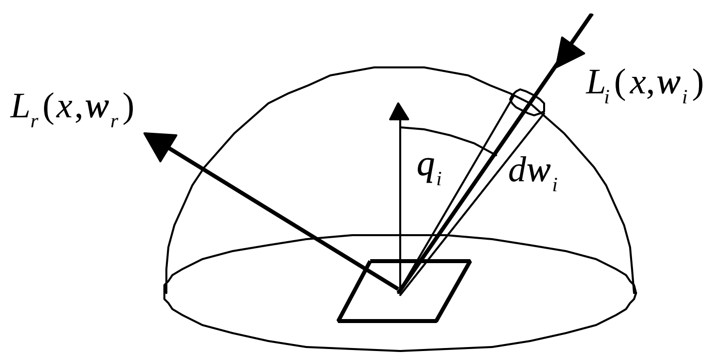
$$f_r(w_i \rightarrow w_r) \equiv \frac{dL_r(w_i \rightarrow w_r)}{dE_i} = \frac{dL_r(w_i \rightarrow w_r)}{L_i(w_i)\cos q_i \, dw_i} \quad \left[\frac{1}{sr}\right]$$

Spherical Goniometer





The Reflection Equation



$$L_r(x,w_r) = \int_{H^2} f_r(x,w_i \to w_r)\, L_i(x,w_i) \cos q_i \, dw_i$$

Note: Point and distant light sources are delta functions
Note: Area light sources with constant radiance may be pulled out

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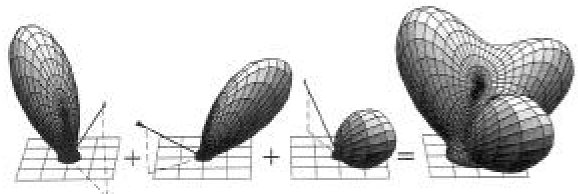
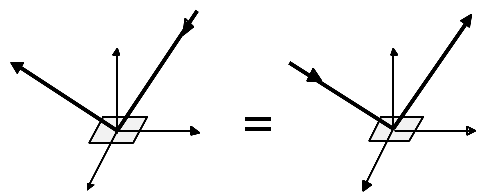
Properties of BRDF's

1. Linearity



From Sillion, Arvo, Westin, Greenberg

2. Reciprocity principle $f_r(w_i \to w_r) = f_r(w_r \to w_i)$



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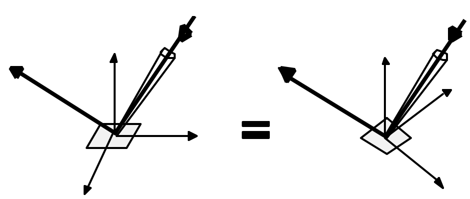
Properties of BRDF's

3. Isotropic vs. anisotropic $f_r(q_i, j_i; q_r, j_r) = f_r(q_i, q_r, j_r - j_i)$



Reciprocity and isotropy

$$f_r(q_i, q_r, j_r - j_i) = f_r(q_r, q_i, j_i - j_r) = f_r(q_i, q_r, |j_r - j_i|)$$

4. Energy conservation

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The Reflectance

Definition: A reflectance is a ratio of reflected to incident power

$$r_r(w_i \rightarrow w_r) \equiv \frac{d\Phi_r}{d\Phi_i} = \frac{\int_{H^2} L_r(w_r)\cos q_r \, dw_r}{\int_{H^2} L_i(w_i)\cos q_i \, dw_i}$$

$$= \frac{\int_{H^2}\int_{H^2} f_r(w_i \rightarrow w_r)\cos q_i \, dw_i \cos q_r \, dw_r}{\int_{H^2} \cos q_i \, dw_i}$$

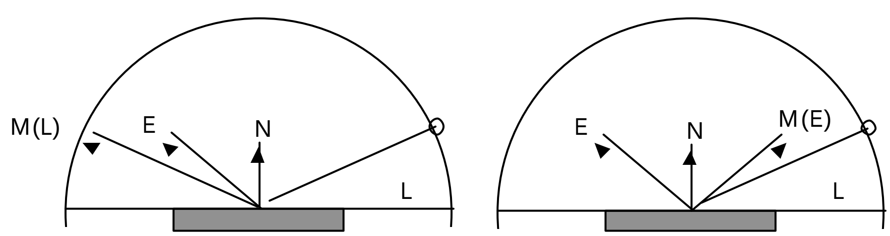
Derivation assumes uniform incident radiance

All experiments measure reflectances

Conservation of energy: 0 £ r £ 1 vs. 0 £ f_r £ ¥

Units: r [dimensionless], f_r [1/steradians]

Phong Model: Distributed Light Source



$$\left(\hat{\mathbf{E}} \bullet \mathbf{R}(\hat{\mathbf{L}})\right)^s \qquad \left(\hat{\mathbf{L}} \bullet \mathbf{R}(\hat{\mathbf{E}})\right)^s$$

Properties of the Phong Model

Normalize Phong Model

$$r(2p \rightarrow w_r) = \int_{H^2} \left(\hat{\mathbf{L}} \bullet \mathbf{R}(\hat{\mathbf{E}})\right)^s \cos q_i \, dw_i$$

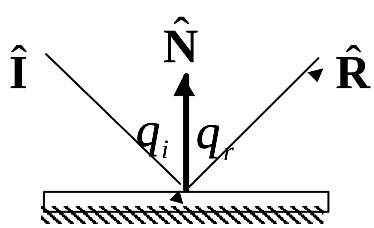
$$\leq \int_{H^2} \left(\hat{\mathbf{L}} \bullet \hat{\mathbf{N}}\right)^s \cos q_i \, dw_i$$

$$\leq \int_{H^2} \cos^{s+1} q_i \, dw_i = \frac{2p}{s+2}$$

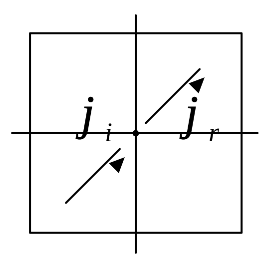
Reciprocal

$$\left(\hat{\mathbf{E}} \bullet \mathbf{R}(\hat{\mathbf{L}})\right)^s = \left(\hat{\mathbf{L}} \bullet \mathbf{R}(\hat{\mathbf{E}})\right)^s$$

Law of Reflection



$$q_r = q_i$$

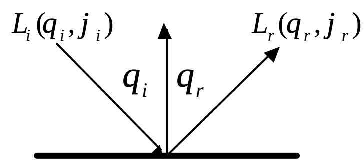


$$j_r = j_i \pm p$$

$$\hat{\mathbf{R}} + (-\hat{\mathbf{I}}) = 2\cos q \; \hat{\mathbf{N}} = -(\hat{\mathbf{I}} \bullet \hat{\mathbf{N}})\hat{\mathbf{N}}$$

$$\hat{\mathbf{R}} = \hat{\mathbf{I}} - (\hat{\mathbf{I}} \bullet \hat{\mathbf{N}})\hat{\mathbf{N}}$$

Ideal Reflection (Mirror)



$$L_{r,m}(q_r, j_r) = L_i(q_i, j_i \pm p)$$

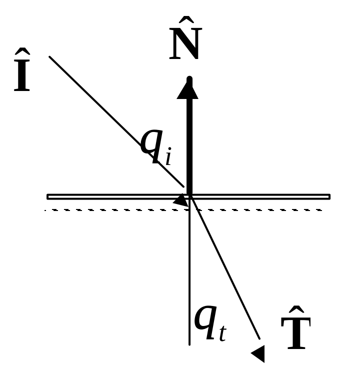
$$L_{r,m}(q_r, j_r) = \int f_{r,m}(q_i, j_i; q_r, j_r) L_i(q_i, j_i) \cos q_i \, d\cos q_i \, dj_i$$

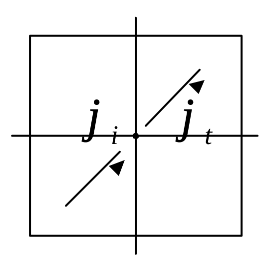
$$= \int \frac{d(\cos q_i - \cos q_r)}{\cos q_i} d(j_i - j_r \pm p) L_i(q_i, j_i) \cos q_i \, d\cos q_i \, dj_i$$

$$= L_i(q_i, j_i \pm p)$$

$$f_{r,m}(q_i, j_i; q_r, j_r) = \frac{d(\cos q_i - \cos q_r)}{\cos q_i} d(j_i - j_r \pm p)$$

Snell’s Law





$$j_t = j_i \pm p$$

$$n_i \sin q_i = n_t \sin q_t$$

$$n_i \hat{\mathbf{N}} \times \hat{\mathbf{T}} = n_t \hat{\mathbf{N}} \times \hat{\mathbf{I}}$$

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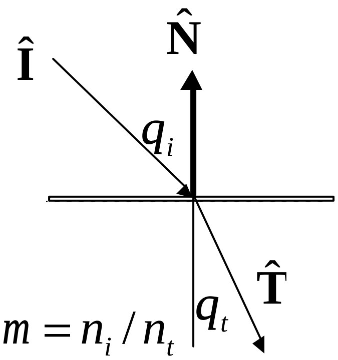
Law of Refraction



$$m = n_i / n_t$$

$$\hat{\mathbf{N}} \times \hat{\mathbf{T}} = m\, \hat{\mathbf{N}} \times \hat{\mathbf{I}}$$

$$\hat{\mathbf{N}} \times (\hat{\mathbf{T}} - m\, \hat{\mathbf{I}}) = 0$$

$$\hat{\mathbf{T}} = m\, \hat{\mathbf{I}} + g\, \hat{\mathbf{N}}$$

$$\hat{\mathbf{T}}^2 = 1 = m^2 + g^2 + 2mg\; \hat{\mathbf{I}} \bullet \hat{\mathbf{N}}$$

$$g = -m\, \hat{\mathbf{I}} \bullet \hat{\mathbf{N}} \pm \left\{1 - m^2\left(1 - \left(\hat{\mathbf{I}} \bullet \hat{\mathbf{N}}\right)^2\right)\right\}^{1/2}$$

$$= m \cos q_i \pm \left\{1 - m^2 \sin^2 q_i\right\}^{1/2}$$

$$= m \cos q_i \pm \cos q_t$$

$$= m \cos q_i - \cos q_t$$

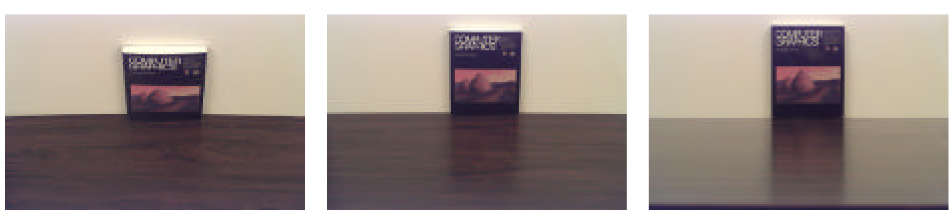
$$\leftarrow g = m - 1$$

Total internal reflection:

$$1 - m^2 (1 - (\hat{\mathbf{I}} \bullet \hat{\mathbf{N}})^2) < 0$$

Examples

Reflections from a shiny floor



From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

Fresnel Equations

Dielectrics (Two polarizations)

$$R_\perp = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} \qquad R_{\|} = \frac{n_1 \cos\theta_2 - n_2 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1}$$

$$T_\perp = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2} \qquad T_{\|} = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1}$$

Metals $n + ik$

$$a^2 + b^2 = n^2(1 - k^2) - \sin^2\theta$$

$$R = \frac{a^2 + b^2 - 2a\cos\theta + \cos^2\theta}{a^2 + b^2 + 2a\cos\theta + \cos^2\theta}$$

$$T = \frac{a^2 + b^2 - 2a\sin\theta\tan\theta + \sin^2\theta\tan^2\theta}{a^2 + b^2 + 2a\sin\theta\tan\theta + \sin^2\theta\tan^2\theta} R$$

Fresnel Equations

Normal incidence

- Dielectrics

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$$

Glass: ***n***=1.5 ***R***=0.04
Diamond: ***n***=2.4 ***R***=0.15

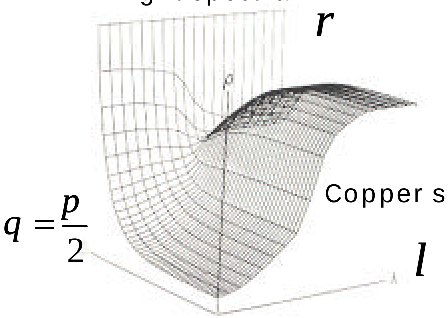
- Metals

$$R = \left(\frac{(n-1)^2 + n^2k^2}{(n+1)^2 + n^2k^2}\right)^2$$

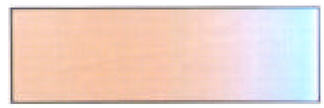
Silver: ***n***<1, k=1 ***R***=0.95
Gold: ***n***<1, k=1 ***R***=0.82

Solve for ***n*** given ***R*** at normal incidence

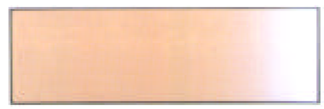
Cook-Torrance Model for Metals



Reflectance of Copper as a function of wavelength and angle of incidence



Measured Reflectance of Copper

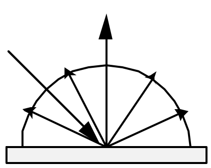


Approxmated Reflectance of Copper

$$R = R(0) + R(\pi/2)\left[\frac{\overline{R}(\theta) - \overline{R}(0)}{\overline{R}(\pi/2) - \overline{R}(0)}\right]$$

Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction (independent of input direction).



$$L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i)\cos\theta_i \, d\omega_i$$
$$= f_{r,d} \int L_i(\omega_i)\cos\theta_i \, d\omega_i$$
$$= f_{r,d} E$$

$$B = \int L_r(\omega_r)\cos\theta_r \, d\omega_r = L_r \int \cos\theta_r \, d\omega_r = \pi L_r$$

$$\rho_d = \frac{B}{E} = \pi f_{r,d}$$

Lambert's Cosine Law $B = \rho_d E = \rho_d E_s \cos\theta_s$

“Diffuse” Reflection

Theoretical

- Bouguer - Special micro-facet distribution
- Seeliger - Subsurface reflection
- Multiple surface or subsurface reflections

Experimental

- Pressed magnesium oxide powder
- Almost never valid at high angles of incidence

Paint manufactures attempt to create ideal diffuse