Reflection Models

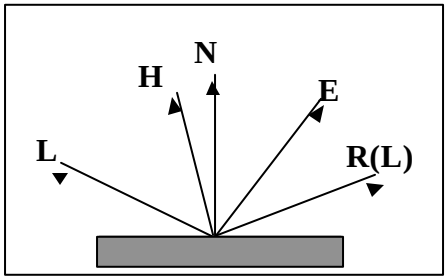
Tuesday

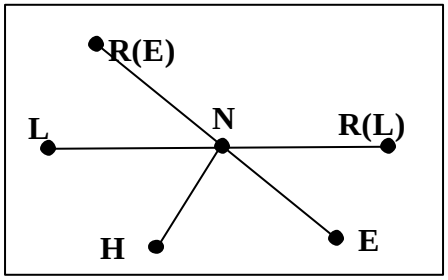
- Reflection models
- The reflection equation and the BRDF
- Ideal reflection, refraction and diffuse

Today

- Glossy reflection models
- Rough surfaces
- Microfacets
- Self-shadowing
- Anisotropic reflection models

Reflection Geometry





$$\cos q_i = \vec{\mathbf{L}} \bullet \vec{\mathbf{N}}$$

$$\cos q_r = \vec{\mathbf{R}} \bullet \vec{\mathbf{N}}$$

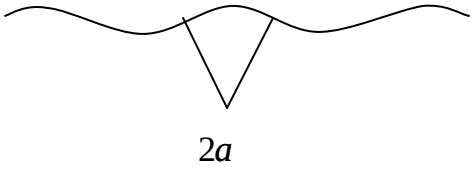
$$\cos q_g = \vec{\mathbf{E}} \bullet \vec{\mathbf{L}}$$

$$\cos q_s = \vec{\mathbf{E}} \bullet \mathbf{R}(\vec{\mathbf{L}}) = \mathbf{R}(\vec{\mathbf{E}}) \bullet \vec{\mathbf{L}}$$

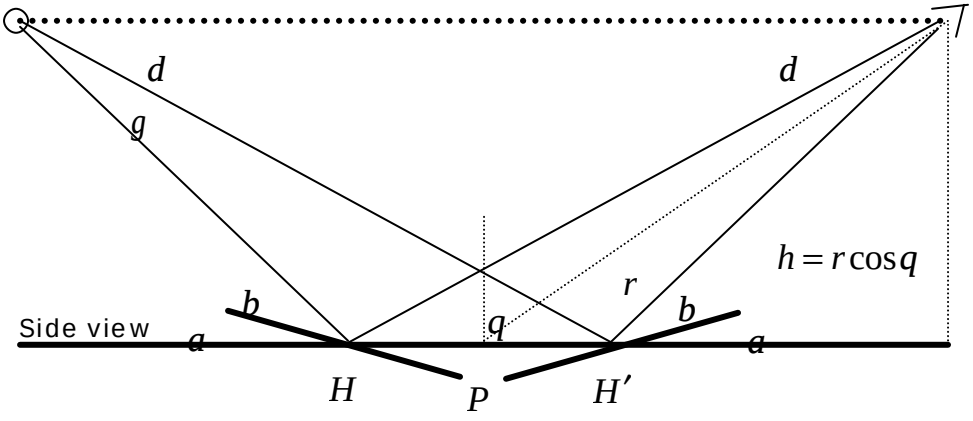
$$\cos q_{s'} = \vec{\mathbf{H}} \bullet \vec{\mathbf{N}}$$

Reflection of the Sun from the Sea

Minnaert p. 28



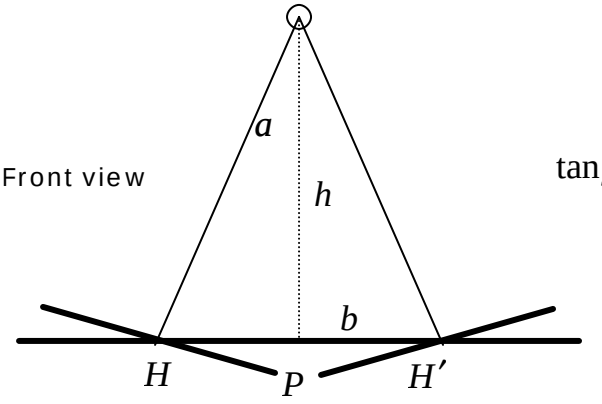
Reflection Angles



$$a + b = g + d \qquad b - a = d$$

$$g = 2a$$

Reflection Angles

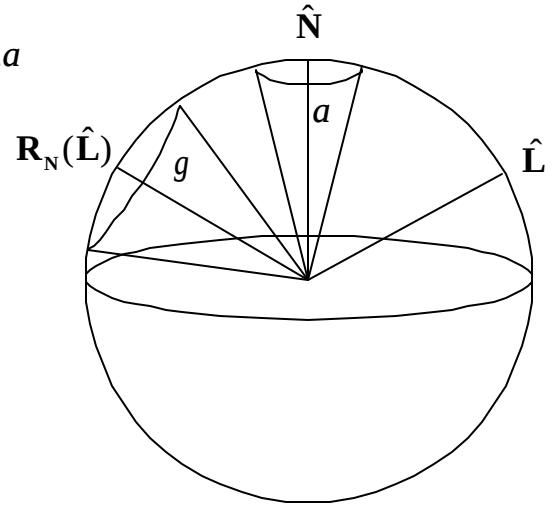


$$\tan y = \frac{b}{r} = \tan a \cos q$$

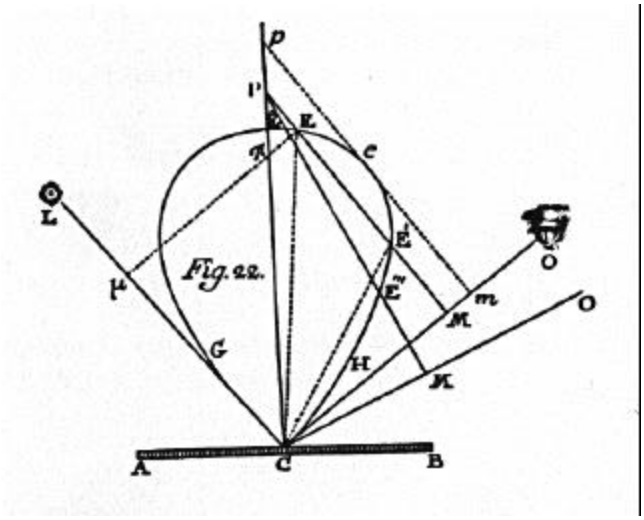
$$\tan a = \frac{b}{h}$$

Analysis on the Sphere

$g = 2a$

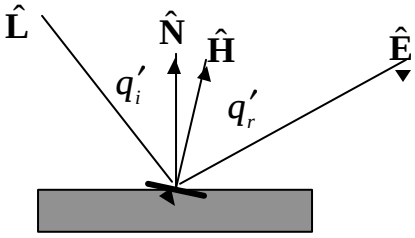


Bouguer's "little faces"



P. Bouguer, *Treatise on Optics*, 1760

Torrance-Sparrow Model



$$d^2\Phi_h = L_i(w_i)\cos q_i' dw_i\, dA'$$
$$= L_i(w_i)\cos q_i' dw_i\, D(w_h)\, dw_h\, dA$$

$$d^2\Phi_r = d^2\Phi_m$$

$$dA' = D(w_h)\, dw_h\, dA$$

$$d^2\Phi_r = dL_r(w_i \to w_r)\cos q_r\, dw_r\, dA$$

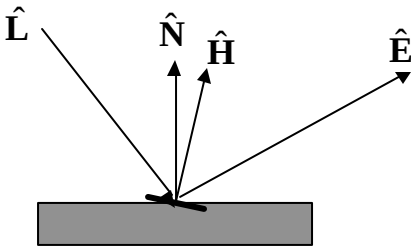
$$\cos q_i = \hat{\mathbf{L}} \bullet \hat{\mathbf{N}}$$

$$\cos q_i' = \hat{\mathbf{L}} \bullet \hat{\mathbf{H}}$$

$$dL_r(w_i \to w_r)\cos q_r\, dw_r\, dA$$
$$= L_i(w_i)\cos q_i' dw_i\, D(w_h)\, dw_h\, dA$$

$$dw_i' = dw_i$$

Torrance-Sparrow Model



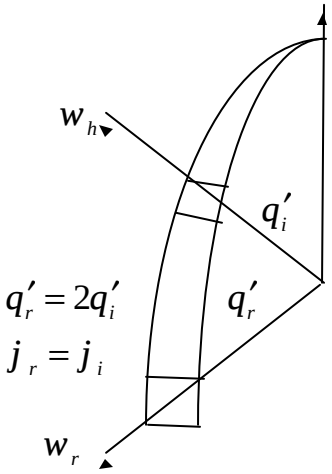
$$f_r(w_i \to w_r) = \frac{dL_r(w_i \to w_r)}{dE_i(w_i)}$$

$$= \frac{L_i(w_i)\cos q_i' \, dw_i D(w_h) \, dw_h \, dA}{(\cos q_r \, dw_r \, dA)(L_i(w_i)\cos q_i \, dw_i)}$$

$$= \frac{D(w_h)}{\cos q_i \cos q_r} \cos q_i' \frac{dw_h}{dw_r}$$

$$= \frac{D(w_h)}{4\cos q_i \cos q_r}$$

Solid Angle Distributions



$$dw_r = \sin q_r' \, dq_r' \, dj_r$$

$$= (\sin 2q_i')2dq_i' \, dj_i$$

$$= (2\sin q_i' \cos q_i')2dq_i' \, dj_i$$

$$= 4\cos q_i' dw_i$$

$$\frac{dw_h}{dw_r} = \frac{1}{4\cos q_i'}$$

Normalizing Microfacet Distributions

$$dA' = D(w_h)\, dw_h\, dA$$

$$\int_{H^2} \cos q_h\, dA' = \int_{H^2} D(w_h)\, dw_h\, dA = dA$$

$$\int_{H^2} D(w_h) \cos q_h\, dw_h = 1$$

Microfacet Distribution Functions

Isotropic distributions $D(w) = D(a)$

Characterize by half-angle $D(b) = \frac{1}{2}$

Examples:

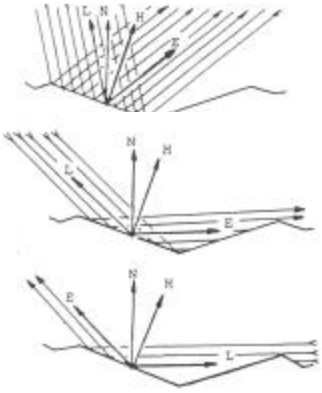
- Blinn $D_1(a) = \cos^{c_1} a$ $\quad c_1 = \frac{\ln 2}{\ln \cos b}$
- Torrance-Sparrow $D_2(a) = e^{-(c_2 a)^2}$ $\quad c_2 = \frac{\sqrt{2}}{b}$
- Trowbridge-Reitz $D_3(a) = \frac{c_3^2}{(1 - c_3^2)\cos^2 a - 1}$ $\quad c_3 = \left(\frac{\cos^2 b - 1}{\cos^2 b - \sqrt{2}} \right)^{1/2}$

Self-Shadowing: V-Groove Model

Assumptions (Torrance-Sparrow)

1. Symmetric, longitudinal, isotropically-distributed
2. Upper edges lie in plane

$$G = \min(G_a, G_b, G_c)$$

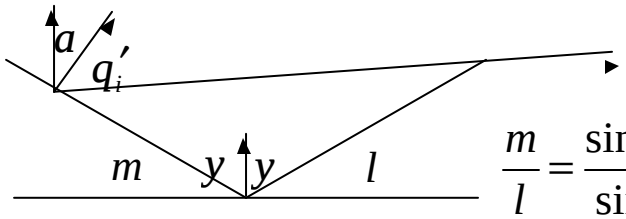


$$G_a = 1$$

$$G_b = \frac{2(\mathbf{N}\bullet\mathbf{H})(\mathbf{N}\bullet\mathbf{E})}{(\mathbf{H}\bullet\mathbf{E})}$$

$$G_c = \frac{2(\mathbf{N}\bullet\mathbf{H})(\mathbf{N}\bullet\mathbf{L})}{(\mathbf{H}\bullet\mathbf{L})}$$

Self-Shadowing: V-Groove Model



$$\frac{m}{l} = \frac{\sin m}{\sin l}$$

$$\sin l = \cos q_i'$$
$$\cos l = \sin q_i'$$
$$\sin y = \cos a$$
$$\cos y = \sin a$$

$$\begin{aligned}
\sin m &= \sin l + 2y \\
&= \sin l \cos 2y + \cos l \sin 2y \\
&= \cos q_i' \cos 2y + \sin q_i' \sin 2y \\
&= \cos q_i' (1 - 2\sin^2 y) + \sin q_i' 2\cos y \sin y \\
&= \cos q_i' (1 - 2\cos^2 a) + \sin q_i' 2\cos a \sin a \\
&= \cos q_i' - 2\cos a (\cos a \cos q_i' - \sin a \sin q_i') \\
&= \cos q_i' - 2\cos a \cos(a + q_i') \\
&= \cos q_i' - 2\cos a \cos q_r \\
&= \mathbf{H}\bullet\mathbf{E} - 2(\mathbf{N}\bullet\mathbf{H})(\mathbf{N}\bullet\mathbf{E})
\end{aligned}$$

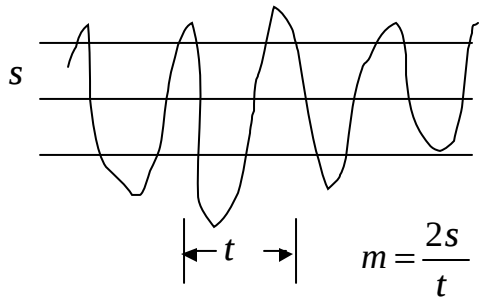
$$\begin{aligned}
G &= 1 - \frac{m}{l} \\
&= 1 - \frac{\sin m}{\sin l} \\
&= \frac{\mathbf{H}\bullet\mathbf{E} - \mathbf{H}\bullet\mathbf{E} + 2(\mathbf{N}\bullet\mathbf{H})(\mathbf{N}\bullet\mathbf{E})}{\mathbf{H}\bullet\mathbf{E}} \\
&= \frac{2(\mathbf{N}\bullet\mathbf{H})(\mathbf{N}\bullet\mathbf{E})}{\mathbf{H}\bullet\mathbf{E}}
\end{aligned}$$

Gaussian Rough Surface

Beckmann

$$p(z) = \frac{1}{\sqrt{2p}\,s} e^{-\frac{z^2}{2s^2}}$$

$$D(a) = \frac{1}{\sqrt{p}\,m^2 \cos^2 a} e^{-\frac{\tan^2 a}{m^2}}$$



$$m = \frac{2s}{t}$$

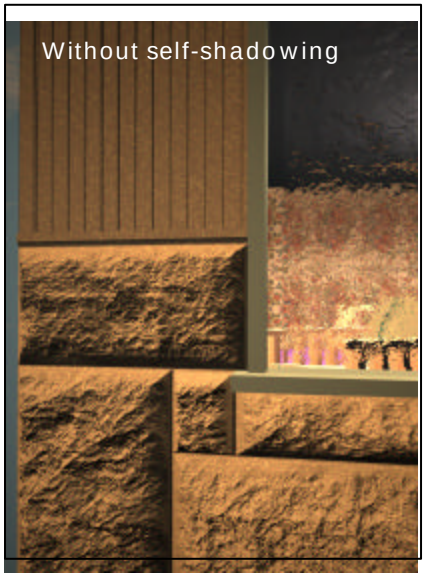
Smith

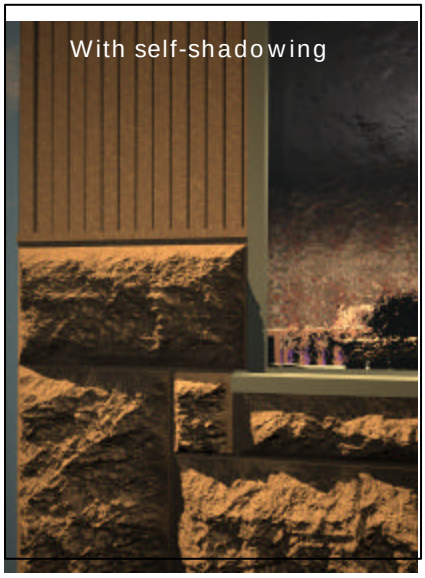
Derives shadowing function probabilistically

Self-consistency condition

The sum of the areas of the illuminated surface projected onto the plane normal to the direction of incidence is independent of the roughness of the surface, and equal to the projected area of the underlying mean plane.

Shadows on Rough Surfaces

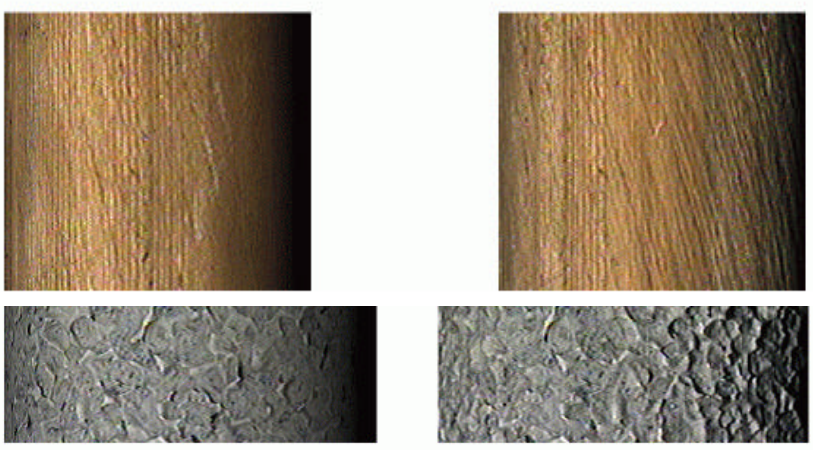




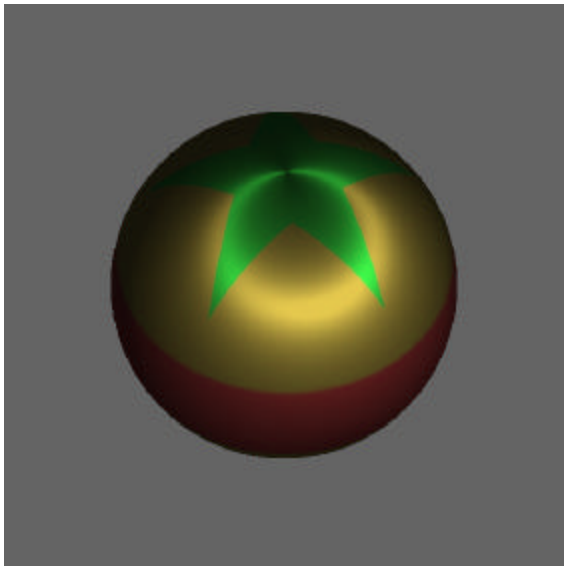
S. Nayar's BTF Experiments

Complex interplay between texture and brdf

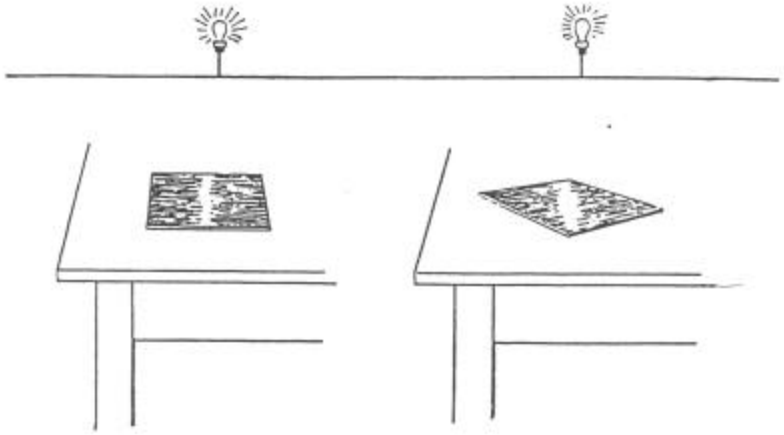
Self-shadowing a major effect



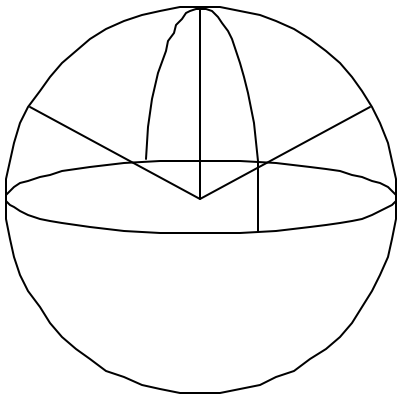
Anisotropic Reflection



Anisotropic Reflection



Anisotropic Reflection



Anisotropic Reflection