Reflection Models

Today

- Types of reflection models
- The BRDF and reflectance
- The reflection equation
- Ideal reflection and refraction
- Fresnel effect
- Ideal diffuse

Thursday

- Glossy and specular reflection models
- Rough surfaces and microfacets
- Self-shadowing
- Anisotropic reflection models

Reflection Models

Definition: Reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident side without change in frequency.

Properties

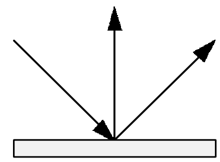
- Spectra and Color [Moon Spectra]
- Polarization
- Directional distribution

Theories

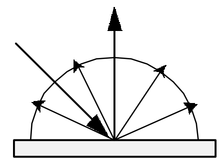
- Phenomenological
- Physical

Types of Reflection Functions

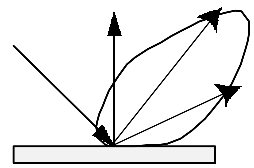
Ideal Specular
- Reflection Law
- Mirror



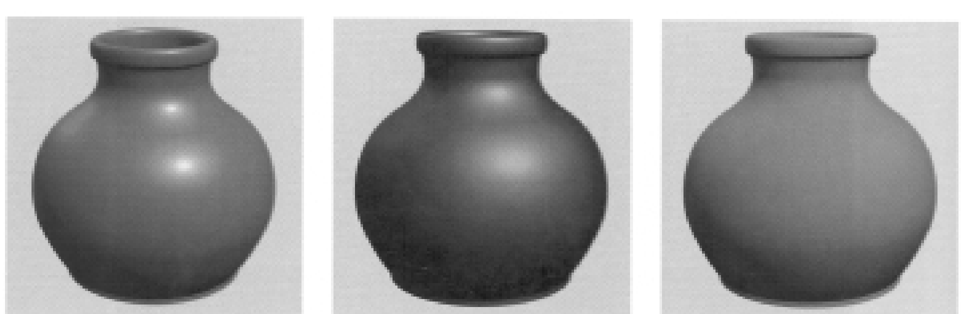
Ideal Diffuse
- Lambert's Law
- Matte



Specular
- Glossy
- Directional diffuse



Materials

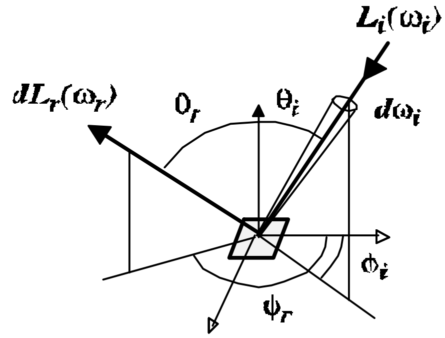


Plastic Metal Matte

Source: Apodaca and Gritz, *Advanced RenderMan*

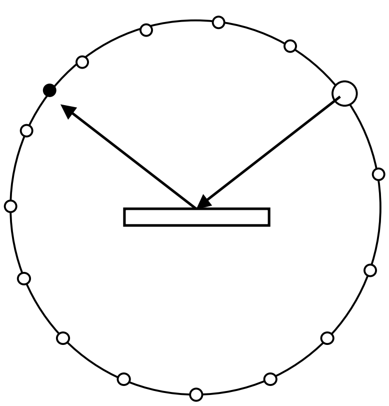
The BRDF

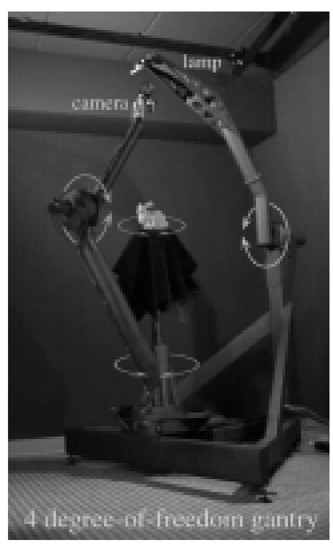
Bidirectional Reflectance-Distribution Function (BRDF)



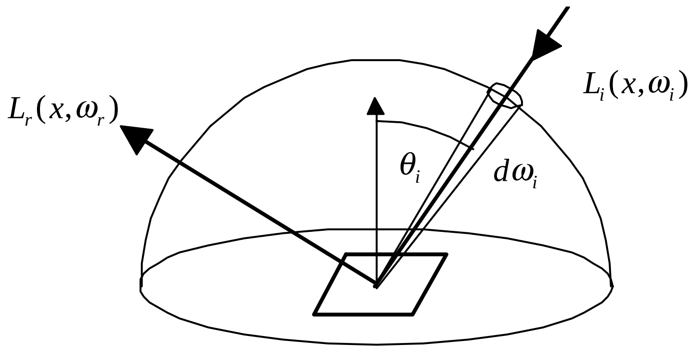
$$f_r(\omega_i \to \omega_r) \equiv \frac{dL_r(\omega_i \to \omega_r)}{dE_i} = \frac{dL_r(\omega_i \to \omega_r)}{L_i(\omega_i)\cos\theta_i \, d\omega_i} \left[\frac{1}{sr}\right]$$

Gonioreflectometer





The Reflection Equation



$$L_r(x,\omega_r) = \int_{H^2} f_r(x,\omega_i \to \omega_r) L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$

Note: Point and distant light sources are delta functions

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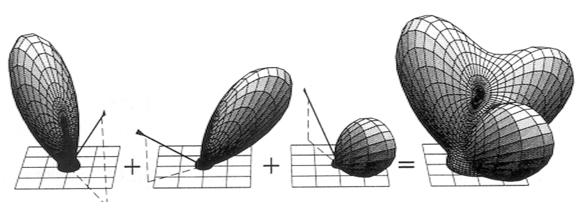
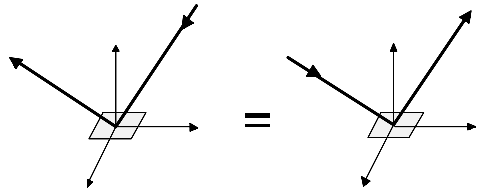
Properties of BRDF's

1. Linearit



Source: Sillion, Arvo, Westin, Greenberg

2. Reciprocity principle $f_r(\omega_i \to \omega_r) = f_r(\omega_r \to \omega_i)$



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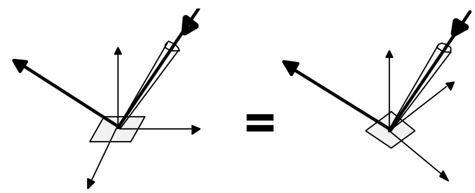
Properties of BRDF's

3. Isotropic vs. anisotropic $f_r(\theta_i,\varphi_i;\theta_r,\varphi_r) = f_r(\theta_i,\theta_r,\varphi_r-\varphi_i)$



Reciprocity *and* isotropy

$$f_r(\theta_i,\theta_r,\varphi_r-\varphi_i) = f_r(\theta_r,\theta_i,\varphi_i-\varphi_r) = f_r(\theta_i,\theta_r,|\varphi_r-\varphi_i|)$$

4. Energy conservation

The Reflectance

Definition: A reflectance is a ratio of reflected to incident power

$$\rho_r(\Omega_i \to \Omega_r) \equiv \frac{d\Phi_r}{d\Phi_i} = \frac{\int_{\Omega_r} L_r(\omega_r)\cos\theta_r\, d\omega_r}{\int_{\Omega_i} L_i(\omega_i)\cos\theta_i\, d\omega_i}$$

$$= \frac{\int_{\Omega_r}\int_{\Omega_i} f_r(\omega_i \to \omega_r)\cos\theta_i\, d\omega_i \cos\theta_r\, d\omega_r}{\int_{\Omega_i} \cos\theta_i\, d\omega_i}$$

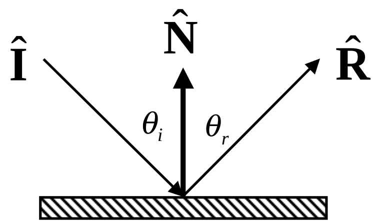
Derivation assumes uniform incident radiance

Experiments measure reflectances

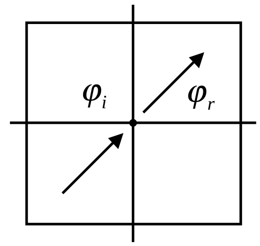
Conservation of energy: 0 < *r* < 1 vs. 0 < f_r < ∞

Units: ρ [dimensionless], f_r [1/steradians]

Law of Reflection



$$\theta_r = \theta_i$$

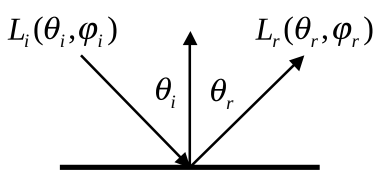


$$\varphi_r = \varphi_i \pm \pi$$

$$\hat{\mathbf{R}} + (-\hat{\mathbf{I}}) = 2\cos\theta\, \hat{\mathbf{N}} = -2(\hat{\mathbf{I}} \bullet \hat{\mathbf{N}})\hat{\mathbf{N}}$$

$$\hat{\mathbf{R}} = \hat{\mathbf{I}} - 2(\hat{\mathbf{I}} \bullet \hat{\mathbf{N}})\hat{\mathbf{N}}$$

Ideal Reflection (Mirror)

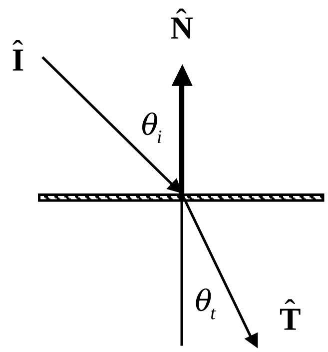


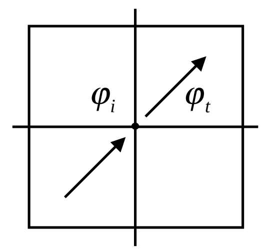
$$L_{r,m}(\theta_r, \varphi_r) = L_i(\theta_r, \varphi_r \pm \pi)$$

$$f_{r,m}(\theta_i, \varphi_i; \theta_r, \varphi_r) = \frac{\delta(\cos\theta_i - \cos\theta_r)}{\cos\theta_i}\delta(\varphi_i - \varphi_r \pm \pi)$$

$$\begin{aligned} L_{r,m}(\theta_r, \varphi_r) &= \int f_{r,m}(\theta_i, \varphi_i; \theta_r, \varphi_r) L_i(\theta_i, \varphi_i)\cos\theta_i \, d\cos\theta_i \, d\varphi_i \\ &= \int \frac{\delta(\cos\theta_i - \cos\theta_r)}{\cos\theta_i}\delta(\varphi_i - \varphi_r \pm \pi) L_i(\theta_i, \varphi_i)\cos\theta_i \, d\cos\theta_i \, d\varphi_i \\ &= L_i(\theta_r, \varphi_r \pm \pi) \end{aligned}$$

Snell's Law



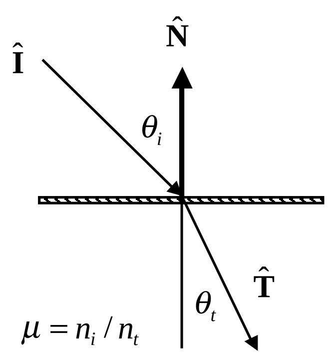


$$\varphi_t = \varphi_i \pm \pi$$

$$n_i \sin\theta_i = n_t \sin\theta_t$$

$$n_i\, \hat{\mathbf{N}} \times \hat{\mathbf{T}} = n_t\, \hat{\mathbf{N}} \times \hat{\mathbf{I}}$$

Law of Refraction



$$\mu = n_i / n_t$$

$$\hat{\mathbf{N}} \times \hat{\mathbf{T}} = \mu \;\; \hat{\mathbf{N}} \times \hat{\mathbf{I}}$$

$$\hat{\mathbf{N}} \times (\hat{\mathbf{T}} - \mu \;\; \hat{\mathbf{I}}) = 0$$

$$\hat{\mathbf{T}} = \mu \;\; \hat{\mathbf{I}} + \gamma \;\; \hat{\mathbf{N}}$$

$$\hat{\mathbf{T}}^2 = 1 = \mu^2 + \gamma^2 + 2\mu\gamma \;\; \hat{\mathbf{I}} \bullet \hat{\mathbf{N}}$$

$$\gamma = -\mu \;\; \hat{\mathbf{I}} \bullet \hat{\mathbf{N}} \pm \left\{ 1 - \mu^2 \left(1 - \left(\hat{\mathbf{I}} \bullet \hat{\mathbf{N}} \right)^2 \right) \right\}^{1/2}$$

$$= \mu \;\; \cos\theta_i \pm \left\{ 1 - \mu^2 \sin^2\theta_i \right\}^{1/2}$$

$$= \mu \;\; \cos\theta_i \pm \cos\theta_t \qquad \leftarrow \gamma = \mu - 1$$

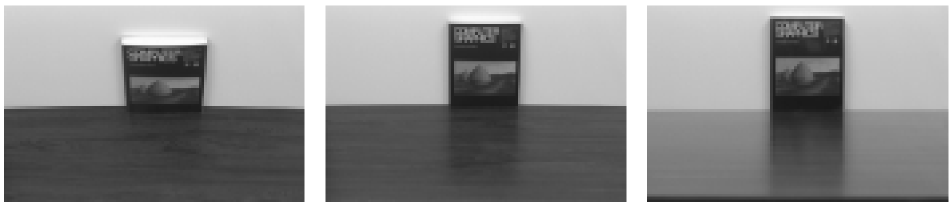
$$= \mu \;\; \cos\theta_i - \cos\theta_t$$

Total internal reflection:

$$1 - \mu^2 (1 - \left(\hat{\mathbf{I}} \bullet \hat{\mathbf{N}} \right)^2) < 0$$

Experiment

Reflections from a shiny floor



Source: Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

Fresnel Equations

Dielectrics (Two polarizations)

$$R_{\perp} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} \quad R_{\parallel} = \frac{n_1 \cos\theta_2 - n_2 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1}$$

$$T_{\perp} = \frac{2 n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2} \quad T_{\parallel} = \frac{2 n_1 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1}$$

Metals $n + i\kappa$

$$a^2 + b^2 = n^2(1-\kappa^2) - \sin^2\theta$$

$$R = \frac{a^2 + b^2 - 2a\cos\theta + \cos^2\theta}{a^2 + b^2 + 2a\cos\theta + \cos^2\theta}$$

$$T = \frac{a^2 + b^2 - 2a\sin\theta\tan\theta + \sin^2\theta\tan^2\theta}{a^2 + b^2 + 2a\sin\theta\tan\theta + \sin^2\theta\tan^2\theta} R$$

Fresnel Equations

Normal incidence

- **Dielectrics**

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2$$

Glass:	n**=1.5** R**=0.04**	
Diamond:	n**=2.4** R**=0.15**	

- **Metals**

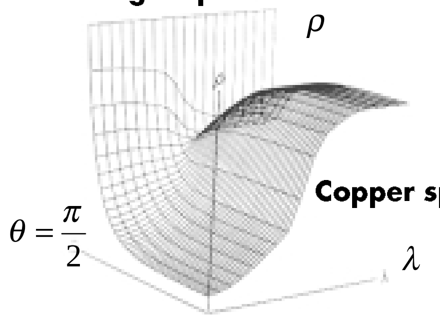
$$R = \left(\frac{(n-1)^2 + n^2\kappa^2}{(n+1)^2 + n^2\kappa^2} \right)^2$$

Silver:	n**<1,** κ**=1** R**=0.95**
Gold:	n**<1,** κ**=1** R**=0.82**

Solve for n given R at normal incidence

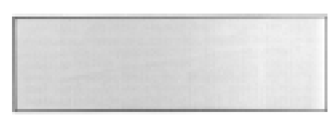
Cook-Torrance Model for Metals

Light spectra

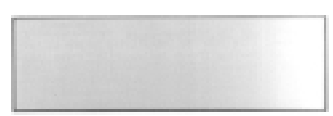


Copper spectra

Reflectance of Copper as a function of wavelength and angle of incidence



Measured Reflectance



Approxmated Reflectance

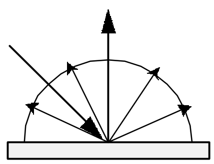
$$R = R(0) + R(\pi/2)\left[\frac{\bar{R}(\theta) - \bar{R}(0)}{\bar{R}(\pi/2) - \bar{R}(0)} \right]$$

Schlick approximate Fresnel

$$F(\theta) = F_0 + (1 - F_0)(1 - \cos\theta)^5$$

Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction (independent of input direction).



$$L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i) \cos\theta_i \, d\omega_i$$
$$= f_{r,d} \int L_i(\omega_i) \cos\theta_i \, d\omega_i$$
$$= f_{r,d} E$$

$$B = \int L_r(\omega_r) \cos\theta_r \, d\omega_r = L_r \int \cos\theta_r \, d\omega_r = \pi \, L_r$$

$$\rho_d = \frac{B}{E} = \pi \, f_{r,d}$$

Lambert's Cosine Law $B = \rho_d E = \rho_d E_s \cos\theta_s$

"Diffuse" Reflection

Theoretical

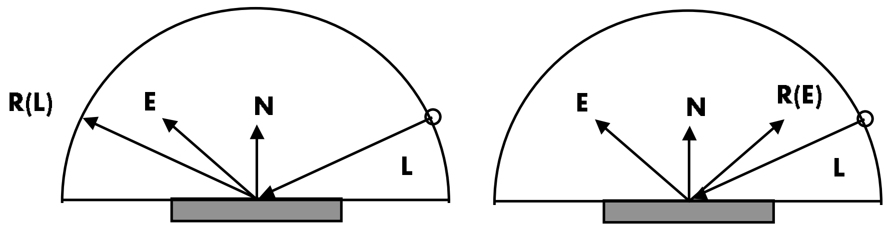
- **Bouguer - Special micro-facet distribution**
- **Seeliger - Subsurface reflection**
- **Multiple surface or subsurface reflections**

Experimental

- **Pressed magnesium oxide powder**
- **Almost never valid at high angles of incidence**

Paint manufactures attempt to create ideal diffuse

Phong Model



$$\left(\hat{\mathbf{E}} \bullet \mathbf{R}(\hat{\mathbf{L}})\right)^s \qquad \left(\hat{\mathbf{L}} \bullet \mathbf{R}(\hat{\mathbf{E}})\right)^s$$

Reciprocity: $\left(\hat{\mathbf{E}} \bullet \mathbf{R}(\hat{\mathbf{L}})\right)^s = \left(\hat{\mathbf{L}} \bullet \mathbf{R}(\hat{\mathbf{E}})\right)^s$

Distributed light source!

Properties of the Phong Model

Normalize Phong Model

$$\begin{aligned}\rho(2\pi \to \omega_r) &= \int_{H^2(\hat{\mathbf{N}})} \left(\hat{\mathbf{L}} \bullet \mathbf{R}(\hat{\mathbf{E}})\right)^s \cos\theta_i \, d\omega_i \\ &\leq \int_{H^2(\hat{\mathbf{R}})} \left(\hat{\mathbf{L}} \bullet \mathbf{R}(\hat{\mathbf{E}})\right)^s \cos\theta_i \, d\omega_i \\ &\leq \int_{H^2} \cos^{s+1}\theta_i \, d\omega_i = \frac{2\pi}{s+2}\end{aligned}$$