Participating Media and Volumetric Scattering

Applications

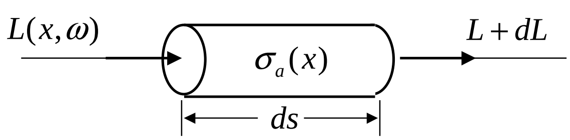
- Clouds, smoke, water, …
- Subsurface scattering: paint, skin, …
- Scientific and medical visualization: CT, MRI, …

Topics

- Absorption
- Scattering and phase functions
- Volume rendering equation
- Volume representations
- Ray tracing volumes

Absorption

$$dL(x,\omega) = -\sigma_a(x) L(x,\omega)\, ds$$



Absorption cross-section : probability of being absorbed per unit length

Beer's Law

Homogenous media: constant σ_a

$$L(x + s\,\omega, \omega) = L(x,\omega)\, e^{-\tau(s)} = L(x,\omega)\, e^{-\sigma_a s}$$

Inhomogenous media: varying $\sigma_a(x)$

$$L(x + s\,\omega, \omega) = T(s)\, L(x,\omega) = e^{-\tau(s)}\, L(x,\omega)$$

Optical distance or depth

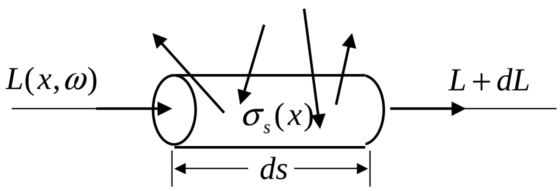
$$\tau(s) = \int_0^s \sigma_a(x + s'\omega)\, ds'$$

Transmittance

$$T(s) = T(x_1, x_2) = e^{-\tau(x_1, x_2)}$$

Out-Scatter

$$dL(x,\omega) = -\sigma_s(x) L(x,\omega)\, ds$$



Scattering cross-section σ_s

Total cross-section $\sigma_t = \sigma_a + \sigma_s$

Albedo $W = \frac{\sigma_s}{\sigma_t} = \frac{\sigma_s}{\sigma_a + \sigma_s}$

$$\tau(s) = \int_0^s \sigma_s(x + s'\omega)\, ds'$$

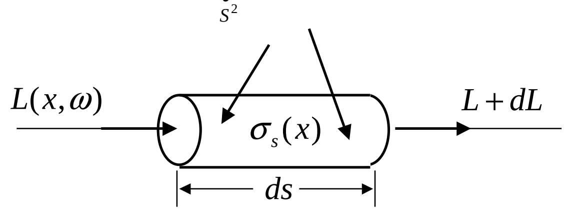
Attenuation due to both absorption and scattering: extinction coefficient

Black Clouds

Source: Greenler, Rainbows, halos and glories

In-Scatter

$$S(x,\omega) = \sigma_s(x) \int_{S^2} p(x, \omega' \to \omega) L(x, \omega')\, d\omega'$$



Scattering or Phase function

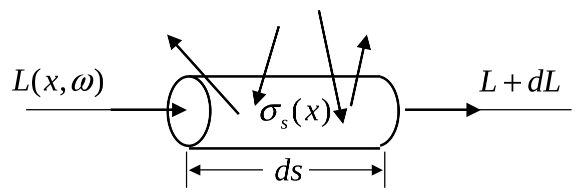
$$p(\omega' \to \omega)$$

$$\int_{S^2} p(\omega' \to \omega)\, d\omega' = 1$$

Scattering Equation

$$dL(x + ds\,\omega, \omega) = -\sigma_t(x) L(x,\omega) + \sigma_s(x) \int_{S^2} p(x, \omega' \to \omega) L(x, \omega')\, d\omega'$$

$$= -\sigma_t(x) L(x,\omega) + \sigma_s(x) S(x,\omega)$$



The Volume Rendering Equation

Integro-differential equation

$$\frac{\partial L(x,\omega)}{\partial s} = -\sigma_t(x)L(x,\omega) + \sigma_s(x)S(x,\omega)$$

Integro-integral equation

$$L(x,\omega) = \int_0^\infty e^{-\int_0^{s'} \sigma_t(x+s''\omega)ds''} \left[\sigma_s(x+s'\omega)S(x+s'\omega)\right]ds'$$

Attenuation: Absorption and scattering **Source: Scatter (+ emission)**

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Simple Atmosphere Model

Assumptions

- **Homogenous media**
- **Constant source term (airlight)**

$$\frac{\partial L(s)}{\partial s} = -\sigma_t L(s) + S$$

$$L(s) = \left(1 - e^{-\sigma_t s}\right)S \quad + e^{-\sigma_t s}C$$

Fog

Haze

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The Sky

Plate 5-16. Fisheye view of clear sky at the South Pole. (Photographed by the author)

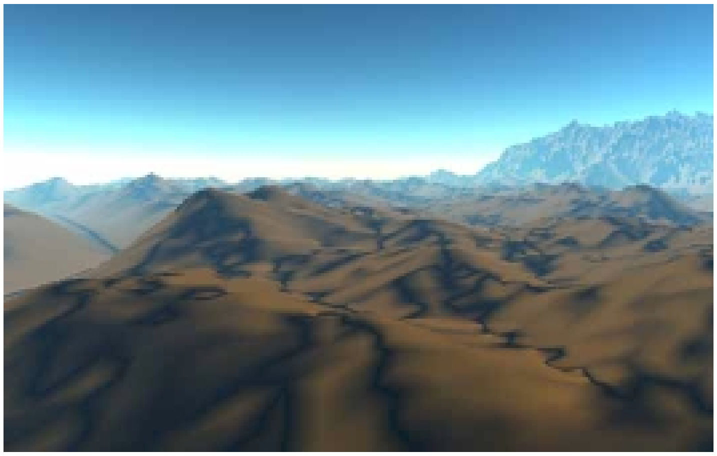
Plate 5-17. View of slightly hazy sky in Wisconsin. (Photographed by the author)

Source: Greenler, Rainbows, halos and glories

Atmospheric Perspective

Source: Greenler, Rainbows, halos and glories

Atmospheric Perspective



Aerial Perspective: loss of contrast and change in color

Source: Musgrave

Semi-Infinite Homogenous Planar Media

Reduced Intensity

$$L(z,\omega_i) = e^{-\tau(z,\omega_i)} L(0,\omega_i) \qquad \tau = \sigma_t z / \cos\theta_i$$

Total first order scattering

$$\frac{\partial L(z,\omega_o)}{\partial z} = -\sigma_t L(z,\omega_o) + \sigma_s S(z,\omega_o)$$

$$S(z,\omega_o) = p(\omega_i \to \omega_o) e^{-\tau(z,\omega_i)} L(0,\omega_i) \cos\theta_i$$

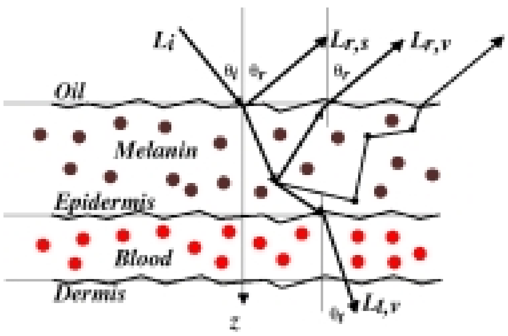
$$L(\omega_o) = \sigma_s p(\omega_i,\omega_o) L(\omega_i) \int_0^\infty e^{-\sigma_t z/\cos\theta_i} e^{-\sigma_t z/\cos\theta_o}\, dz$$

$$= W p(\omega_i,\omega_o) L(\omega_i) \frac{\cos\theta_i}{\cos\theta_i + \cos\theta_o}$$

Seeliger's Law
Law of Diffuse Reflection

Subsurface Scattering

Skin



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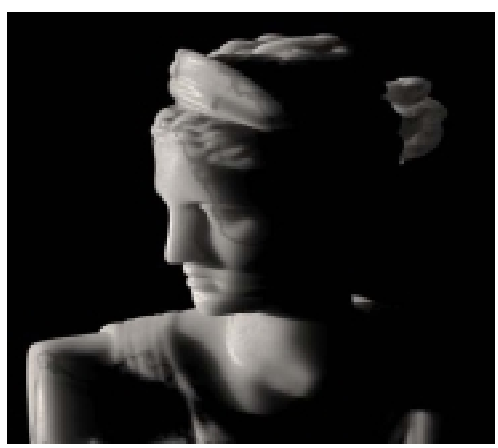
Translucent Materials



Surface Reflection **Subsurface Reflection**

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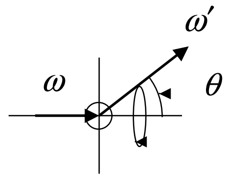
Phase Functions

Phase angle $\cos\theta = \omega \bullet \omega'$



Phase functions

(from the phase of the moon)

1. **Isotropic**
   -simple
2. **Rayleigh**
   -molecules
3. **Mie scattering**
   - small spheres

... Huge literature ...

$$p(\cos\theta) = \frac{1}{4\pi}$$

$$p(\cos\theta) = \frac{3}{4}\frac{1+\cos^2\theta}{\lambda^4}$$

Blue Sky = Red Sunset

Source: Greenler, Rainbows, halos and glories

Coronas and Halos

Moon Corona

Sun Halos

Source: Greenler, Rainbows, halos and glories

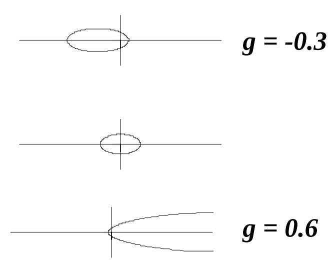
Henyey-Greenstein Phase Function

Empirical phase function

$$p(\cos\theta) = \frac{1}{4\pi}\frac{1-g^2}{\left(1+g^2-2g\cos\theta\right)^{3/2}}$$

$$2\pi\int_0^{\pi} p(\cos\theta)\cos\theta\, d\theta = g$$

***g*: average phase angle**



Volume Representations

3D arrays (uniform rectangular)

- **CT data**

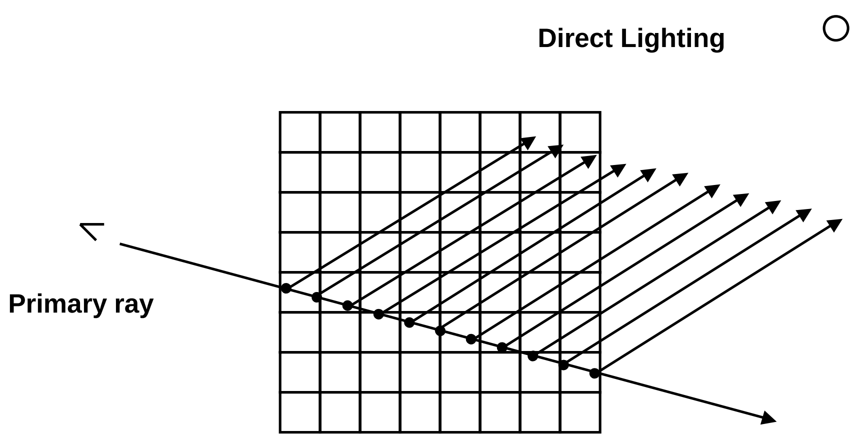
3D meshes

- **CFD, mechanical simulation**

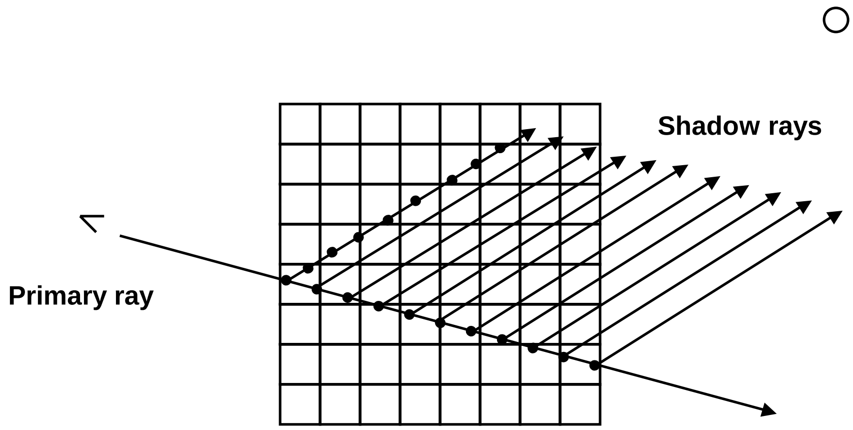
Simple shapes with solid texture

- **Ellipsoidal clouds with sum-of-sines densities**
- **Hypertexture**

Ray Marching



Ray Marching with Shadows



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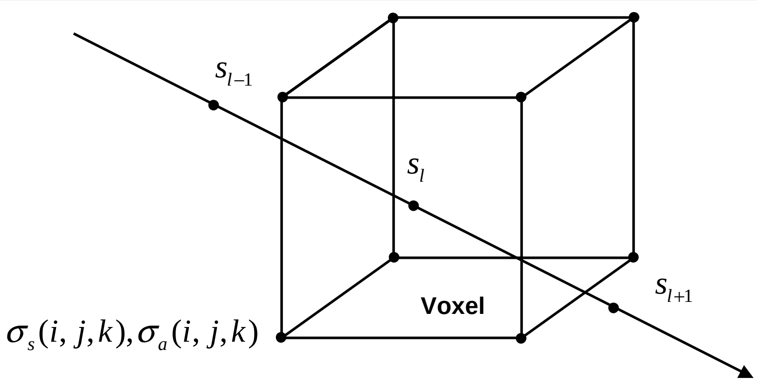
Ray Marching



$$S(s_l) = \sigma_t(x(s_l))\, p(\omega, \omega'(x(s_l)))\, L_s(x(s_l), \omega'(x(s_l))\, \Delta s$$

$$L(s_l) = L(s_{l-1}) + T(s_{l-1}) S(s_l)$$

$$T(s_l) = T(s_{l-1})(1 - \sigma_t(x(s_l)))\, \Delta s$$

$$\sigma(s_l) = \text{trilinear}(\sigma, i, j, k, x(s_l))$$

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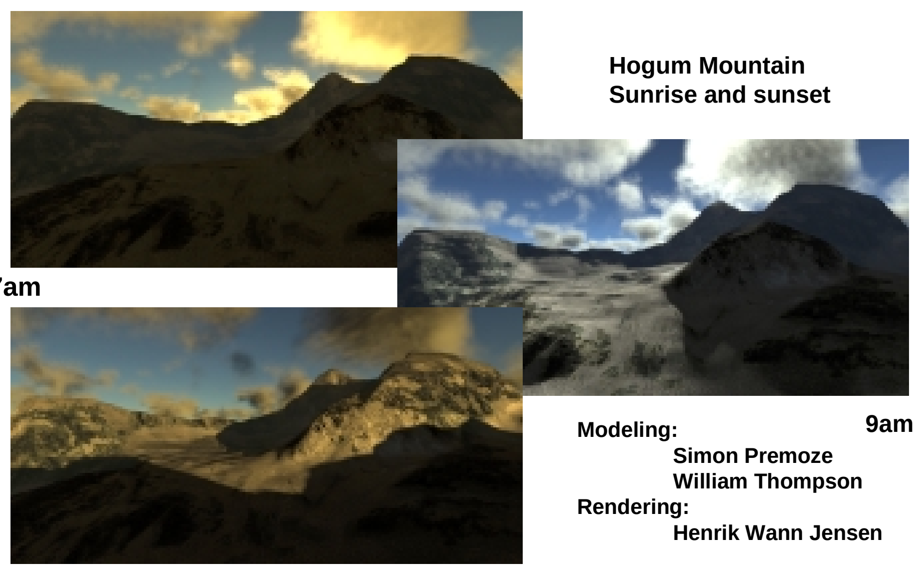
Beams of Light

Source: Greenler, Rainbows, halos and glories

Source: Minneart, Color and light in the open air

Clouds and Atmospheric Phenomena

Hogum Mountain
Sunrise and sunset



7am

9am

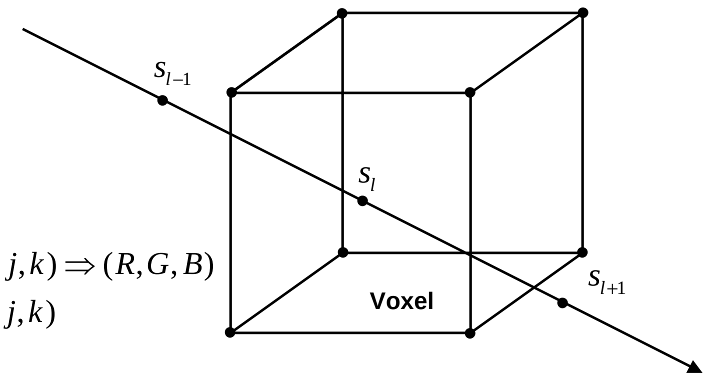
6:30pm

Modeling:
Simon Premoze
William Thompson

Rendering:
Henrik Wann Jensen

Ray Marching – Color and Opacity

M. Levoy, Ray tracing volume densities



$C(i, j, k) \Rightarrow (R, G, B)$

$A(i, j, k)$

$$C_l = C_{l-1} + (1 - A_{l-1})c(x(s_l))$$

Note: 4-component over

$$c(i, j, k) = \left(C(i, j, k) * A(i, j, k), A(i, j, k)\right)$$

Note: Must use premultiplied colors

$$c(x(s_l)) = \text{trilinear}(c, i, j, k, x(s_l))$$

Examples

Participating media

- **Texels**
- **Hypertextures**

Visualization

- **Visible human**
- **Finite element**

Multiple Scattering

- **Translucent materials**
- **Clouds and smoke**
- **Rendering equation**