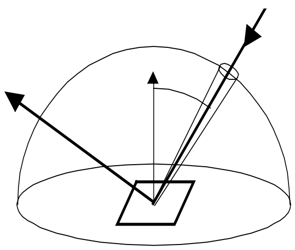
Illumination Models

To evaluate the reflection equation the illumination must be specified or computed.

$$L_r(x,\omega_r) = \int_{H^2} f_r(x,\omega_i \to \omega_r) L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$



- **Direct (*local*) illumination**
  - **Light directly from light sources**
  - **No shadows**
- **Indirect (*global*) illumination**
  - **Shadows due to blocking light**
  - **Interreflections**
  - **Complete accounting of light energy**

To The Rendering Equation

Questions

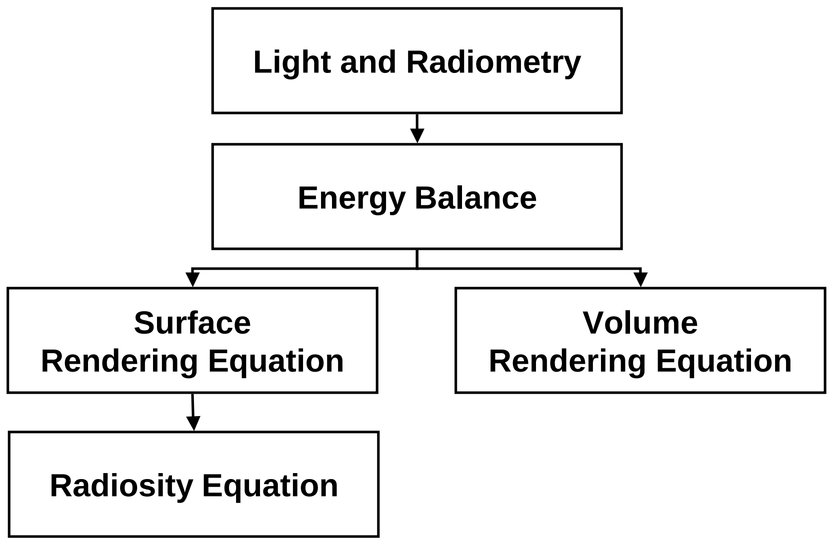
1. How is light measured?

2. How is the spatial distribution of light energy described?

3. How is reflection from a surface characterized?

4. What are the conditions for equilibrium flow of light in an environment?

The Grand Scheme



Balance Equation

Accountability

[*outgoing*] - [*incoming*] = [*emitted*] - [*absorbed*]

- **Macro level**

  The total light energy put into the system must equal the energy leaving the system (usually, via heat).

$$\Phi_o - \Phi_i = \Phi_e - \Phi_a$$

- **Micro level**

  The energy flowing into a small region of phase space must equal the energy flowing out.

$$B(x) - E(x) = B_e(x) - E_a(x)$$

$$L_o(x,\omega) - L_i(x,\omega) = L_e(x,\omega) - L_a(x,\omega)$$

Surface Balance Equation

[*outgoing*] = [*emitted*] + [*reflected*]

$$L_o(x,\omega_o) = L_e(x,\omega_o) + L_r(x,\omega_o)$$
$$= L_e(x,\omega_o) + \int_{H^2} f_r(x,\omega_i \to \omega_o) L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$

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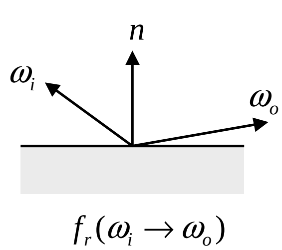
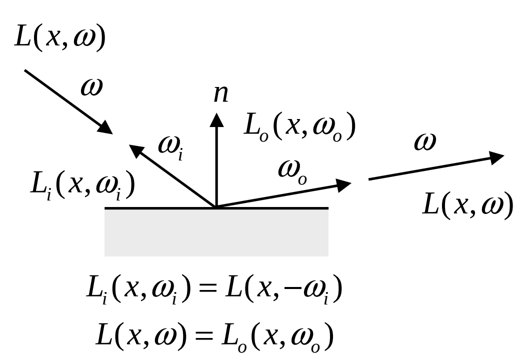
Direction Conventions



BRDF



Surface vs. Field Radiance

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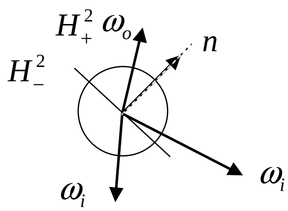
Surface Balance Equation

[*outgoing*] = [*emitted*] + [*reflected*] + [*transmitted*]

$$L_o(x,\omega_o) = L_e(x,\omega_o) + L_r(x,\omega_o) + L_t(x,\omega_o)$$



$$L_r(x,\omega_o) = \int_{H_+^2} f_r(x,\omega_i \to \omega_o) L_i(x,\omega_i)\cos\theta_i \, d\omega_i$$

$$L_t(x,\omega_o) = \int_{H_-^2} f_t(x,\omega_i \to \omega_o) L_i(x,\omega_i)\cos\theta_i \, d\omega_i$$

BTDF

$H_+^2(x) \quad \omega_i \bullet n(x) > 0$

$H_-^2(x) \quad \omega_i \bullet n(x) < 0$

The Rendering Equation

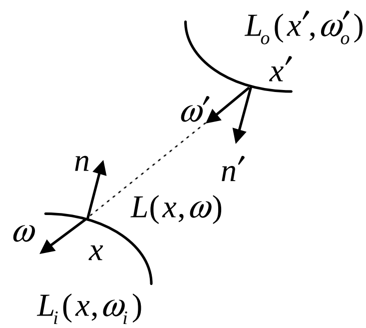
Ray functions

$x' = x^*(x, -\omega)$

$x = x^*(x', \omega')$

$\omega(x, x') = \dfrac{x' - x}{|x' - x|} \qquad \omega_i = \omega(x, x')$

$\qquad \omega_o = \omega(x', x)$



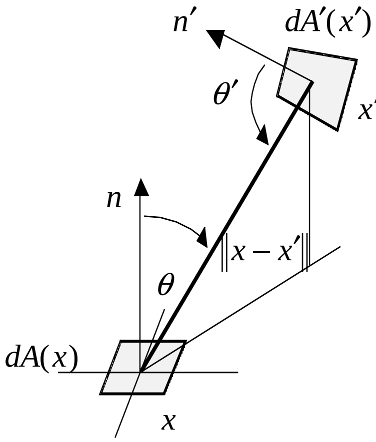
Couple balance equations

$L_i(x, \omega_i) = L(x, -\omega_i)$

$L(x, \omega) = L(x^*(x, -\omega)), \omega)$

$L(x^*(x, -\omega)), \omega) = L_o(x', \omega_o')$

Two Point Geometry



$$d\omega = \frac{\cos\theta'}{\|x - x'\|^2} dA'(x')$$

$$\cos\theta\, d\omega = \frac{\cos\theta \cos\theta'}{\|x - x'\|^2} dA'(x')$$

The Rendering Equation

$$L(x,\omega) = L_e(x,\omega) + \int_{M^2} f_r(x, \omega(x,x') \to \omega)\, L(x', \omega(x',x))\, G(x,x')\, dA'(x')$$

Integrate over All surfaces

Geometry term

$$G(x,x') = \frac{\cos\theta_i \cos\theta_o'}{\|x - x'\|^2} V(x,x')$$

Visibility term

$$V(x,x') = \begin{cases} 1 & \text{visible} \\ 0 & \text{not visible} \end{cases}$$

Note:
-*x, x'* and ω are knowns
-*L* is the surface radiance
-Have dropped outgoing subscript

The Radiosity Equation

Assume diffuse reflection

1. $f_r(x,\omega_i \to \omega_o) = f_r(x) \Rightarrow \rho(x) = \pi f_r(x)$
2. $L(x,\omega) = B(x)/\pi$

$$B(x) = B_e(x) + \rho(x) E(x)$$

$$B(x) = B_e(x) + \rho(x) \int_{M^2} F(x,x') B(x')\, dA'(x')$$

$$F(x,x') = \frac{G(x,x')}{\pi}$$

<u>Form factor</u>: The percentage of light leaving *dA'* that arrives at *dA*

Integral Equations

Integral equations of the 1st kind

$$f(x) = \int k(x,x') g(x')\, dx'$$

Integral equations of the 2nd kind

$$f(x) = g(x) + \int k(x,x') f(x')\, dx'$$

Linear Operators

Linear operators act on functions like matrices act on vectors

$$h(x) = (L \circ f)(x)$$

They are linear in that

$$L \circ (af + bg) = a(L \circ f) + b(L \circ g)$$

Types of linear operators

$$(K \circ f)(x) \equiv \int k(x, x') f(x') \, dx'$$
$$(D \circ f)(x) \equiv \frac{\partial f}{\partial x}(x)$$

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Formal Solution of Integral Equations

Integral equation

$$B = B_e + K \circ B$$
$$(I - K) \circ B = B_e$$

Formal solution

$$B = (I - K)^{-1} \circ B_e$$

Neumann series

$$\frac{1}{1 - x} = 1 + x + x^2 + \ldots$$
$$\frac{1}{I - K} = I + K + K^2 + \ldots$$

$$\begin{aligned}(I-K)\frac{1}{I-K} &= (I-K)(I+K+K^2+\ldots) \\ &= (I+K+K^2+\ldots) - (K+K^2+K^3+\ldots) \\ &= I\end{aligned}$$

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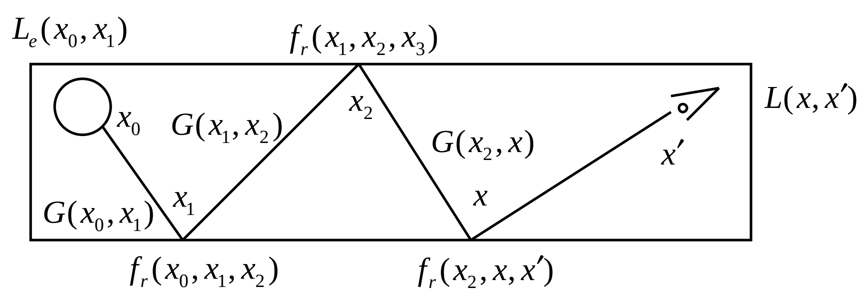
Formal Solution of Integral Equations

Successive Approximation

$$\frac{1}{I-K} B_e = B_e + K \circ B_e + K^2 \circ B_e + \ldots$$
$$= (B_e + K \circ (B_e + K \circ (B_e + \ldots$$

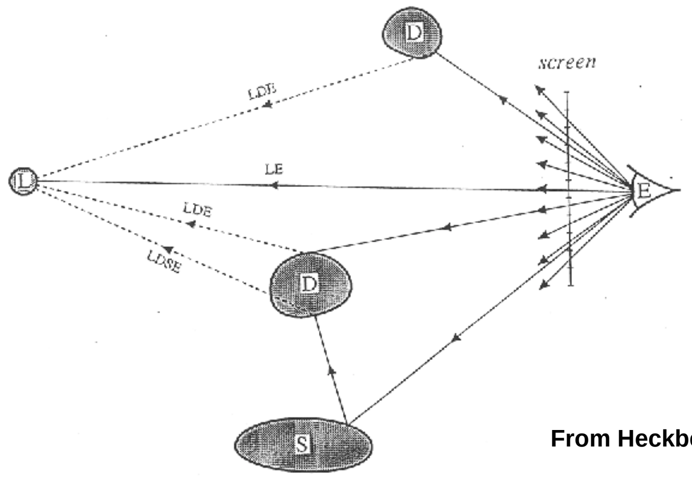
$$B_1 = B_e$$
$$B_2 = B_e + K \circ B_1$$
$$B_3 = B_e + K \circ B_2$$
$$\ldots$$
$$B_n = B_e + K \circ B_{n-1}$$

Light Paths



$$L(x, x') = \sum_{k=1}^{\infty} \int_{M^2} \int_{M^2} \ldots \int_{M^2} L_e(x_0, x_1) G(x_o, x_1)\, f_r(x_0, x_1, x_2) \cdots f_r(x_k, x, x')\ dA_0\, dA_1 \ldots dA_k$$

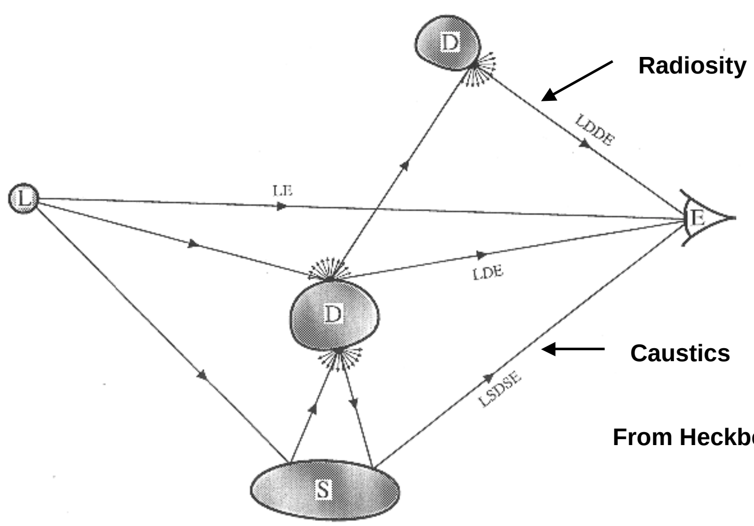
Classic Ray Tracing



From Heckbert

Forward (from eye): E S* (D|G) L

Photon Paths



From Heckbert

How to Solve It?

Finite element methods

- Classic radiosity
  - Mesh surfaces
  - Piecewise constant basis functions
  - Solve matrix equation
- Not practical for rendering equation

Monte Carlo methods

- Distributed ray tracing
  - Randomly traces ray from the eye
- Path tracing
- Bidirectional ray tracing
- Photon tracing