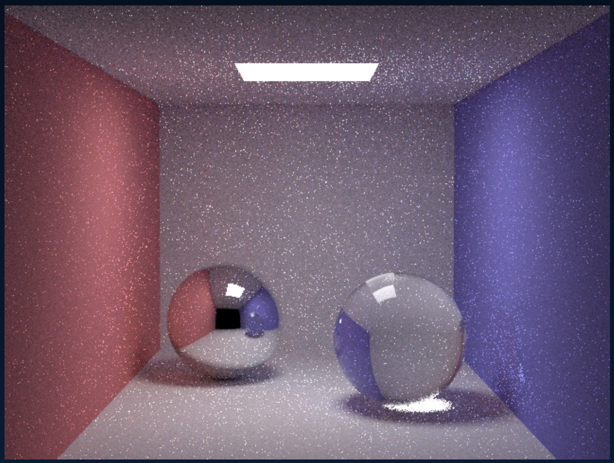
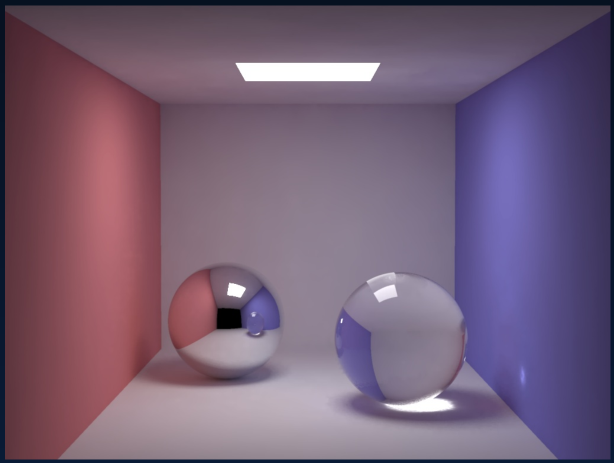
Biased Monte Carlo Ray Tracing:

Filtering, Irradiance Caching and Photon Mapping



$\longrightarrow$



Dr. Henrik Wann Jensen

Stanford University

May 24, 2001

Unbiased and consistent Monte Carlo methods

Unbiased estimator:

$$E\{X\} = \int \ldots$$

Consistent estimator:

$$\lim_{N \to \infty} E\{X\} \to \int \ldots$$

Unbiased and consistent: A very simple example

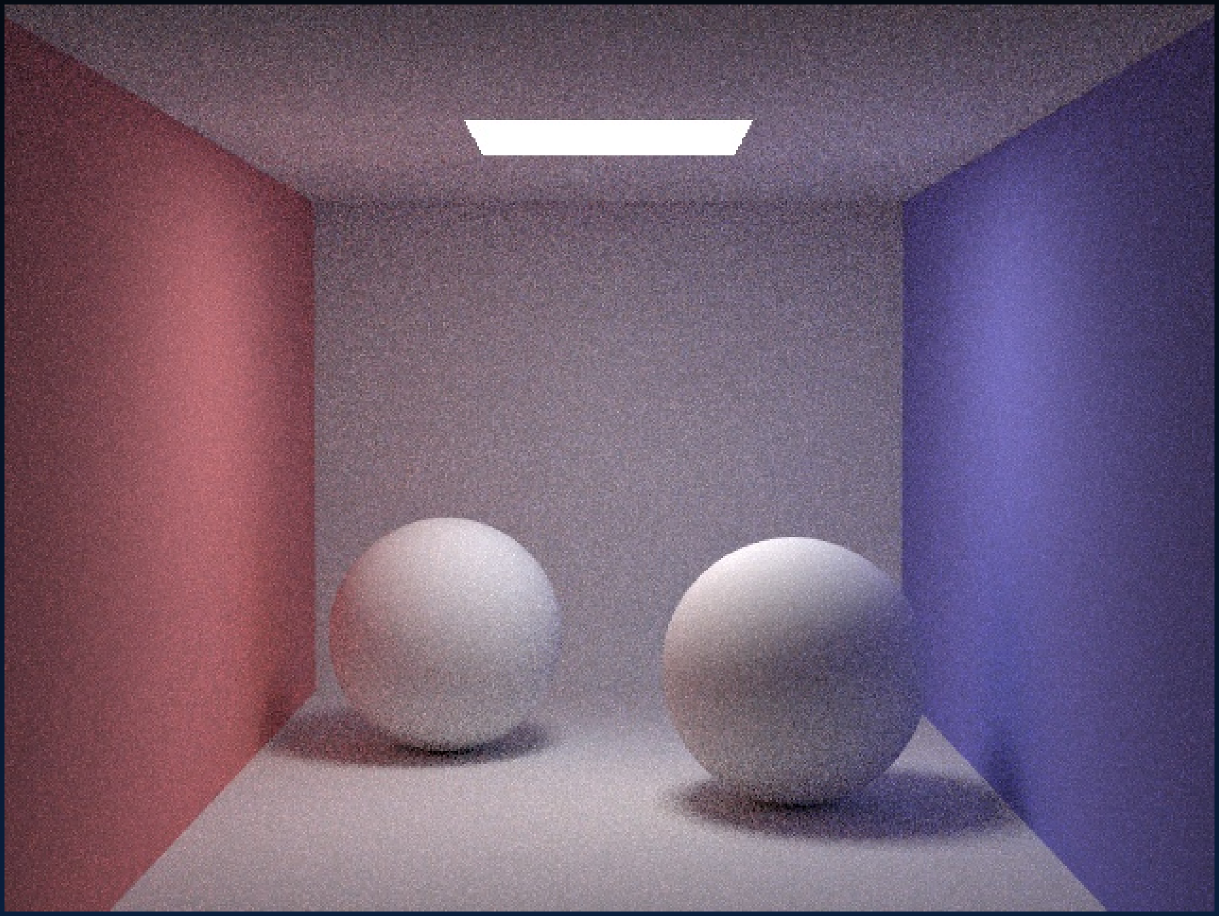
Unbiased estimator:

$$\frac{1}{N}\sum_{i=1}^{N} f(\xi_i)$$

Consistent estimator:

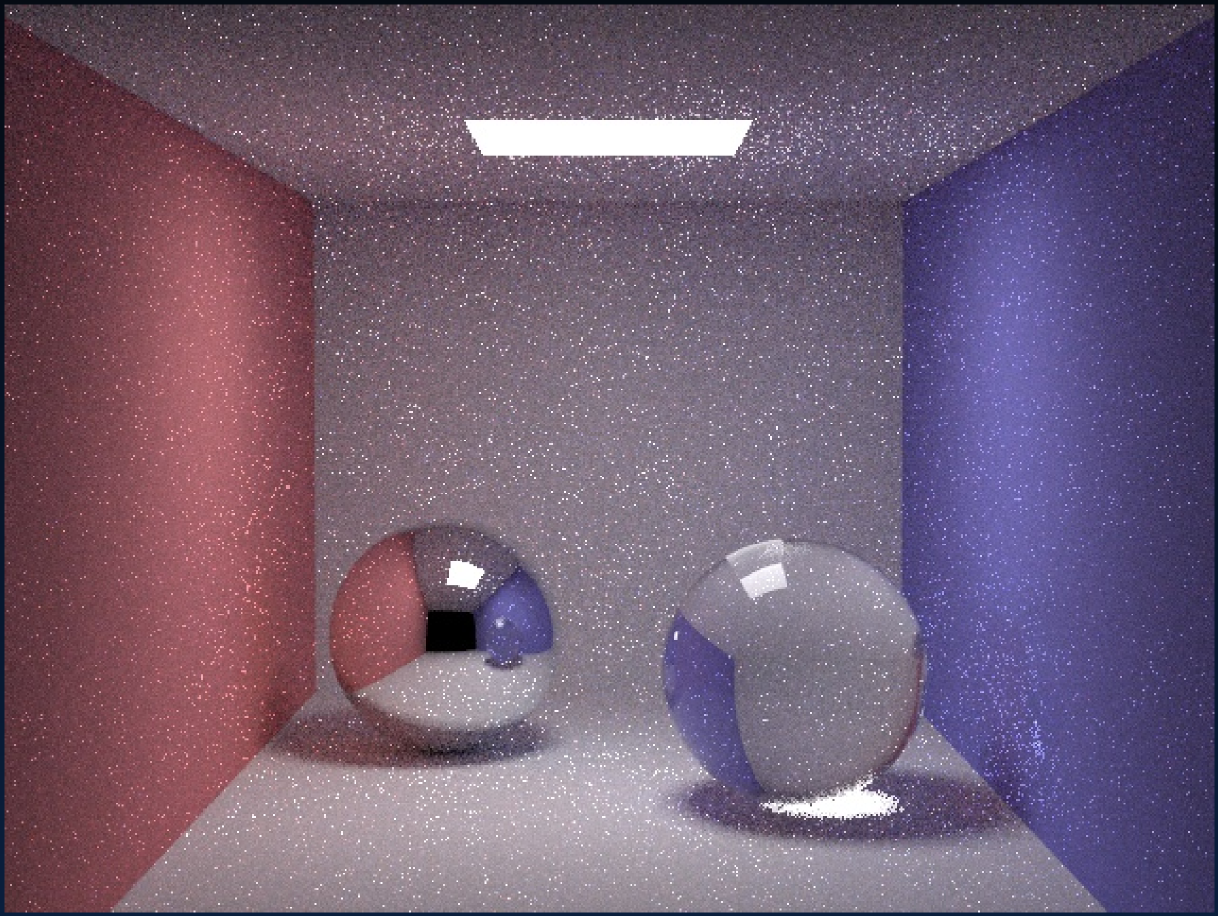
$$\frac{1}{N+1}\sum_{i=1}^{N} f(\xi_i)$$

Path tracing (unbiased)



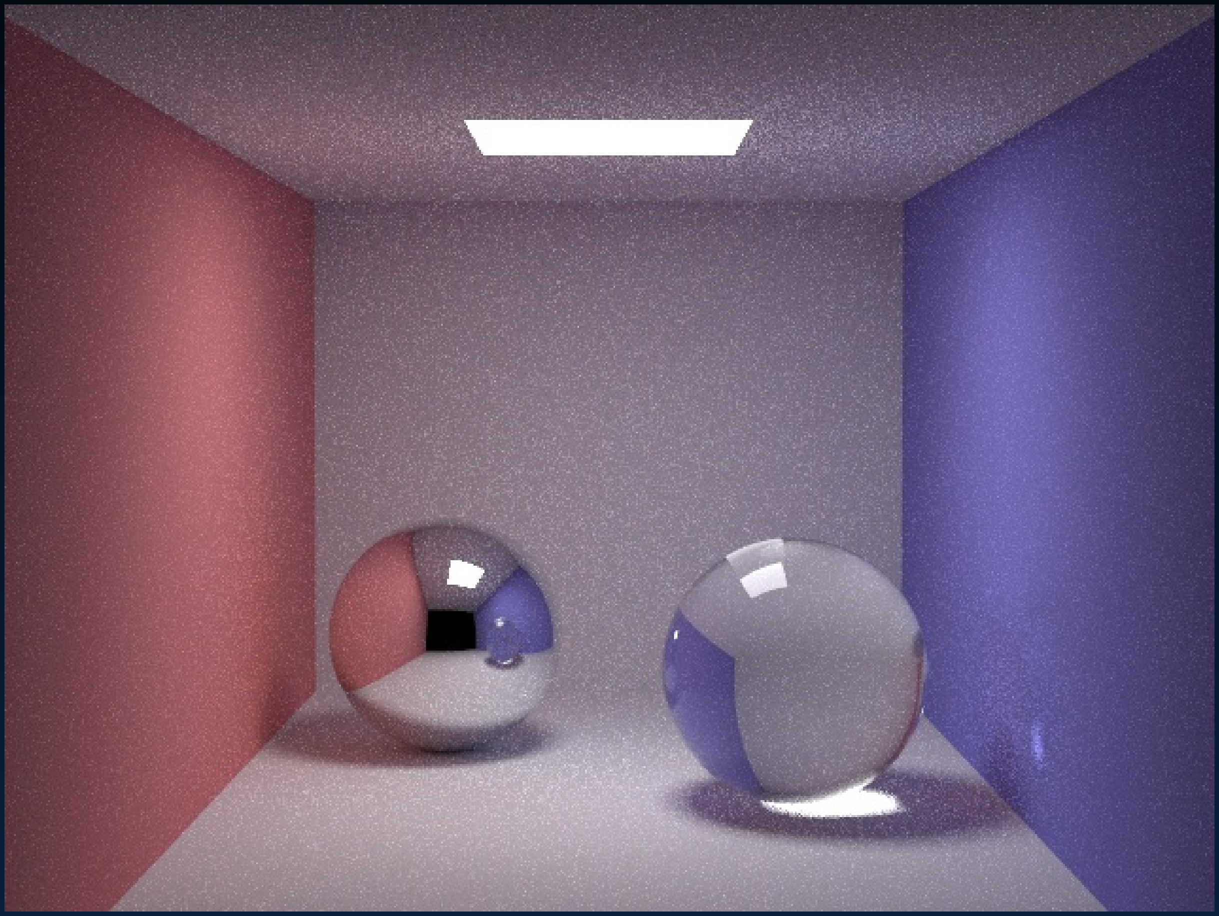
10 paths/pixel

Path tracing (unbiased)



10 paths/pixel

Path tracing (unbiased)



100 paths/pixel

How can we remove this noise?)

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- More samples (slow convergence, $\sigma \propto 1/\sqrt{N}$)

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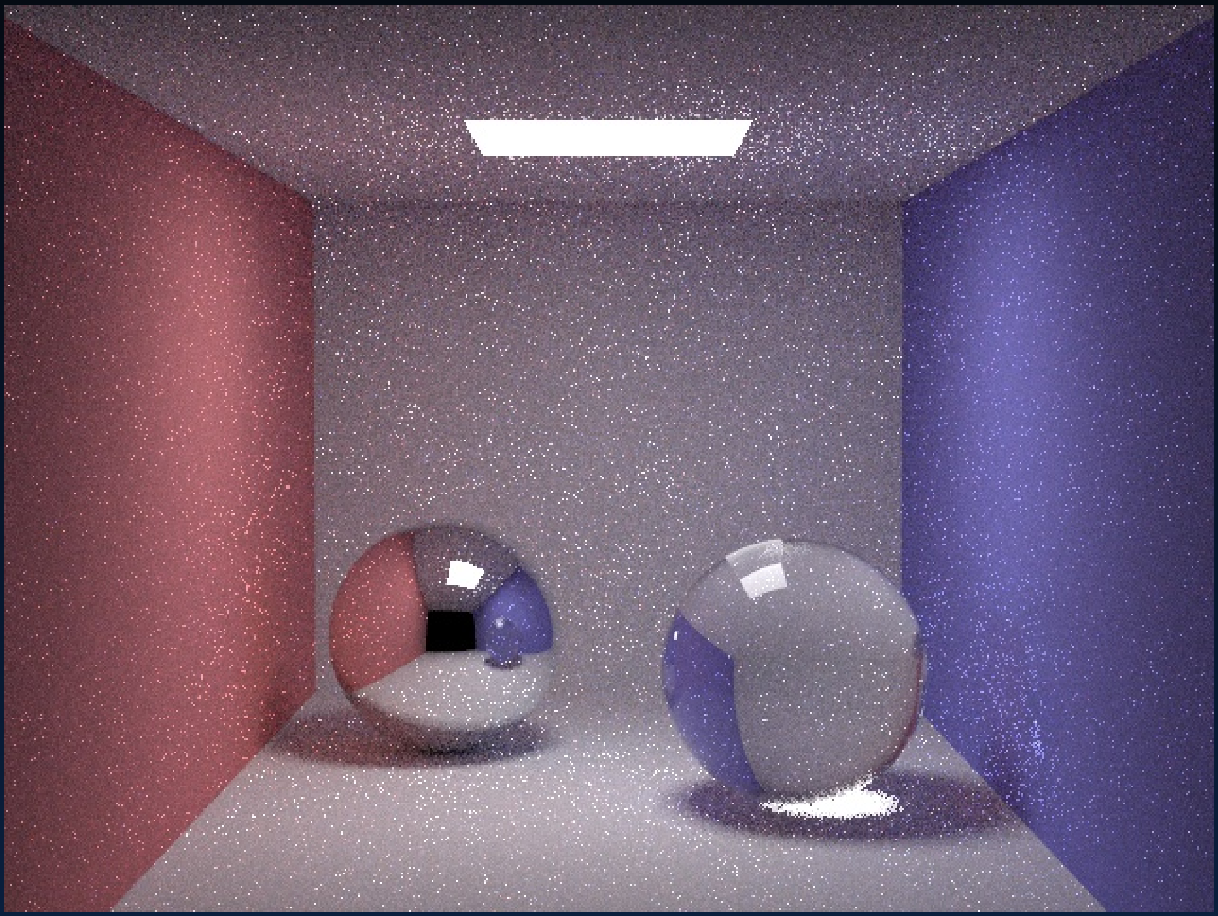
- Adaptive sampling
- Filtering

How can we remove this noise?)

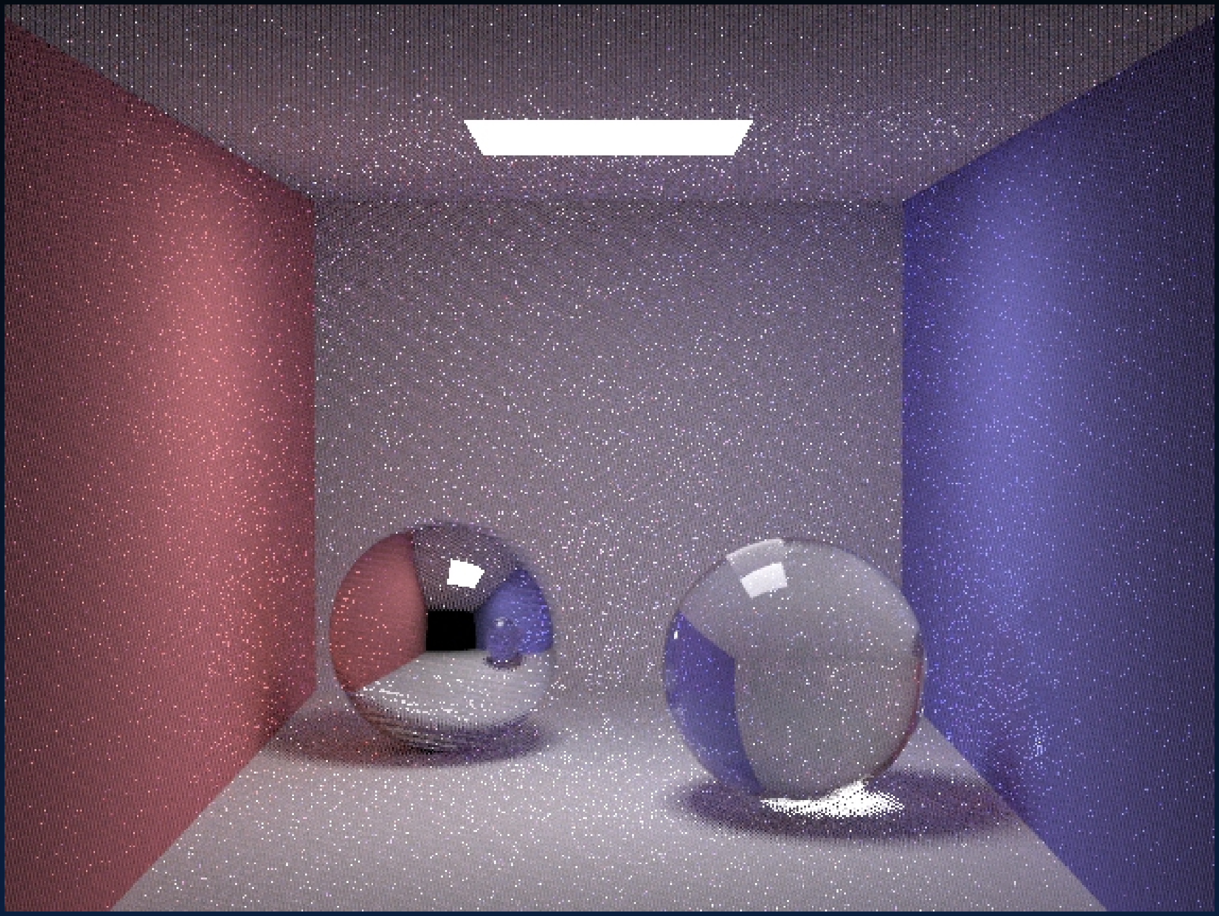
- More samples (slow convergence, $\sigma \propto 1/\sqrt{N}$)
- Better sampling (stratified, importance, qmc etc.)
- Adaptive sampling
- Filtering
- Caching and interpolation

Stratified sampling



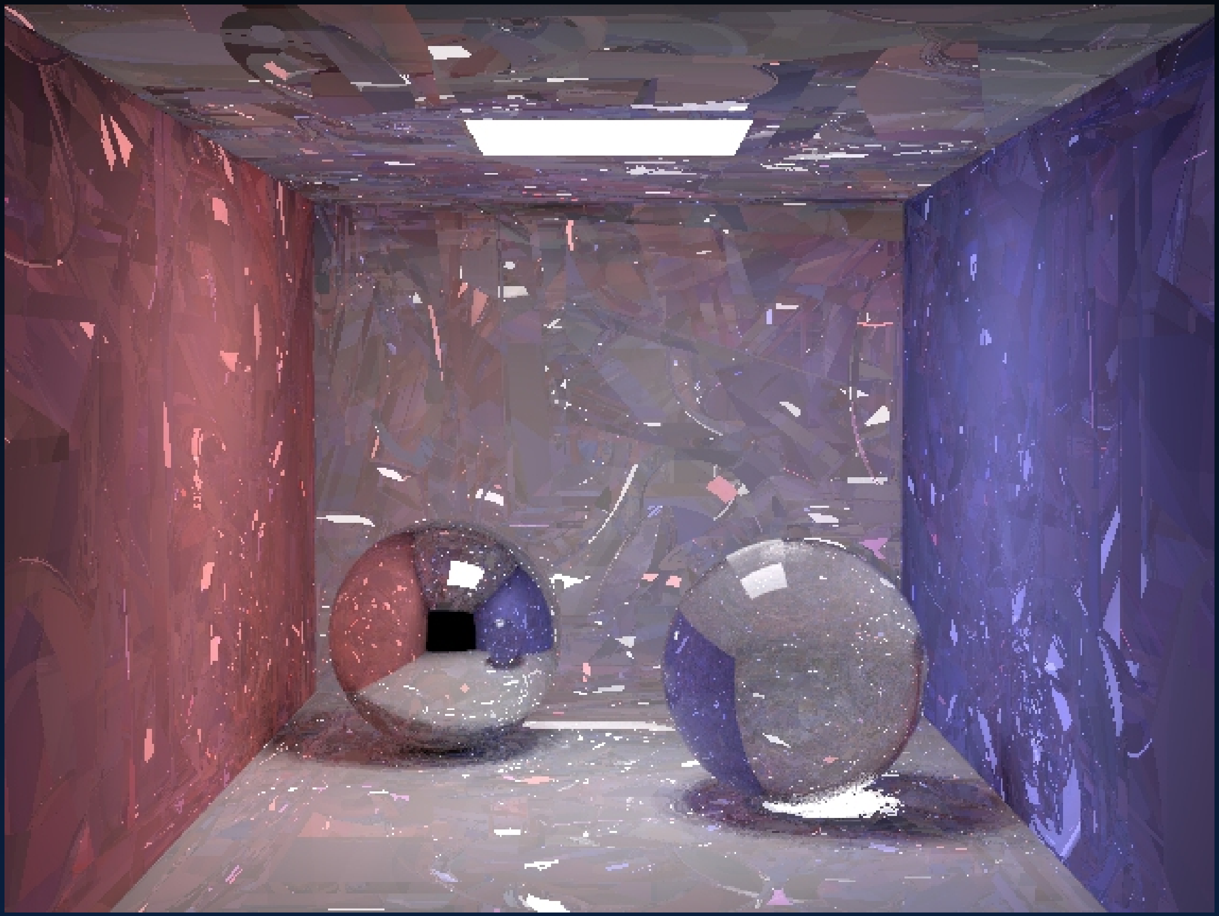
10 paths/pixel

Quasi Monte-Carlo (Halton sequence)



10 paths/pixel

Fixed (Random) Sequence



10 paths/pixel

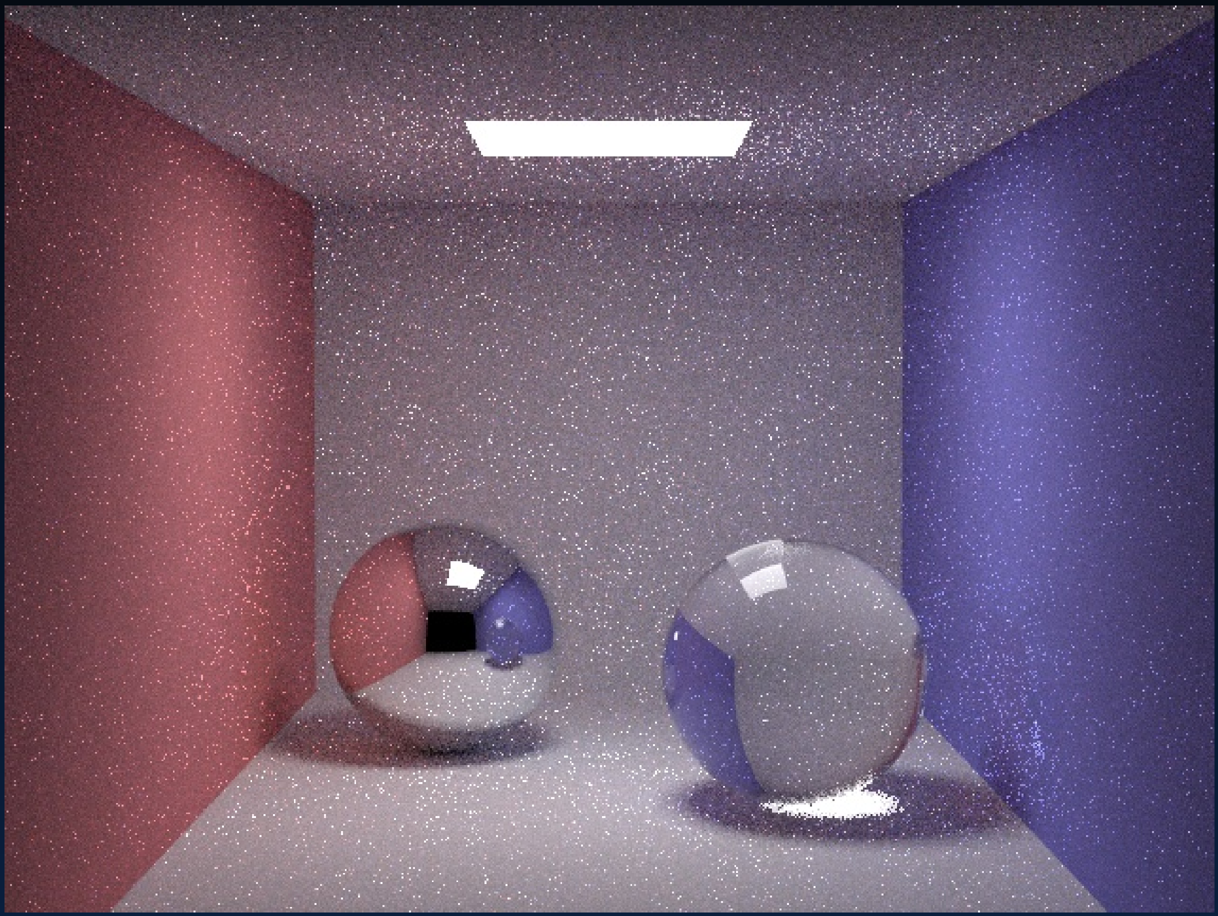
Filtering: idea

- Noise is high frequency

Filtering: idea

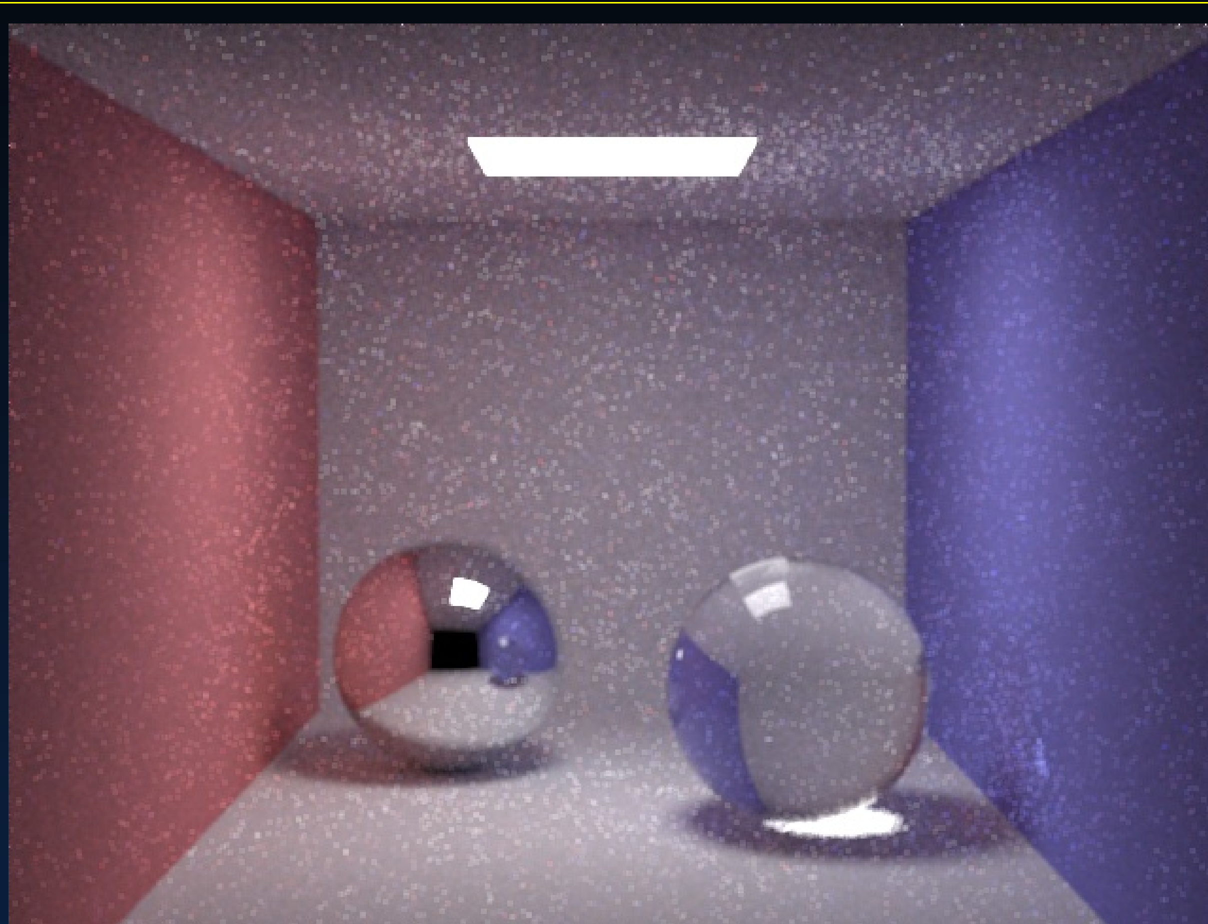
- Noise is high frequency
- Remove high frequency content

Unfiltered image



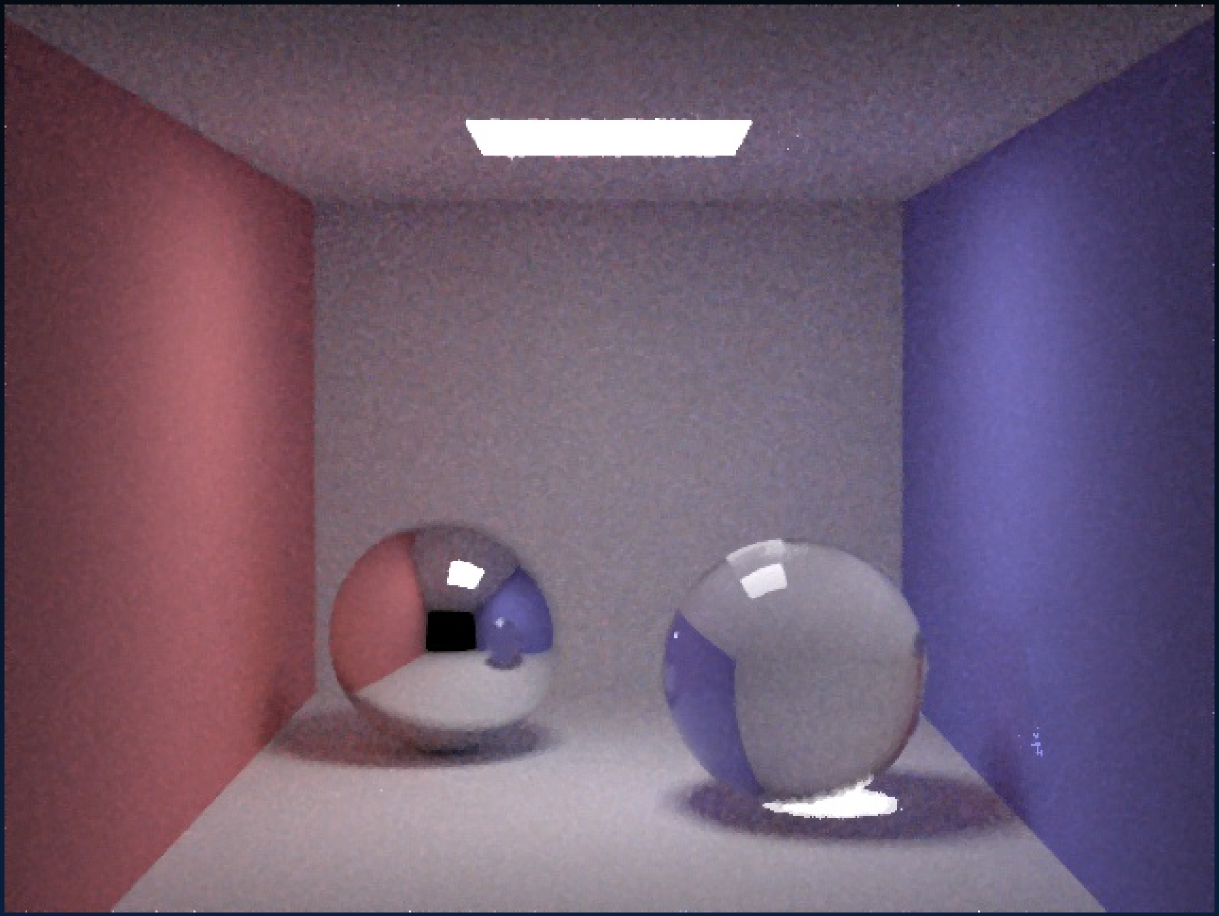
10 paths/pixel

3x3 lowpass filter



10 paths/pixel

3x3 median filter



10 paths/pixel

Energy preserving filters

Energy preserving filters

- Distribute noisy energy over several pixels

Energy preserving filters

- Distribute noisy energy over several pixels
- Adaptive filter width
- Diffusion style filters
- Splatting style filters

Problems with filtering

- Everything is filtered (blurred)
  - Textures
  - Highlights
  - Caustics
  - . . .

Problems with filtering

- Everything is filtered (blurred)
  - ⋆ Textures
  - ⋆ Highlights
  - ⋆ Caustics
  - ⋆ . . .

Solution: Try to filter the noisy part of the illumination

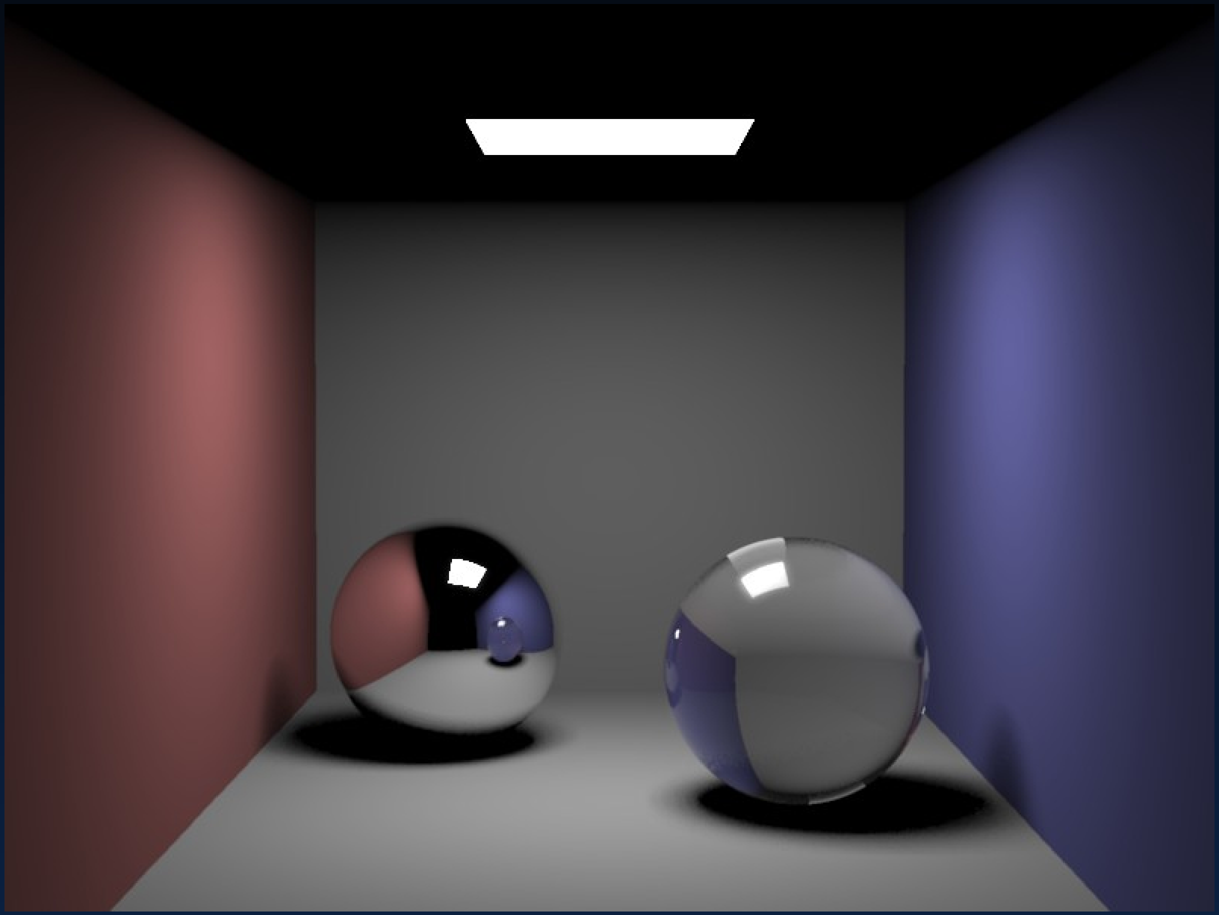
Caching Techniques

Caching Techniques

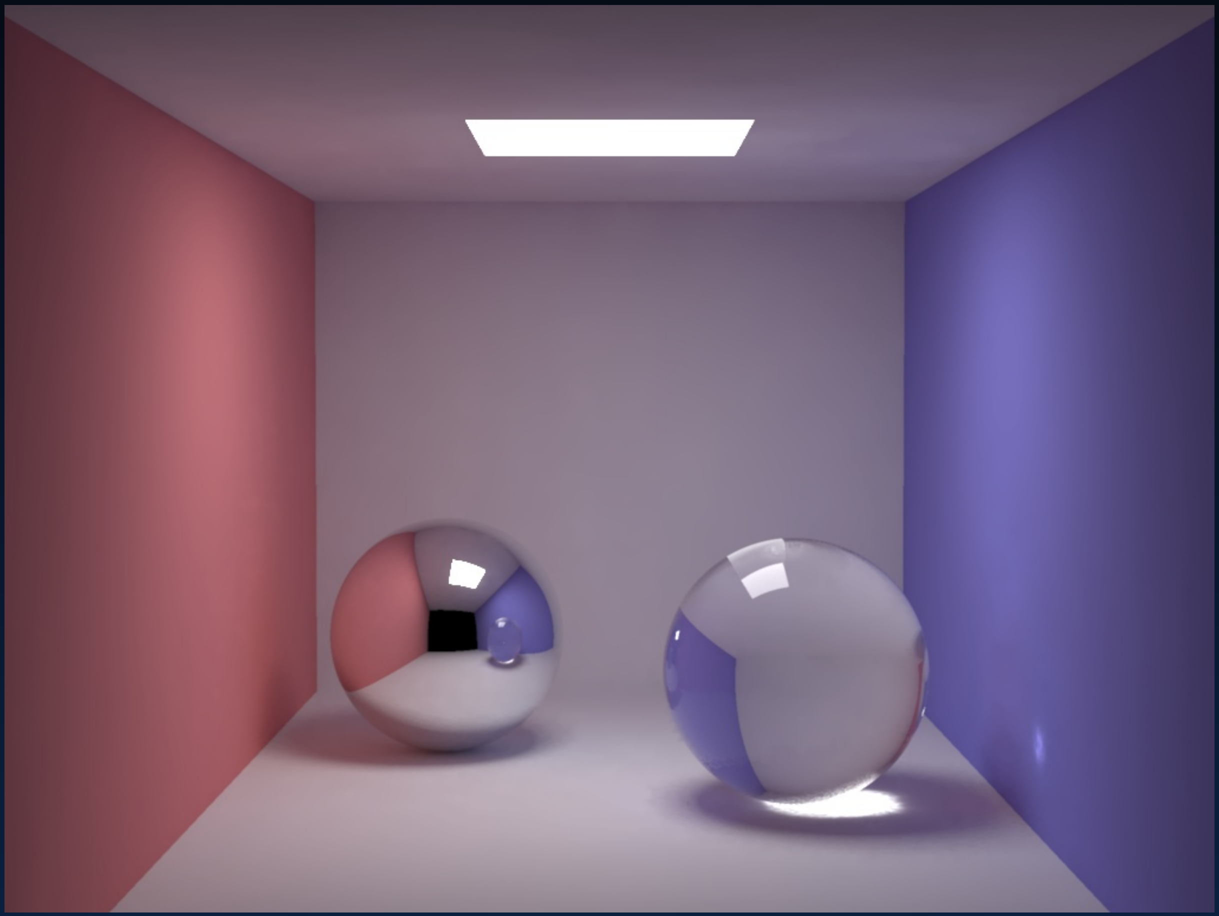
Irradiance caching : Compute irradiance at selected points and interpolate.

Photon mapping : Density estimation and importance sampling using a precomputed flux representation.

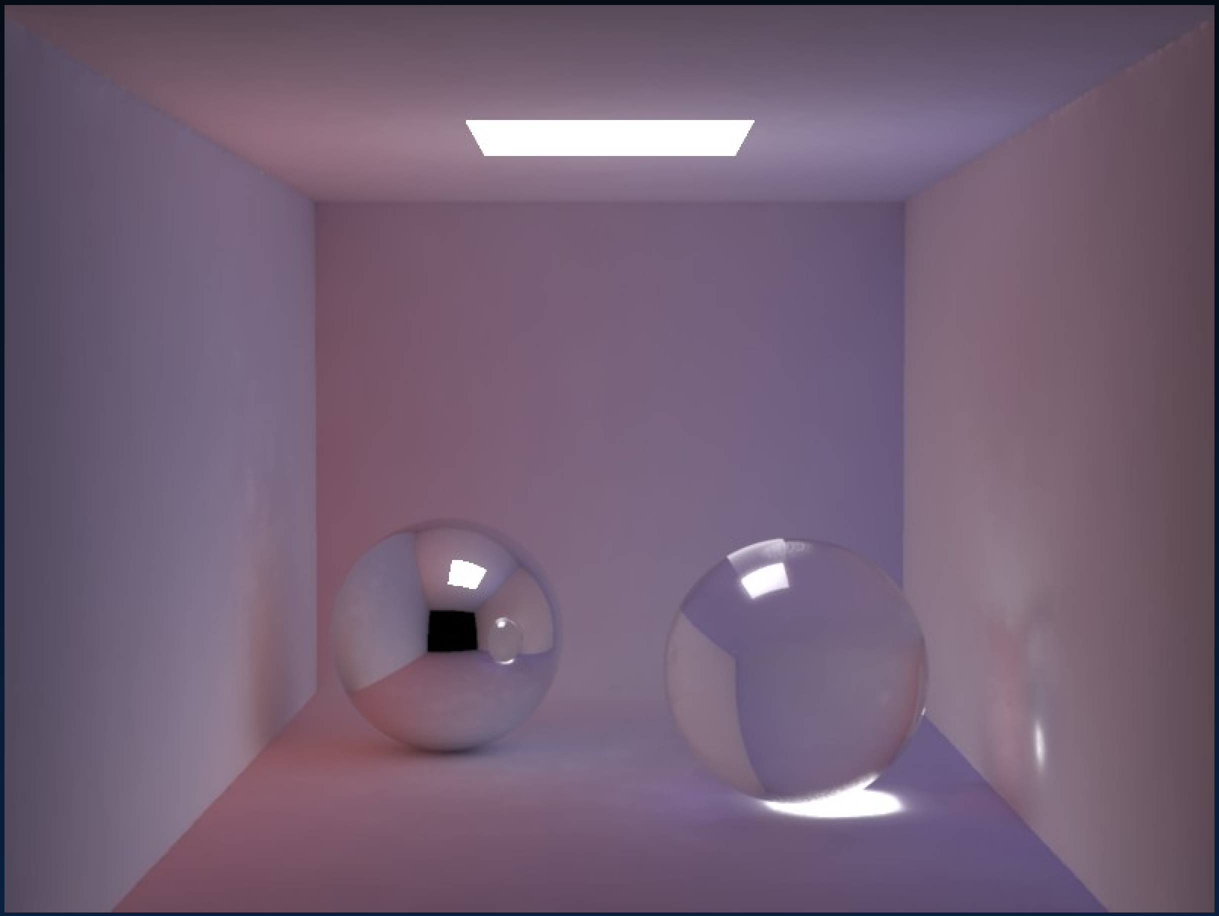
Box: direct illlumination



Box: global illlumination



Box: indirect irradiance



Irradiance caching: idea

Greg Ward, Francis Rubinstein and Robert Clear:
"A Ray Tracing Solution for Diffuse Interreflection".
Proceedings of SIGGRAPH 1988.

Idea: Irradiance changes slowly $\rightarrow$ interpolate.

Irradiance sampling

$$E(x) \;=\; \int_{2\pi} L'(x,\omega') \cos\theta \, d\omega'$$

Irradiance sampling

$$
\begin{aligned}
E(x) &= \int_{2\pi} L'(x,\omega') \cos\theta \, d\omega' \\
&= \int_0^{2\pi} \int_0^{\pi/2} L'(x,\theta,\phi) \cos\theta \sin\theta \, d\theta \, d\phi
\end{aligned}
$$

Irradiance sampling

$$
\begin{aligned}
E(x) &= \int_{2\pi} L'(x, \omega') \cos\theta \, d\omega' \\
&= \int_0^{2\pi} \int_0^{\pi/2} L'(x, \theta, \phi) \cos\theta \sin\theta \, d\theta \, d\phi \\
&\approx \frac{\pi}{TP} \sum_{t=1}^{T} \sum_{p=1}^{P} L'(\theta_t, \phi_p)
\end{aligned}
$$

$\theta_t = \sin^{-1}\left(\sqrt{\frac{t-\xi}{T}}\right)$ and $\phi_p = 2\pi \frac{p-\psi}{P}$

Irradiance change

$$\epsilon(x) \;\leq\; \left| \underbrace{\frac{\partial E}{\partial x}(x - x_0)}_{\text{position}} + \underbrace{\frac{\partial E}{\partial \theta}(\theta - \theta_0)}_{\text{rotation}} \right|$$

Irradiance change

$$\epsilon(x) \leq \left| \underbrace{\frac{\partial E}{\partial x}(x - x_0)}_{\text{position}} + \underbrace{\frac{\partial E}{\partial \theta}(\theta - \theta_0)}_{\text{rotation}} \right|$$

$$\leq E_0 \left(\underbrace{\frac{4}{\pi} \frac{||x - x_0||}{x_{avg}}}_{\text{position}} + \underbrace{\sqrt{2 - 2\vec{N}(x) \cdot \vec{N}(x_0)}}_{\text{rotation}} \right)$$

Irradiance interpolation

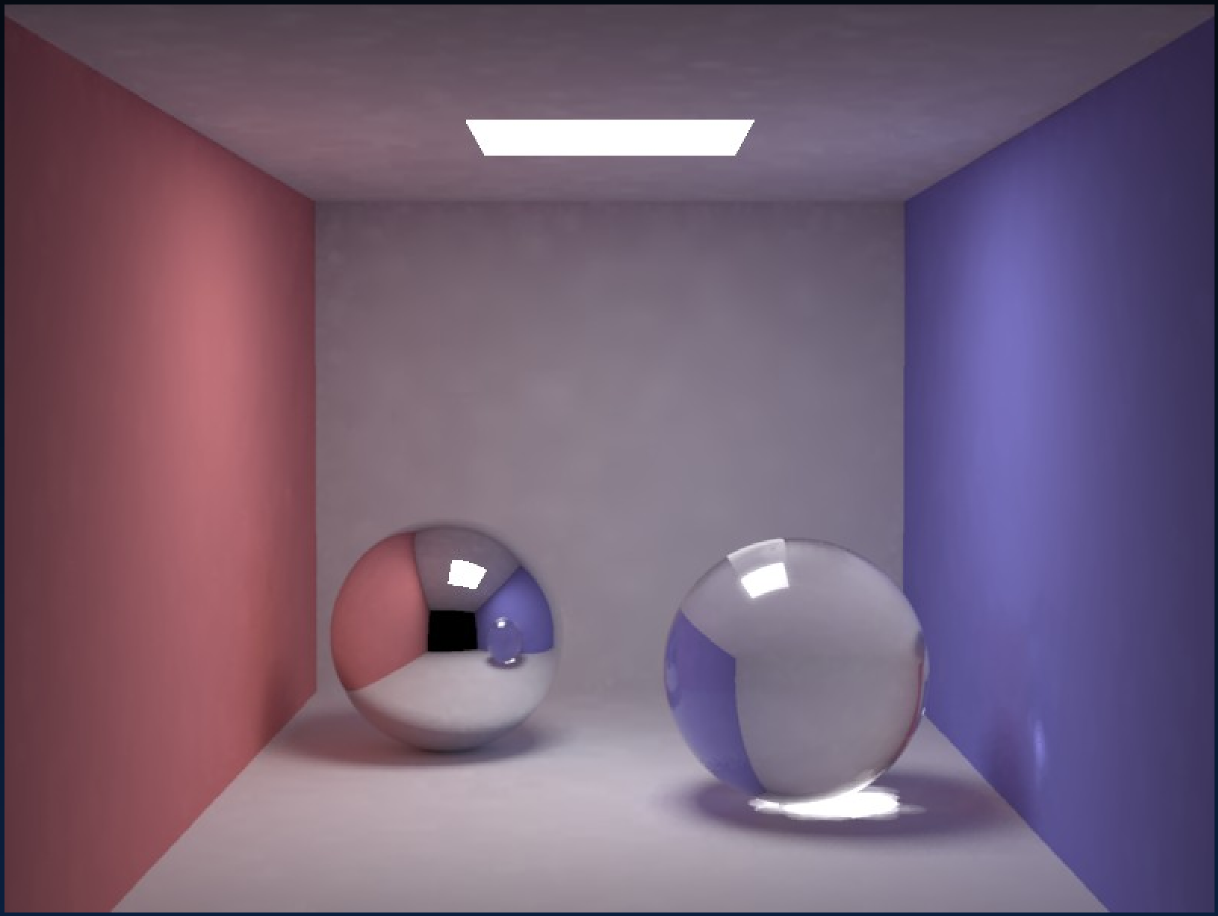
$$w(x) = \frac{1}{\epsilon(x)} \approx \frac{1}{\frac{||x - x_0||}{x_{avg}} + \sqrt{1 - \vec{N}(x) \cdot \vec{N}(x_0)}}$$

$$E_i(x) = \frac{\sum_i w_i(x) E(x_i)}{\sum_i w_i(x)}$$

Irradiance caching algorithm

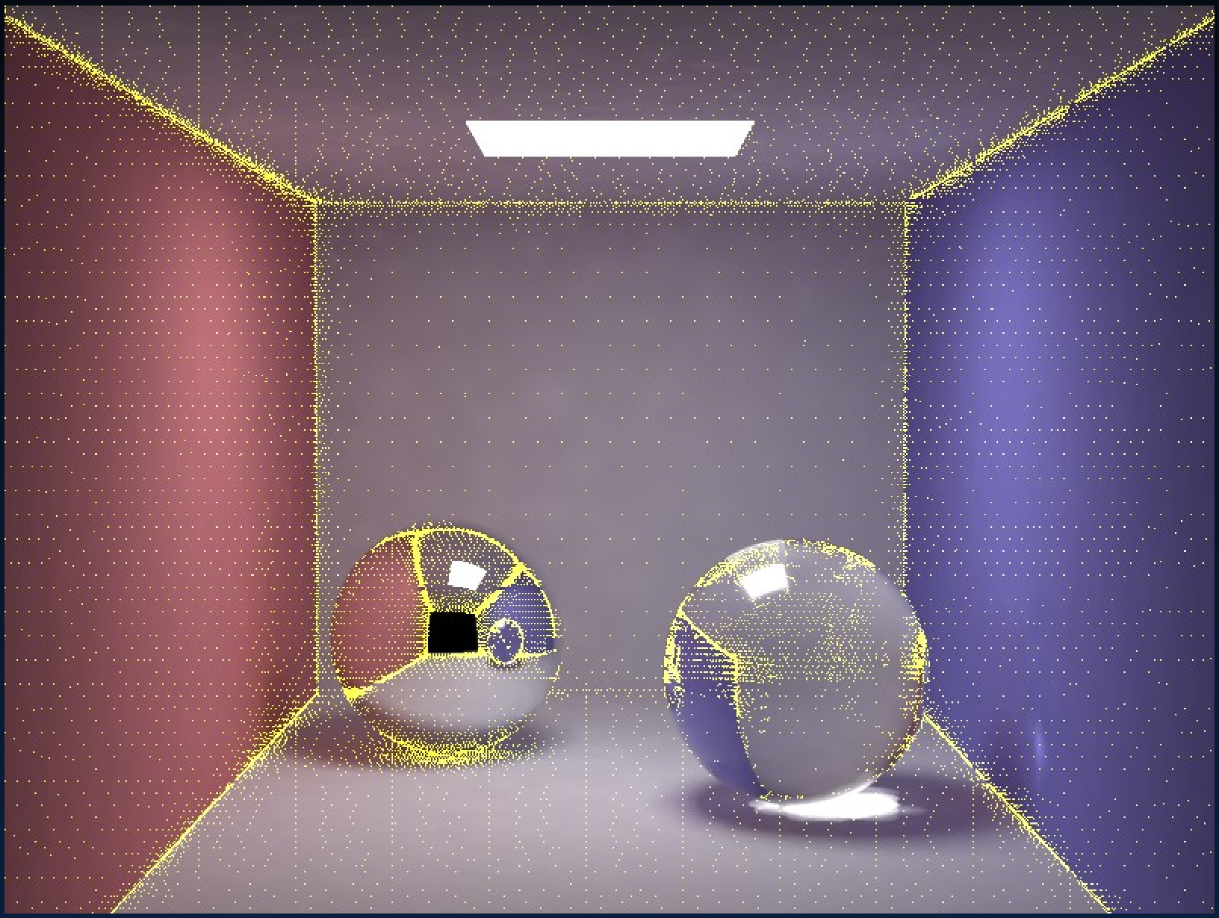
```
Find all irradiance samples with w(x) > q

if (samples found)
  interpolate
else
  compute new irradiance sample
```

Box: irradiance gradients



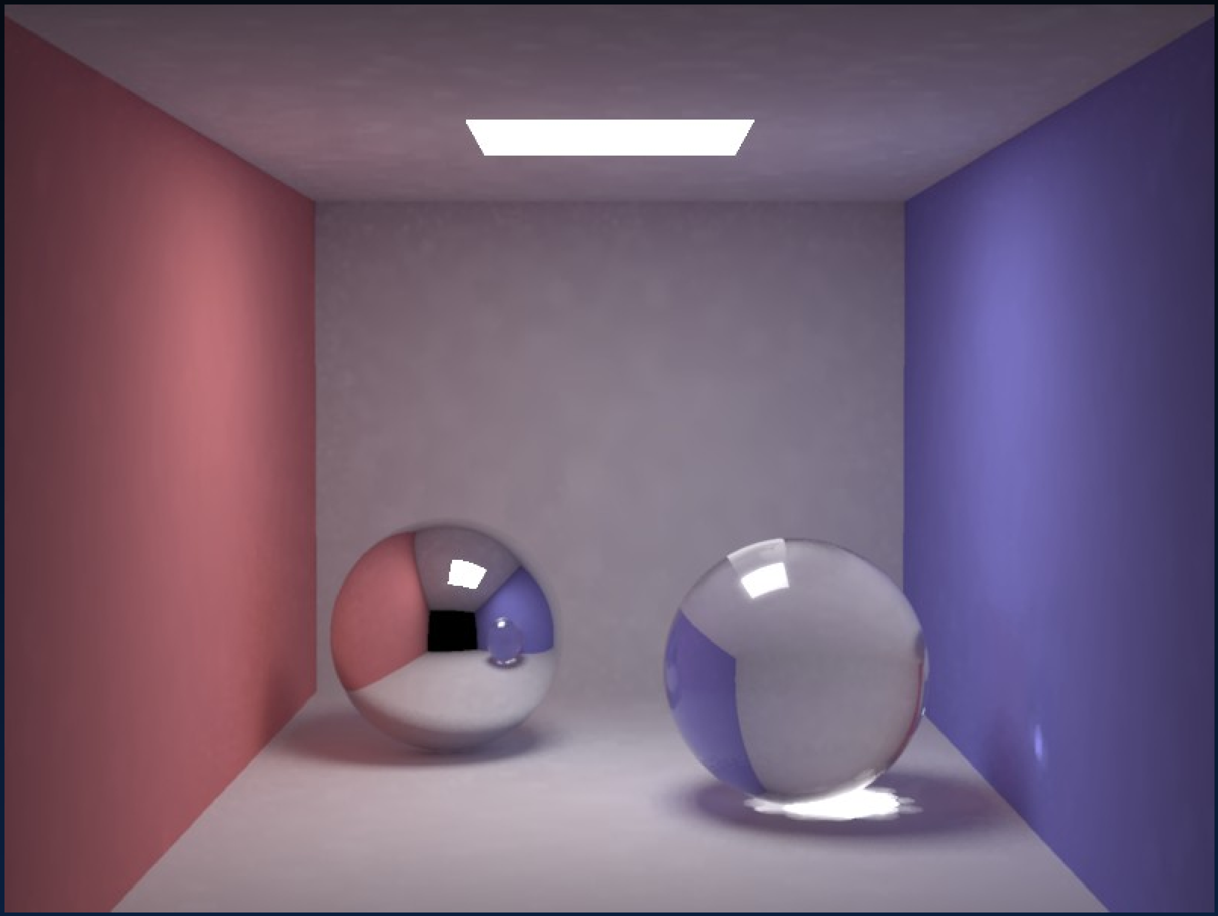
1000 sample rays, $w > 10$

Box: irradiance cache positions



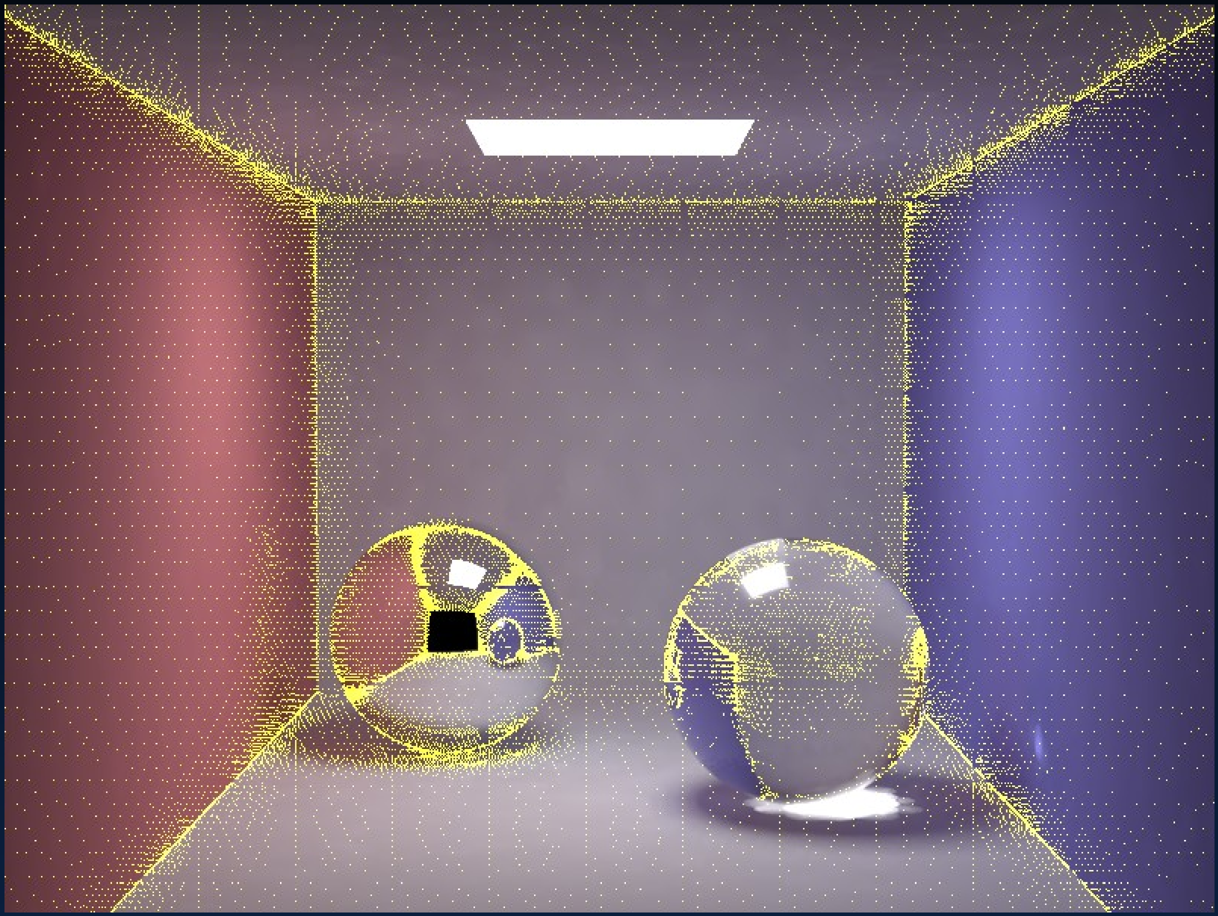
1000 sample rays, w$>$10

Box: irradiance gradients



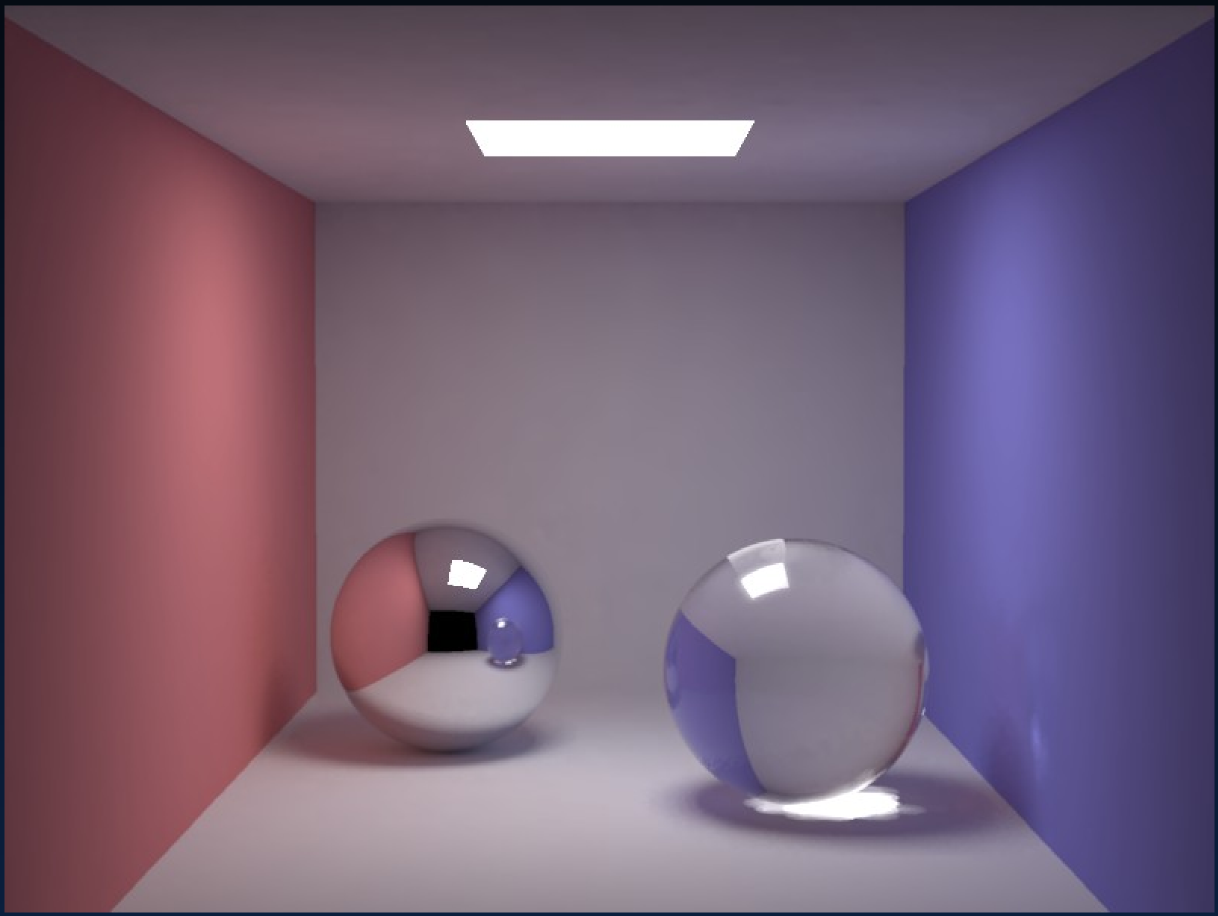
1000 sample rays, $w > 20$

Box: irradiance cache positions



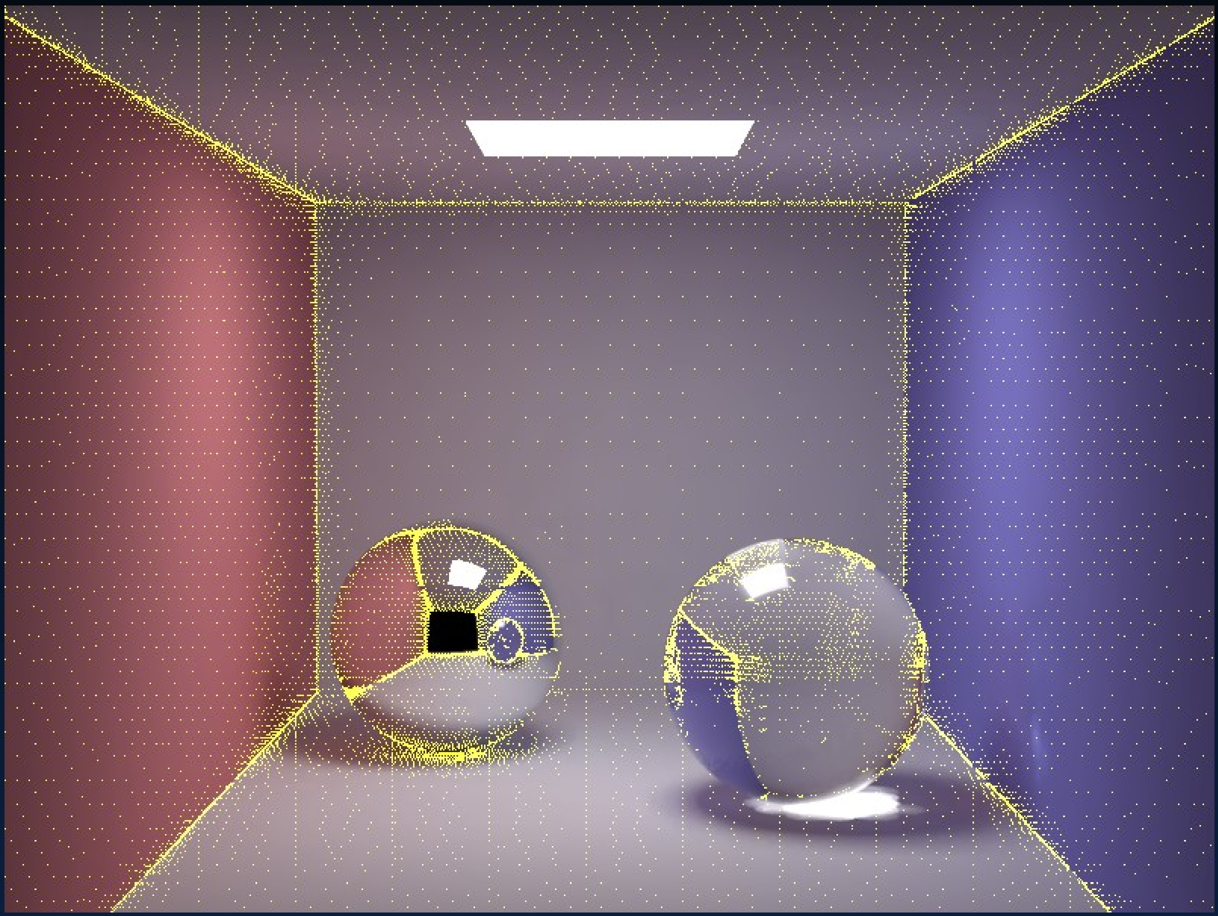
1000 sample rays, $w > 20$

Box: irradiance gradients



5000 sample rays, w>10

Box: irradiance cache positions



5000 sample rays, $w > 10$

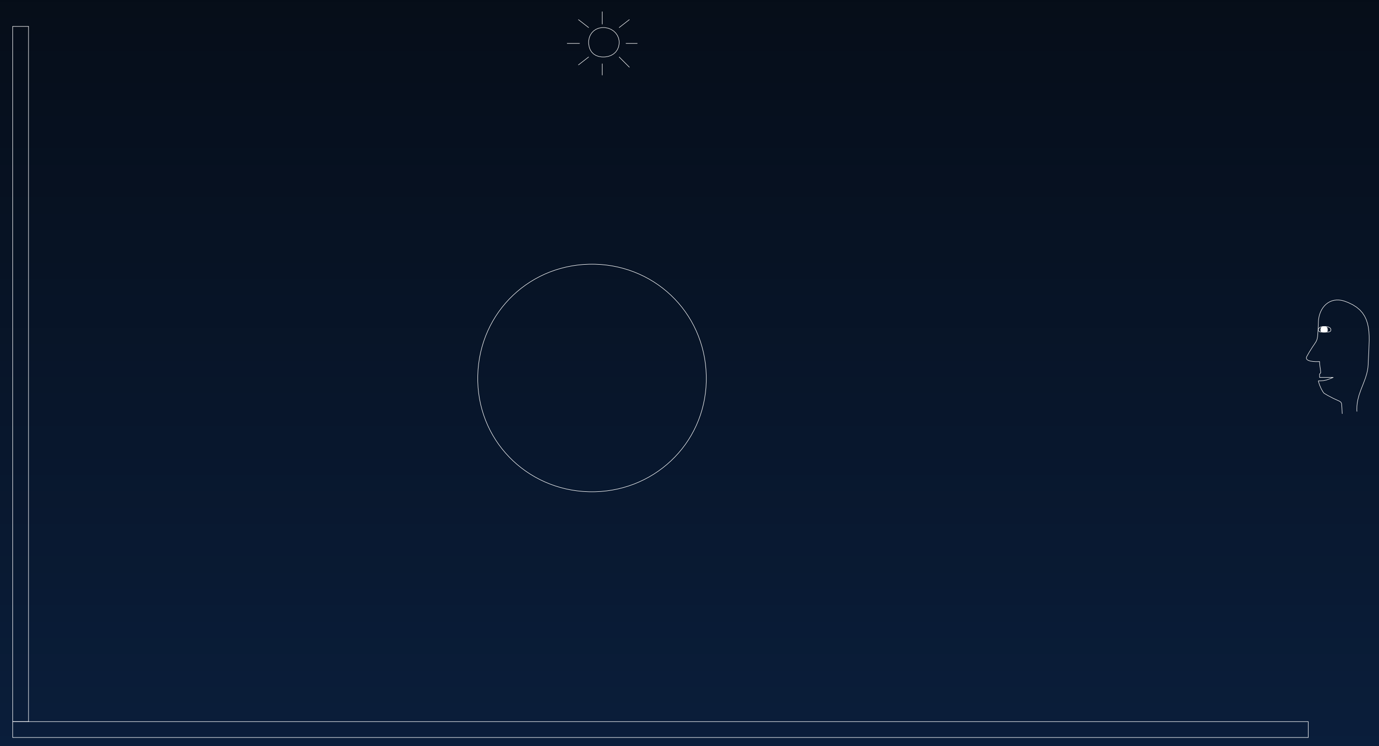
Photon Mapping

Two-pass method:

Pass 1 : Build a *photon map* using photon tracing

Pass 2 : Render the image using the photon map

A simple test scene



Building the Photon Map: Photon Tracing

Photons

The photon map datastructure

The photons are stored in a left balanced kd-tree

```
struct photon = {
  float position[3];
  rgbe power;          // power packed as 4 bytes
  char phi, theta;     // incoming direction
  short flags;
}
```

Radiance estimate

$$L(x,\vec{\omega}) \;=\; \int_{\Omega} f_r(x,\vec{\omega}',\vec{\omega})L'(x,\vec{\omega}')\cos\theta'\,d\omega$$

Radiance estimate

$$\begin{aligned}
L(x,\vec{\omega}) &= \int_\Omega f_r(x,\vec{\omega}',\vec{\omega}) L'(x,\vec{\omega}') \cos\theta' \, d\omega \\
&= \int_\Omega f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi^2(x,\vec{\omega}')}{d\omega \, \cos\theta' dA} \, \cos\theta' d\omega
\end{aligned}$$

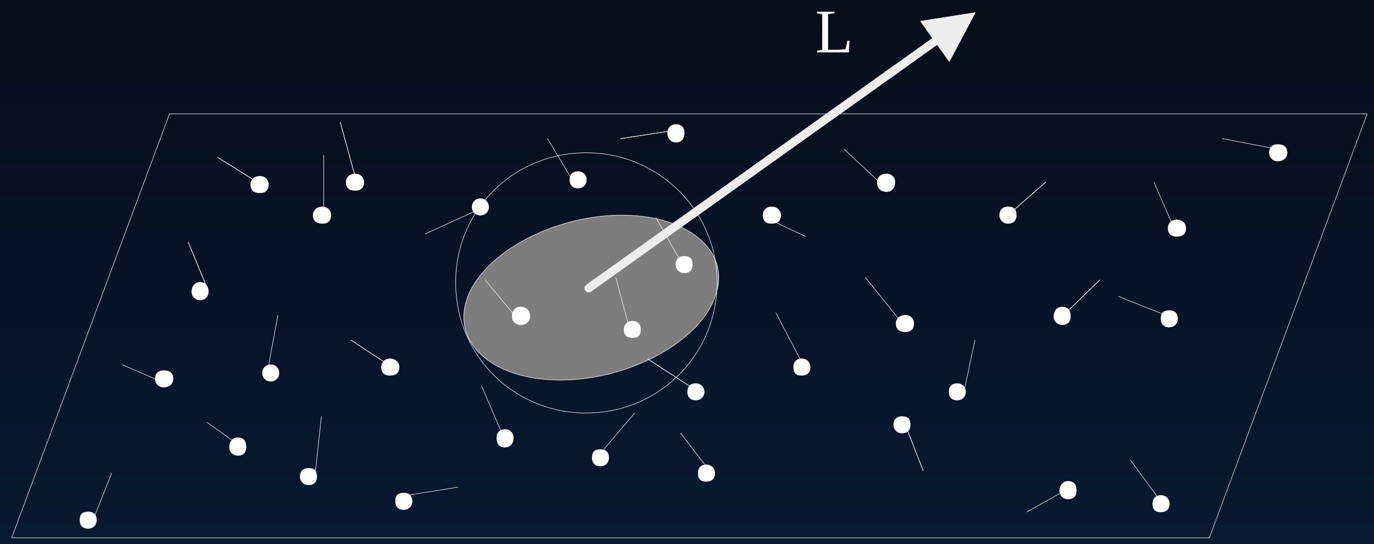
Radiance estimate

$$\begin{aligned} L(x,\vec{\omega}) &= \int_{\Omega} f_r(x,\vec{\omega}',\vec{\omega}) L'(x,\vec{\omega}') \cos\theta' \, d\omega \\ &= \int_{\Omega} f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi^2(x,\vec{\omega}')}{d\omega \, \cos\theta' dA} \, \cos\theta' d\omega \\ &= \int_{\Omega} f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi^2(x,\vec{\omega}')}{dA} \end{aligned}$$

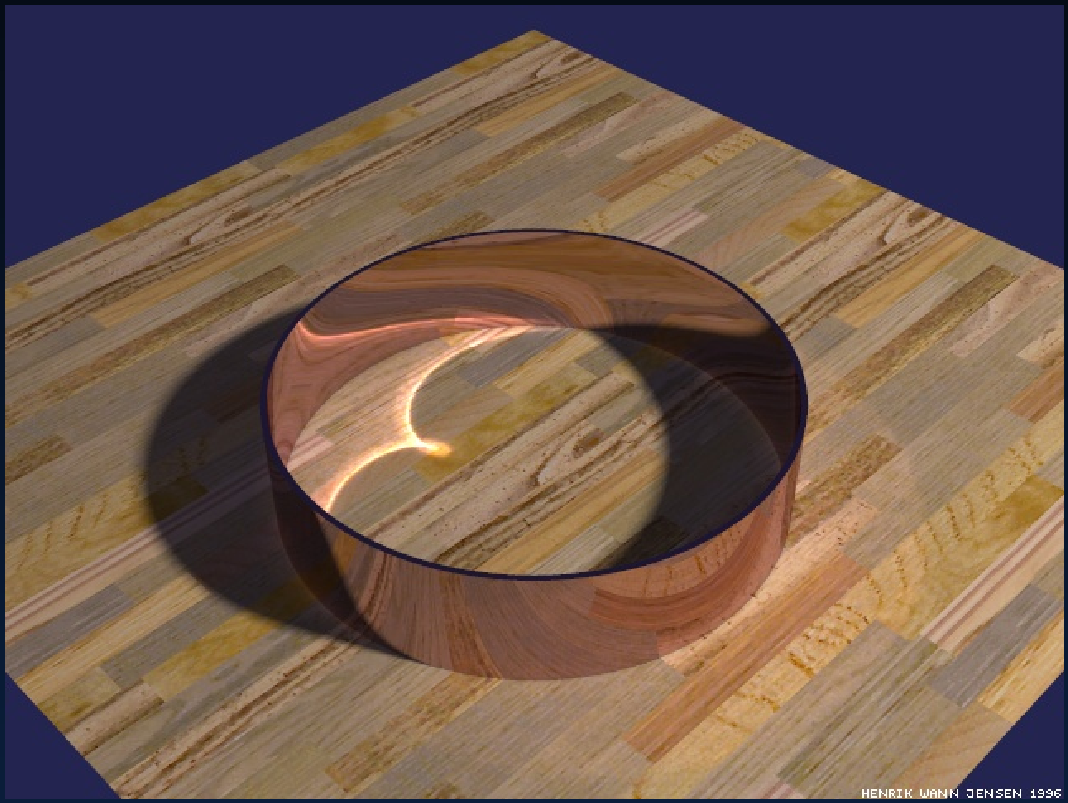
Radiance estimate

$$
\begin{aligned}
L(x,\vec{\omega}) &= \int_\Omega f_r(x,\vec{\omega}',\vec{\omega}) L'(x,\vec{\omega}') \cos\theta' \, d\omega \\
&= \int_\Omega f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi^2(x,\vec{\omega}')}{d\omega \, \cos\theta' dA} \cos\theta' d\omega \\
&= \int_\Omega f_r(x,\vec{\omega}',\vec{\omega}) \frac{d\Phi^2(x,\vec{\omega}')}{dA} \\
&\approx \sum_{p=1}^{n} f_r(x,\vec{\omega}'_p,\vec{\omega}) \frac{\Delta\Phi_p(x,\vec{\omega}'_p)}{\pi r^2}
\end{aligned}
$$

Radiance estimate



Reflection inside a metal ring



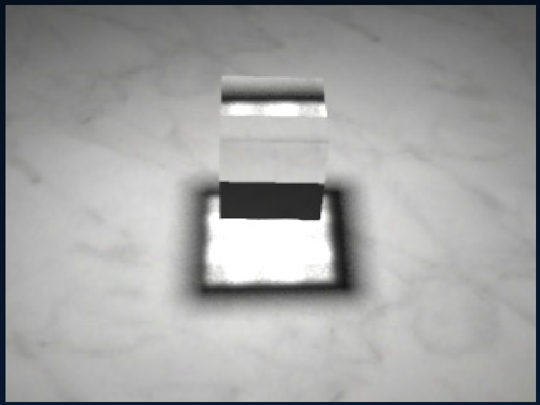
50000 photons / 50 photons in radiance estimate

Caustics on glossy surfaces

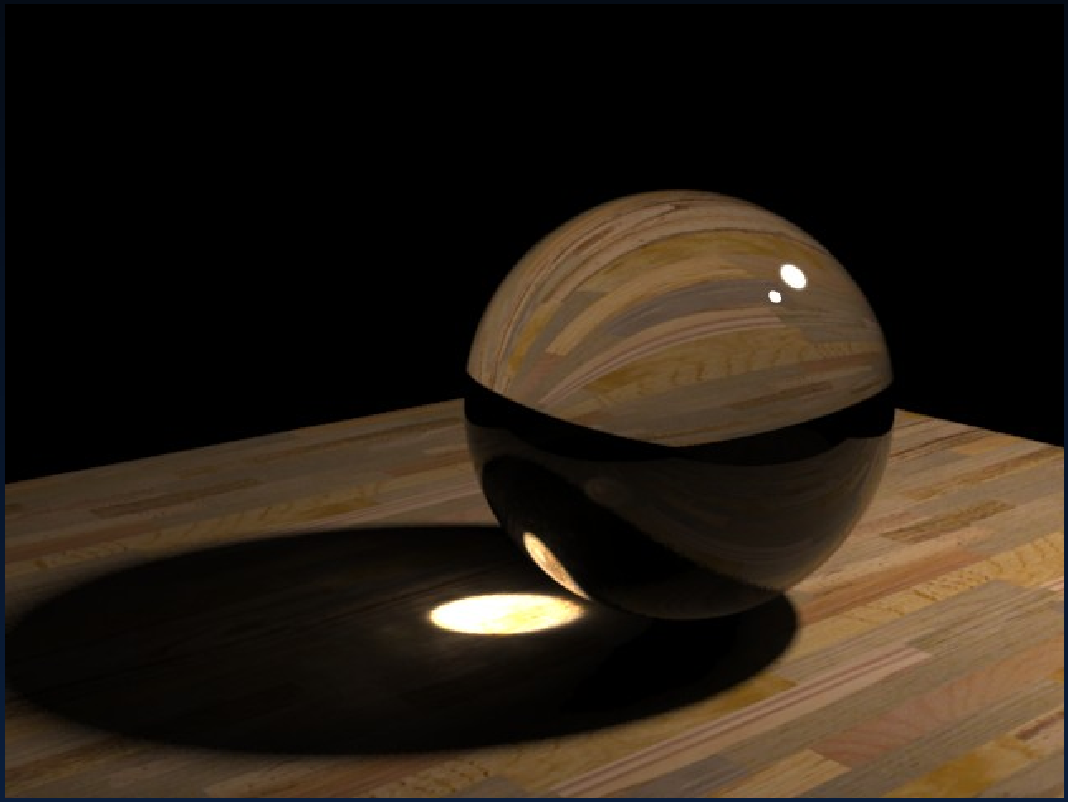
340000 photons / $\approx$ 100 photons in radiance estimate

Cognac glass

Cube caustic

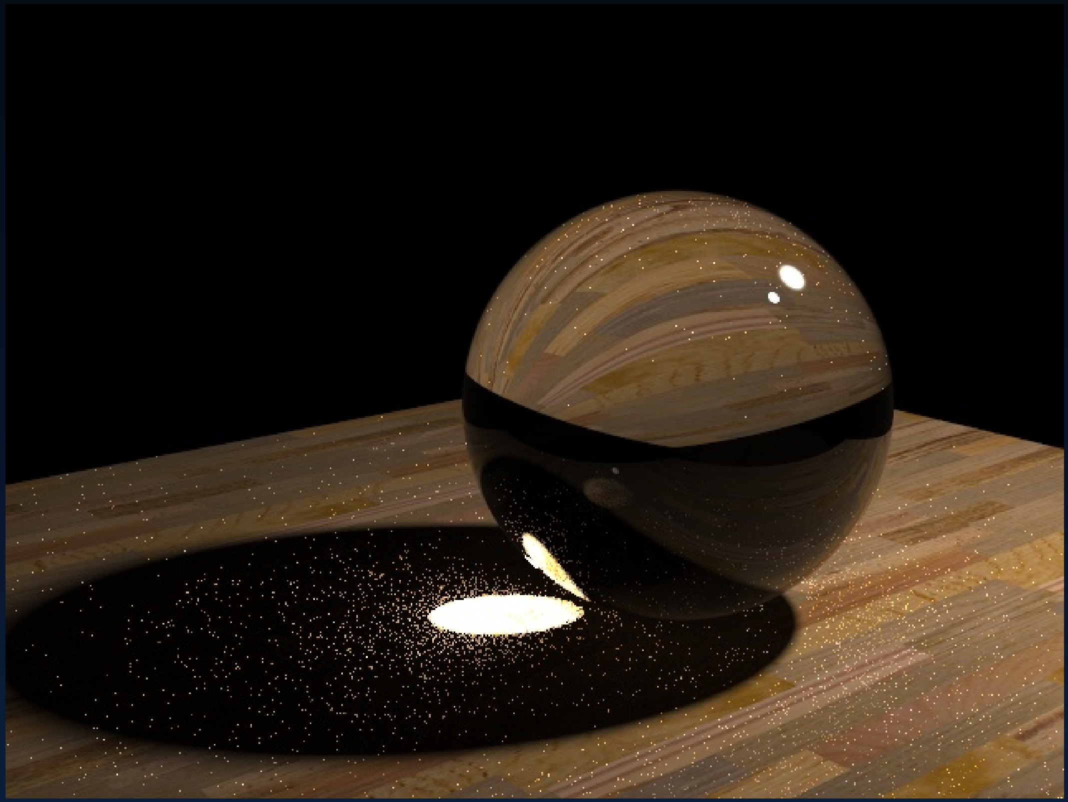


Caustic from a glass sphere



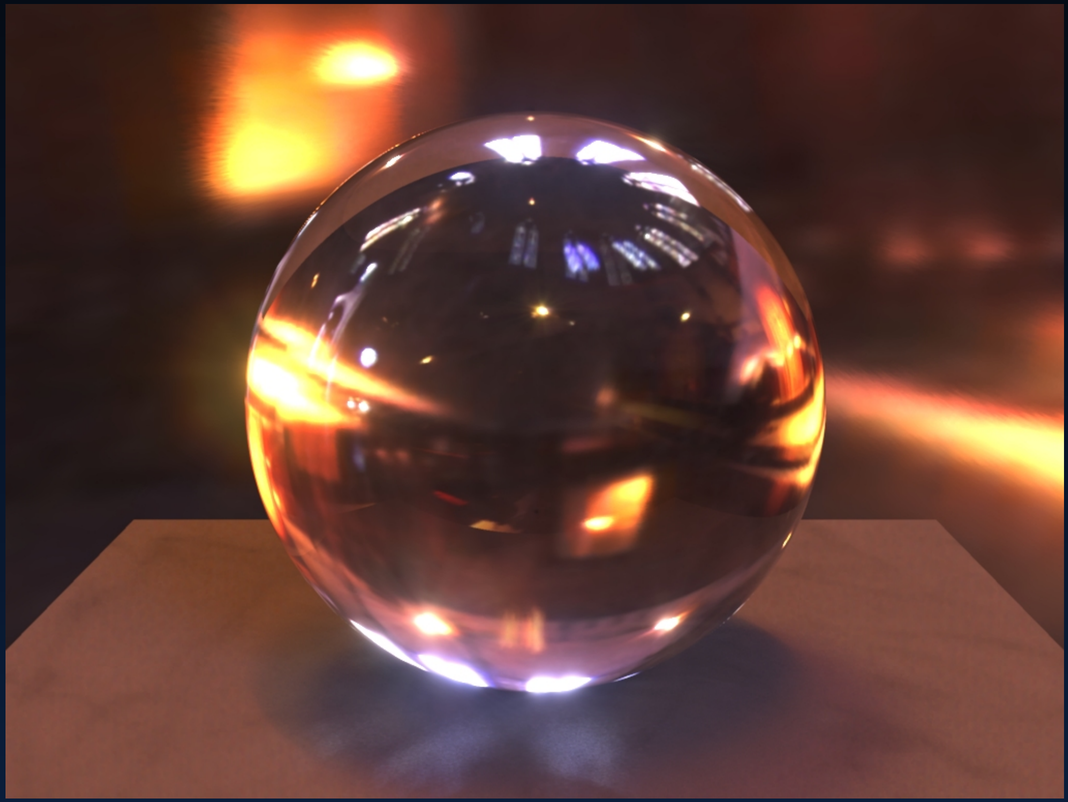
10000 photons / 50 photons in radiance estimate

Caustic from a glass sphere
Path tracing



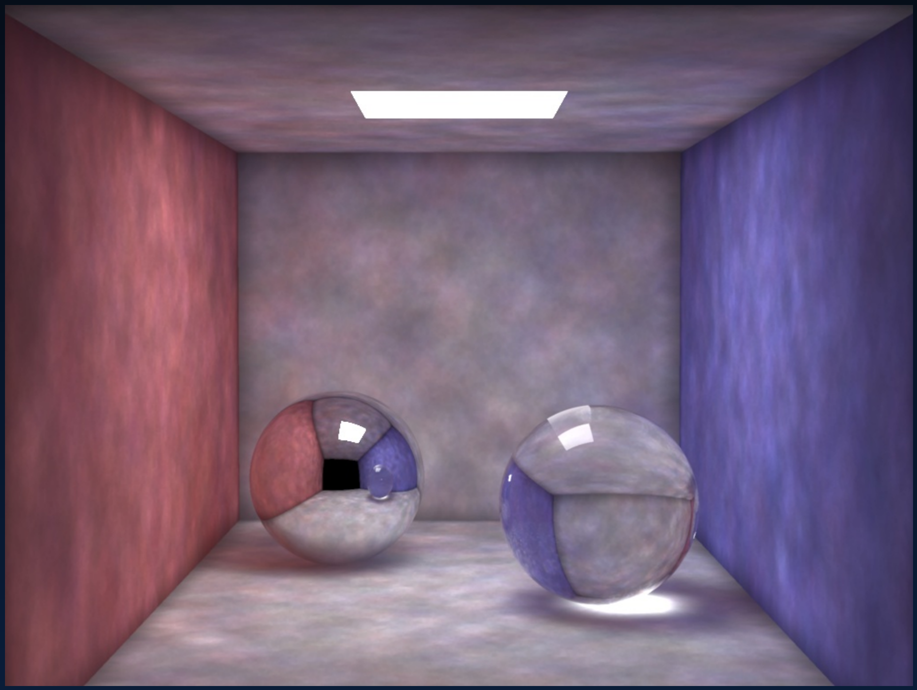
1000 paths/pixel

Caustic from a glass sphere in Grace Cathedral



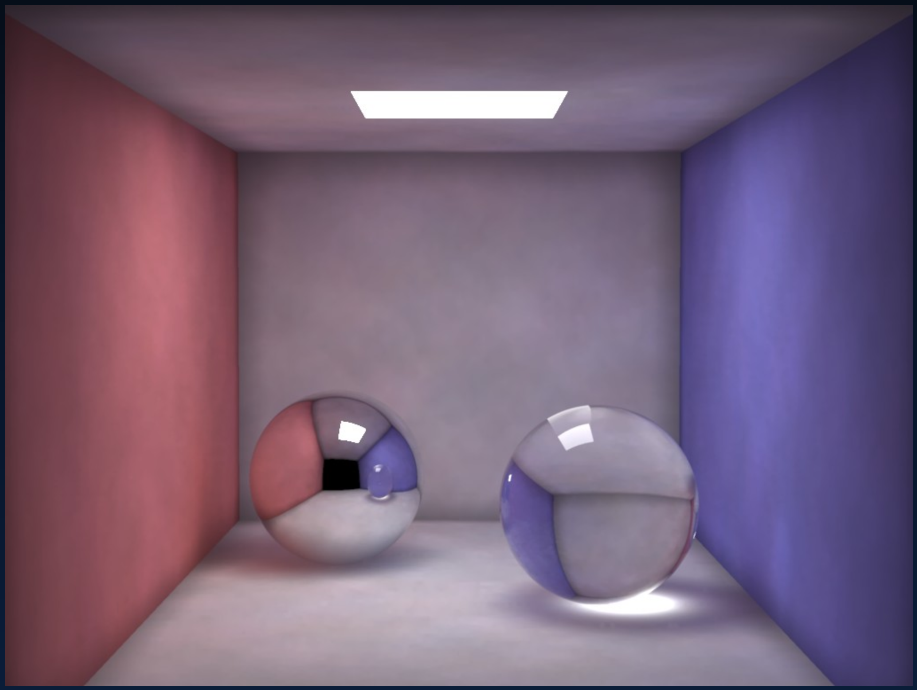
Using lightprobe from www.debevec.org

Direct visualization of the radiance estimate



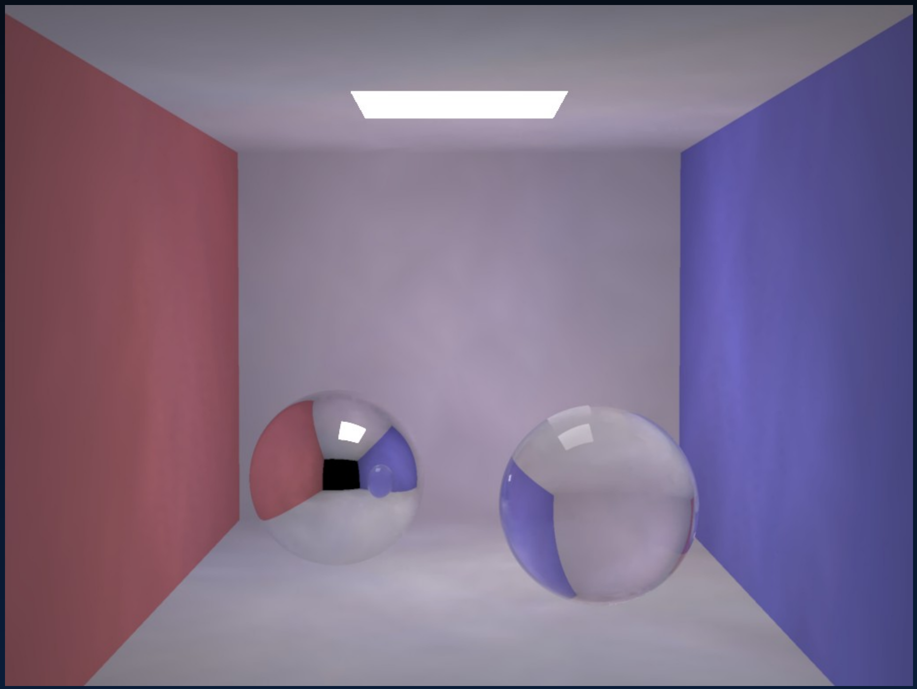
100000 photons / 50 photons in radiance estimate

Direct visualization of the radiance estimate



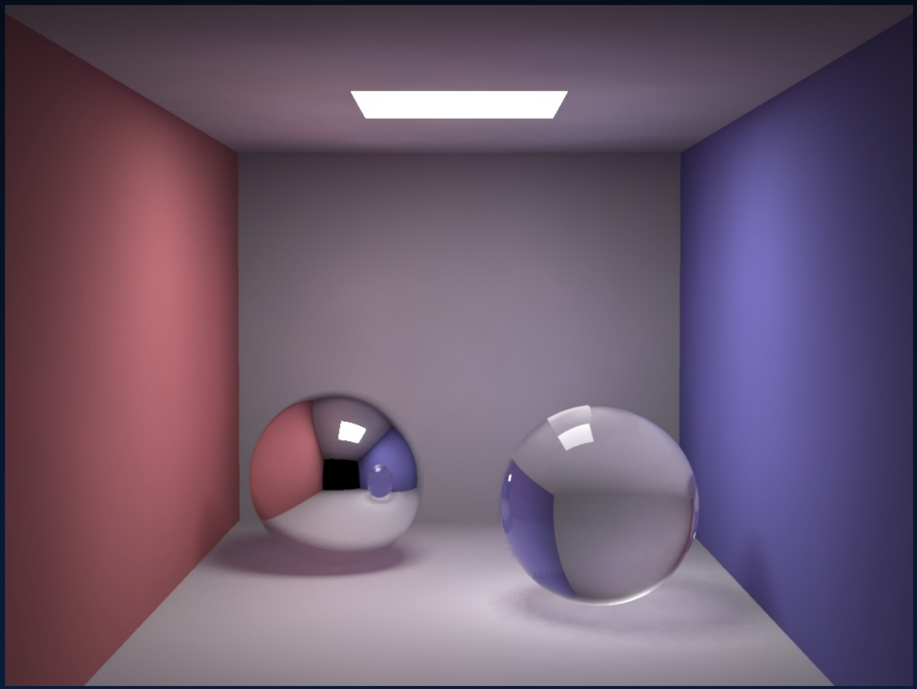
500000 photons / 500 photons in radiance estimate

Fast estimate



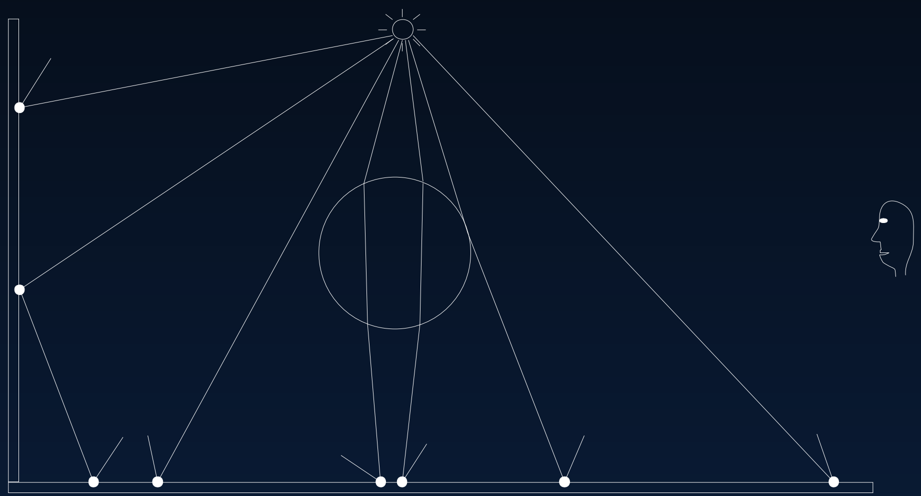
200 photons / 50 photons in radiance estimate

Only use photons for indirect irradiance

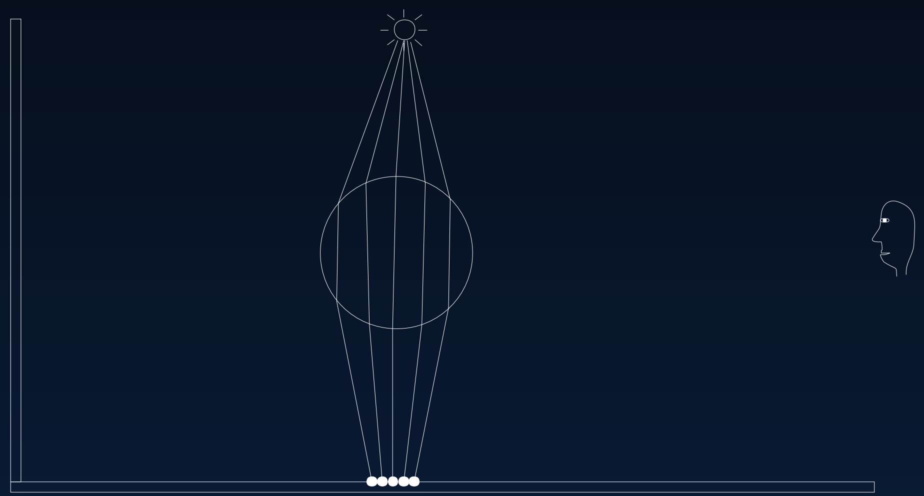


10000 photons / 500 photons in radiance estimate

Two photon maps

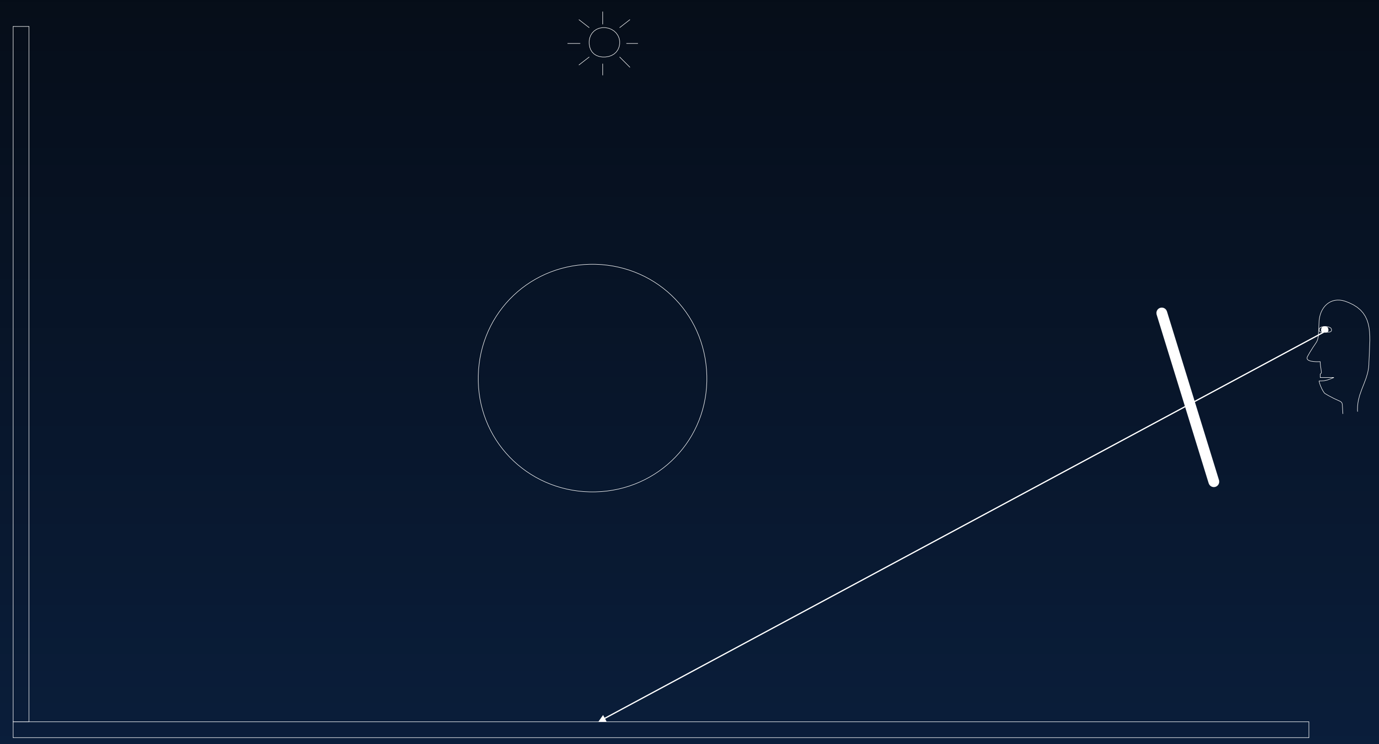


global photon map



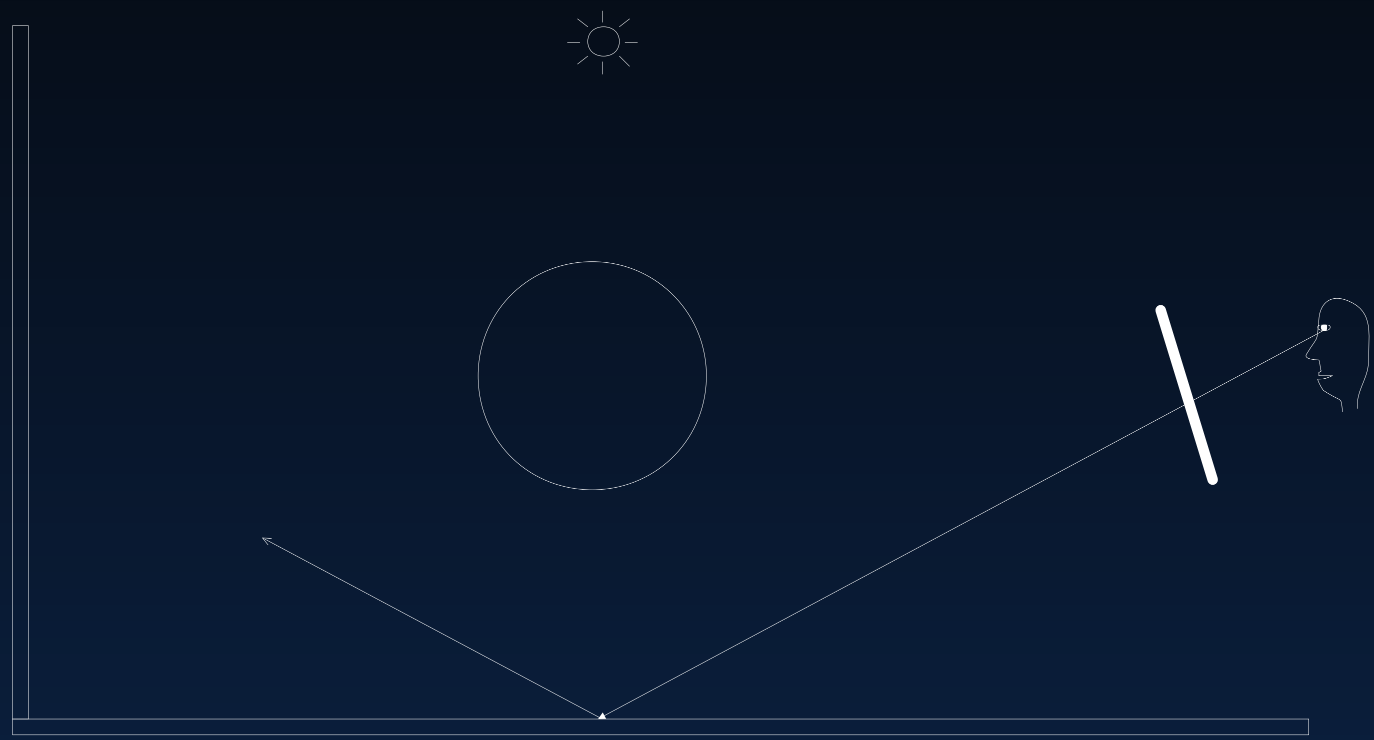
caustics photon map

Rendering

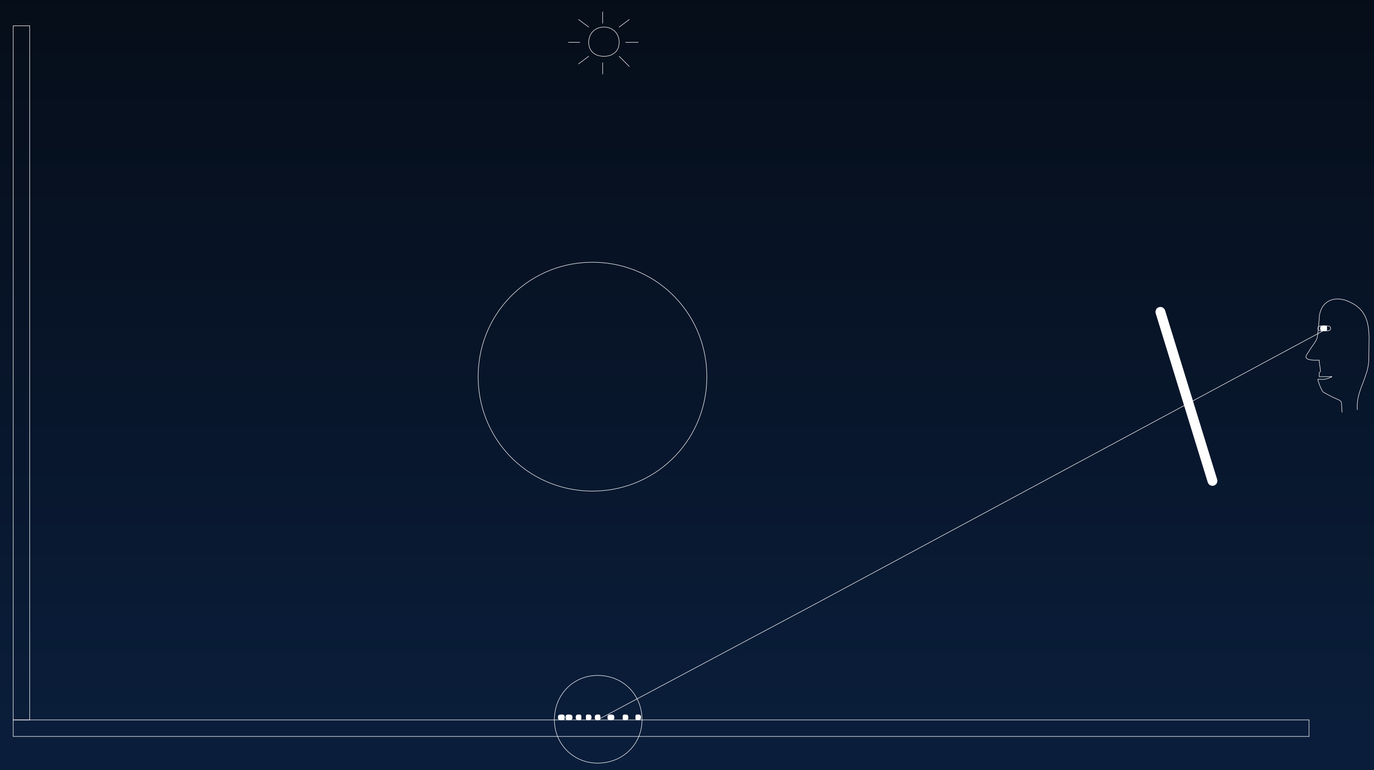


Rendering: direct illumination

Rendering: specular reflection



Rendering: caustics



Rendering: indirect illumination

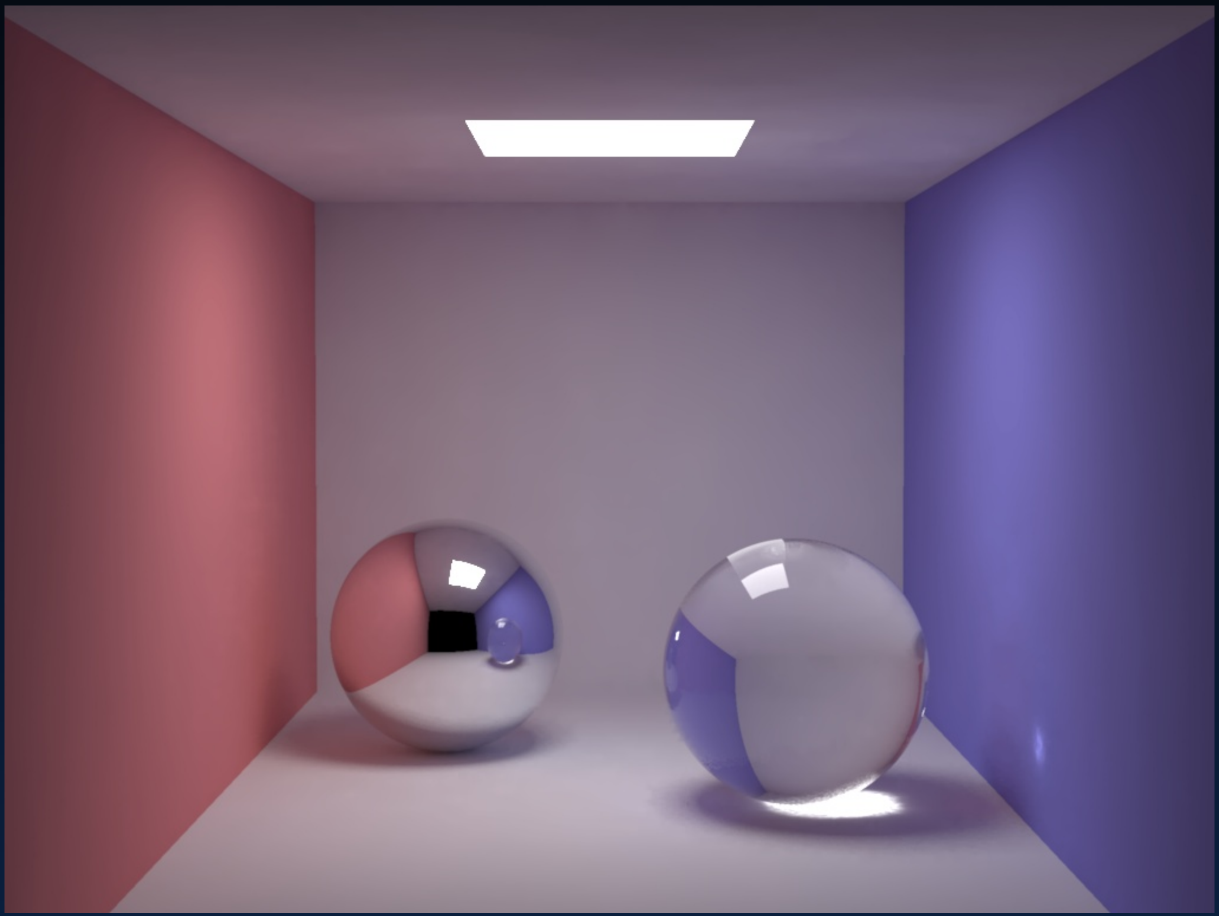
Two-pass method

Radiance = direct illumination +
specular reflection/transmission +
caustics +
soft indirect irradiance

Rendering Equation Solution

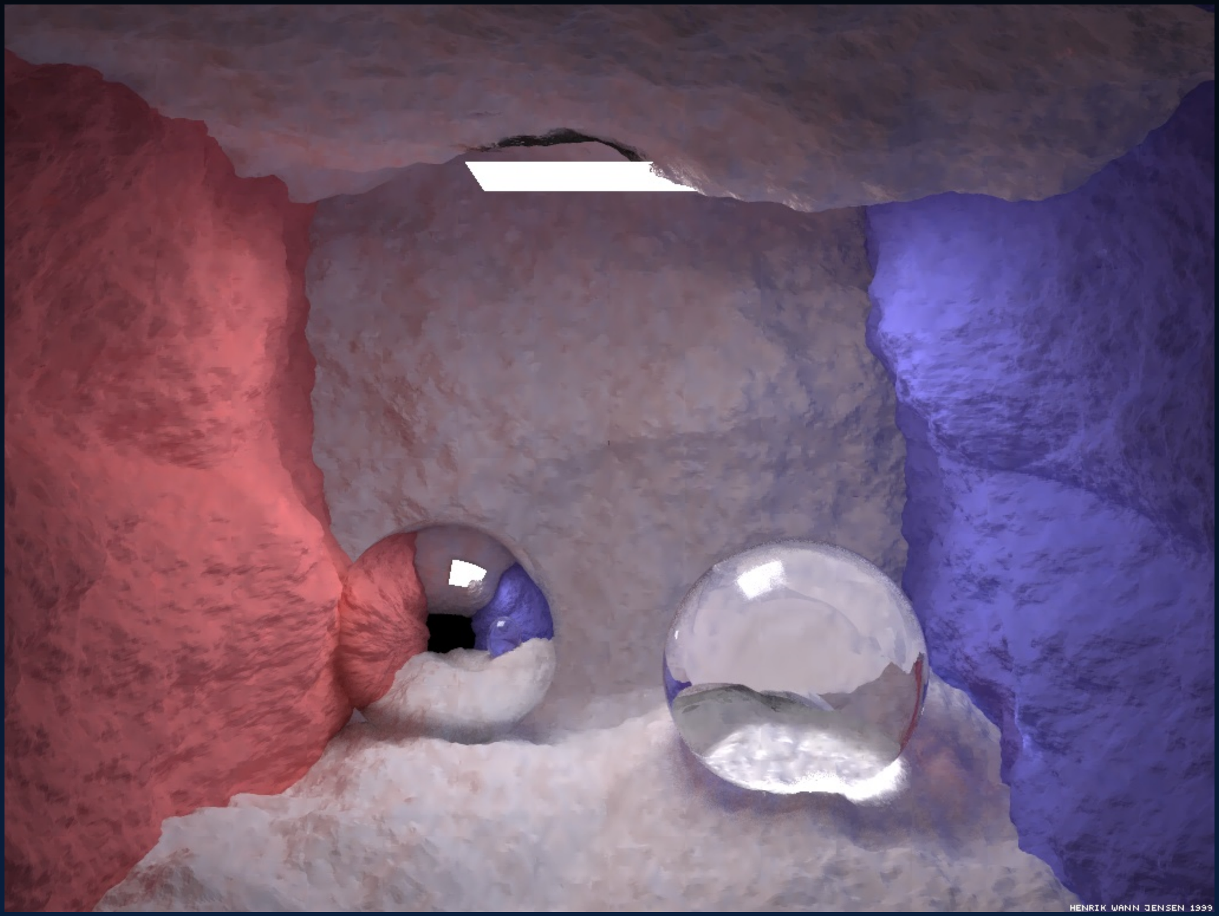
$$
\begin{aligned}
L_r(x,\vec{\omega}) &= \int_{\Omega_x} f_r(x,\vec{\omega}',\vec{\omega}) L_i(x,\vec{\omega}') \cos\theta_i \, d\omega_i' \\
&= \int_{\Omega_x} f_r(x,\vec{\omega}',\vec{\omega}) L_{i,l}(x,\vec{\omega}') \cos\theta_i \, d\omega_i' + \\
&\quad \int_{\Omega_x} f_{r,s}(x,\vec{\omega}',\vec{\omega}) (L_{i,c}(x,\vec{\omega}') + L_{i,d}(x,\vec{\omega}')) \cos\theta_i \, d\omega_i' + \\
&\quad \int_{\Omega_x} f_{r,d}(x,\vec{\omega}',\vec{\omega}) L_{i,c}(x,\vec{\omega}') \cos\theta_i \, d\omega_i' + \\
&\quad \int_{\Omega_x} f_{r,d}(x,\vec{\omega}',\vec{\omega}) L_{i,d}(x,\vec{\omega}') \cos\theta_i \, d\omega_i' \, .
\end{aligned}
$$

Box



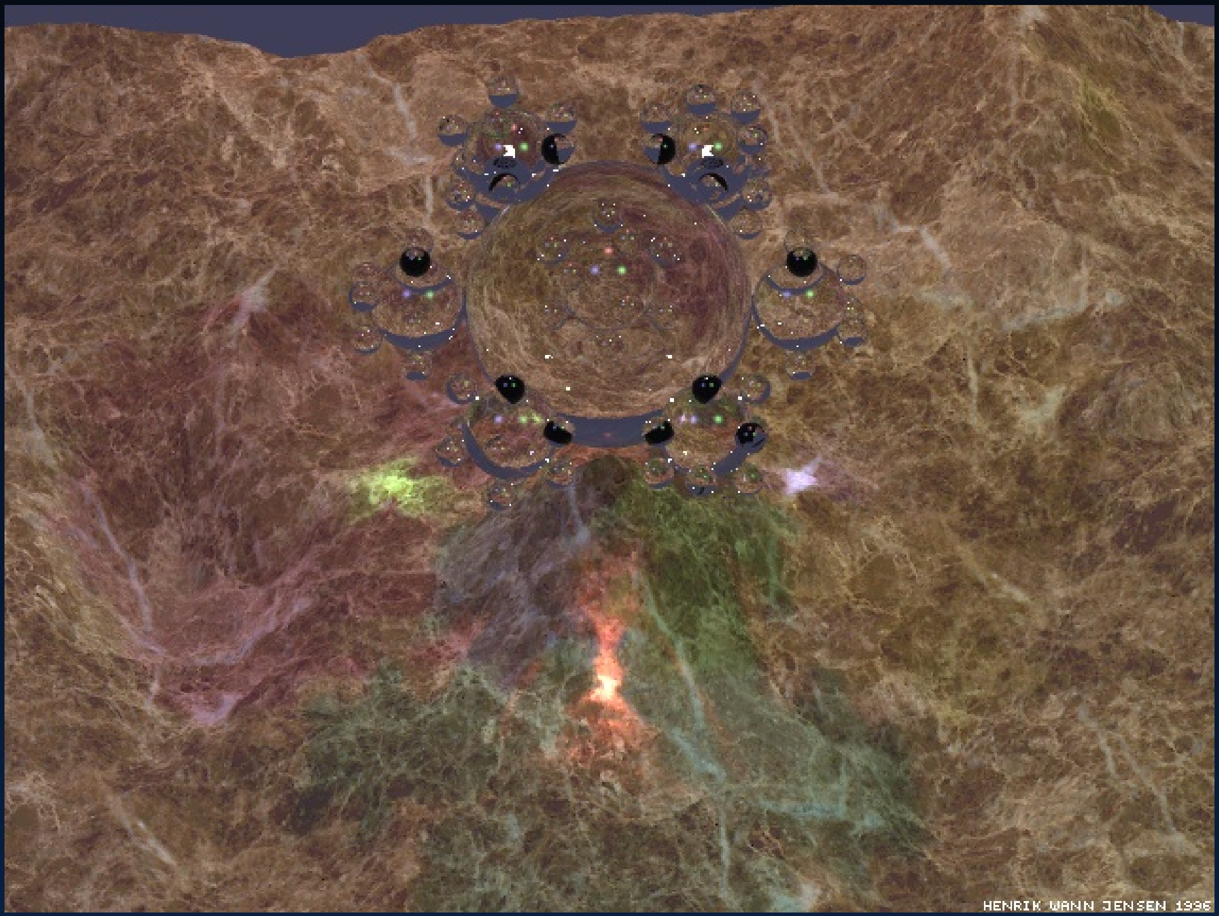
200000 global photons, 50000 caustic photons

Fractal box

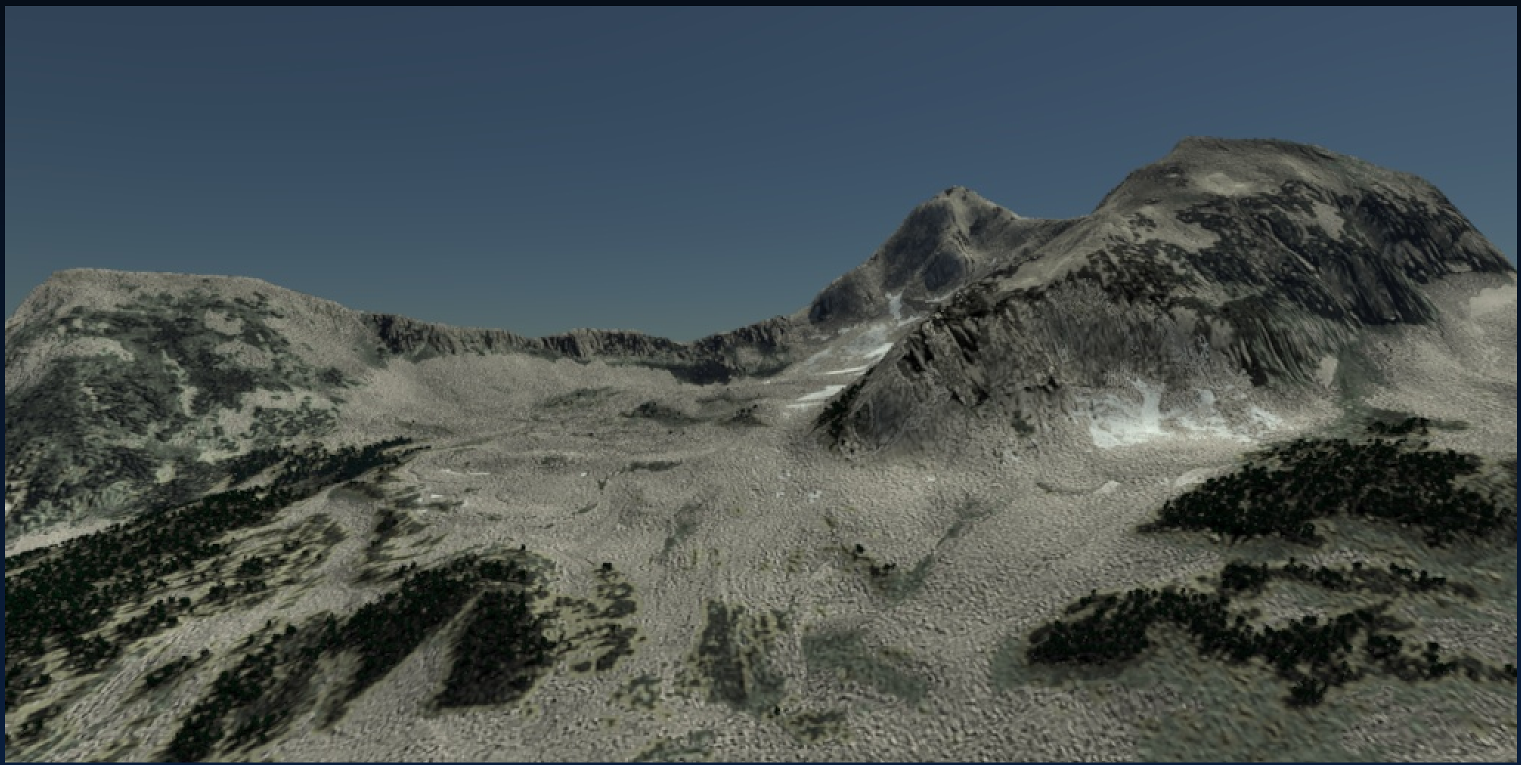


200000 global photons, 50000 caustic photons

Sphereflake caustic



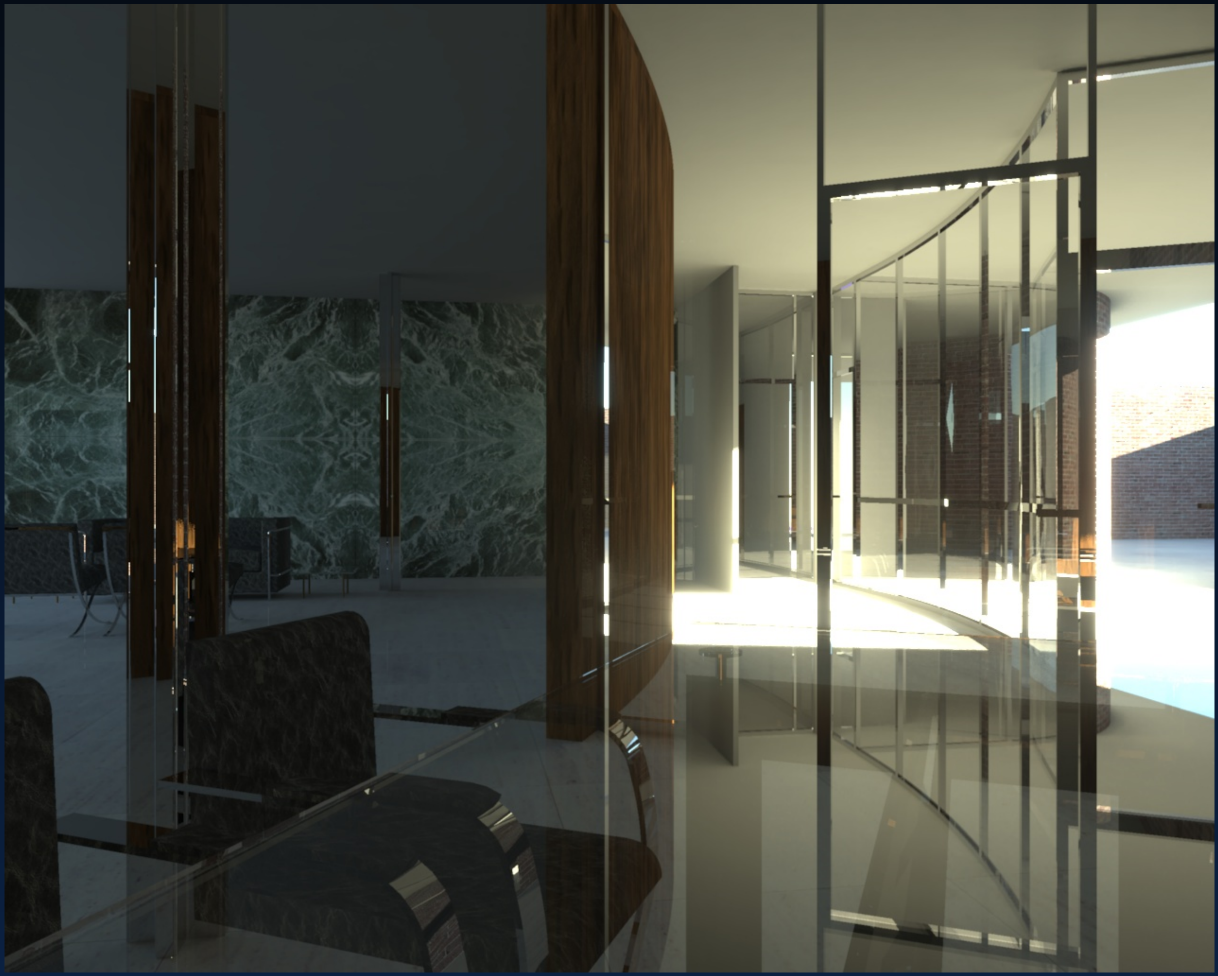
Little Matterhorn



Mies house (swimmingpool)

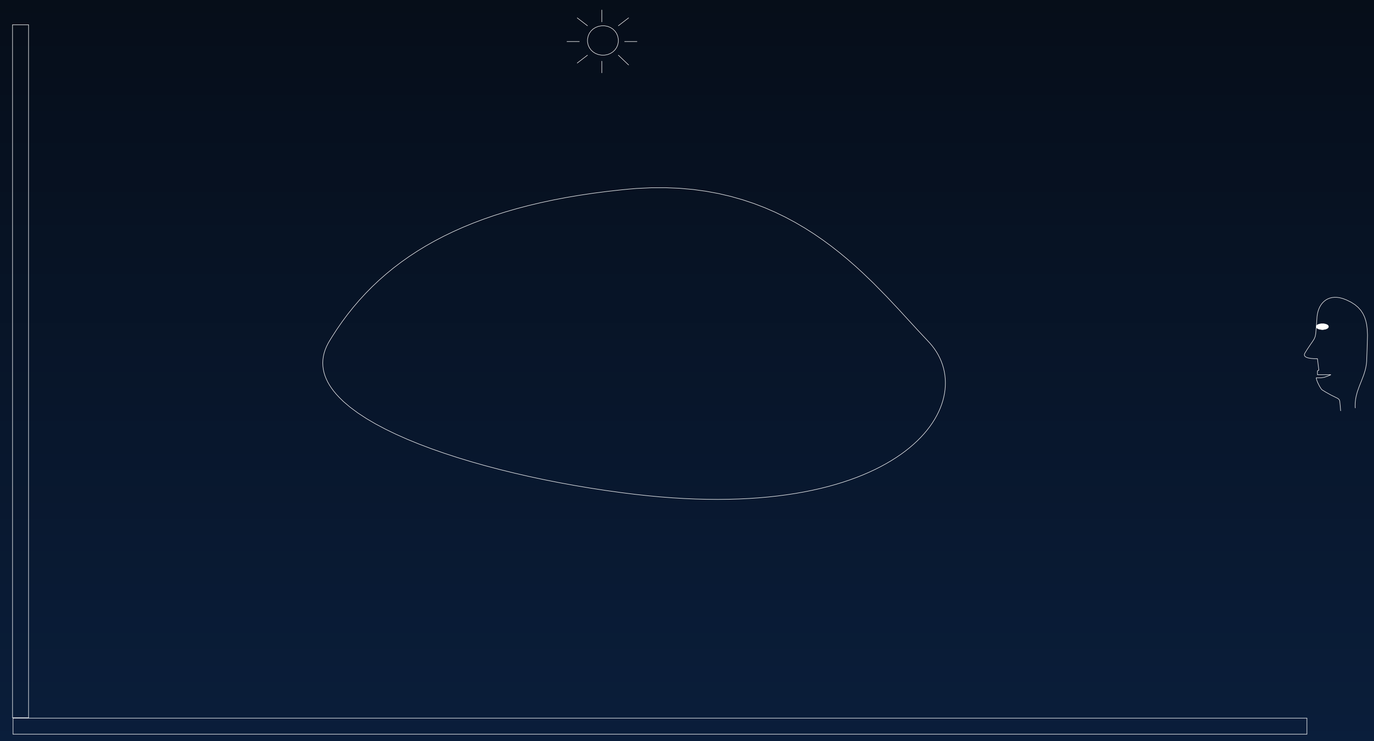
Mies house (3pm)

Mies house (6pm)



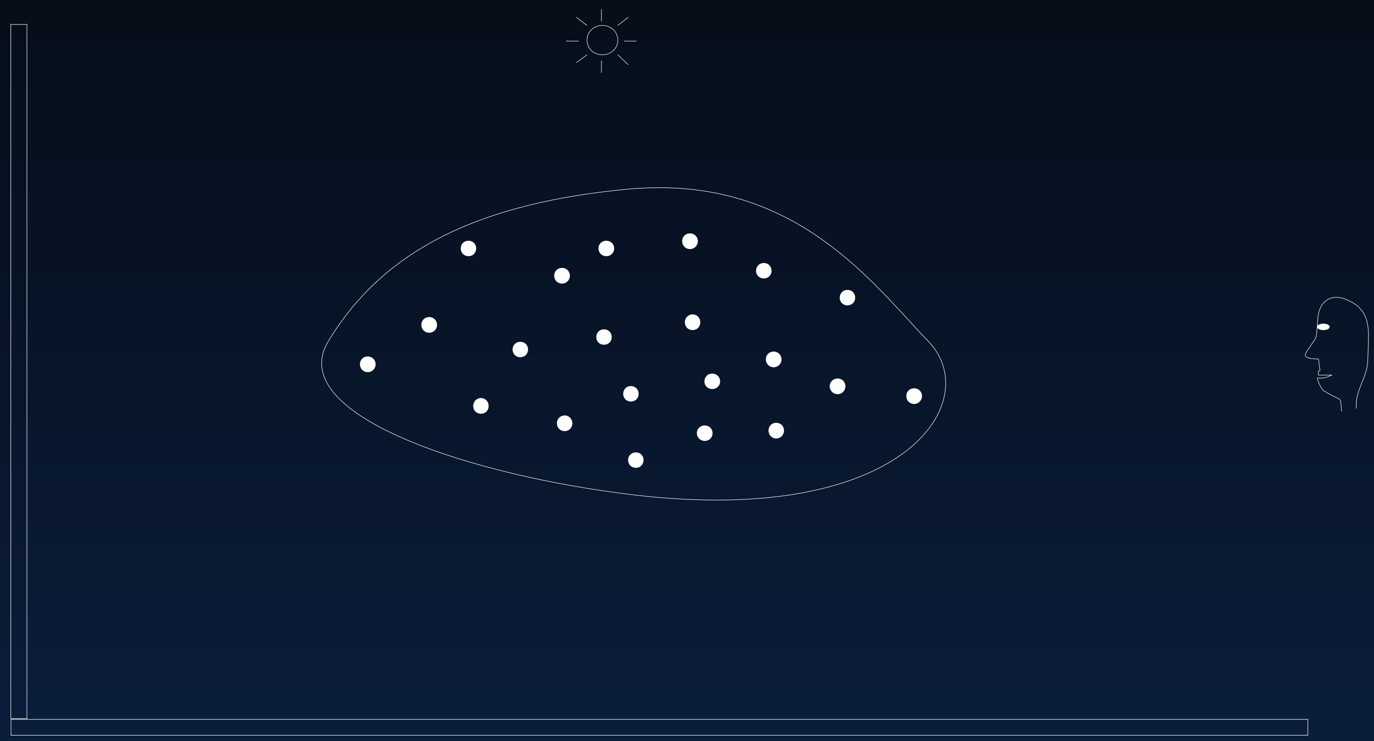
Mies house (7pm)

Participating media

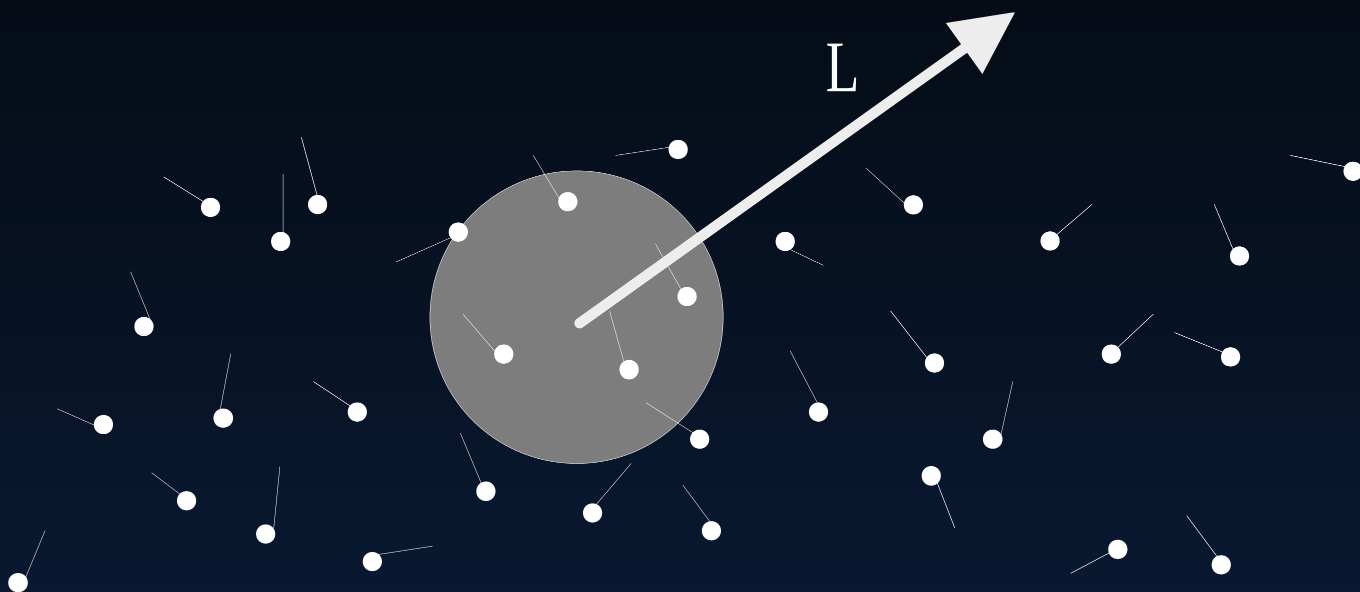


Participating media: photon tracing

The volume photon map

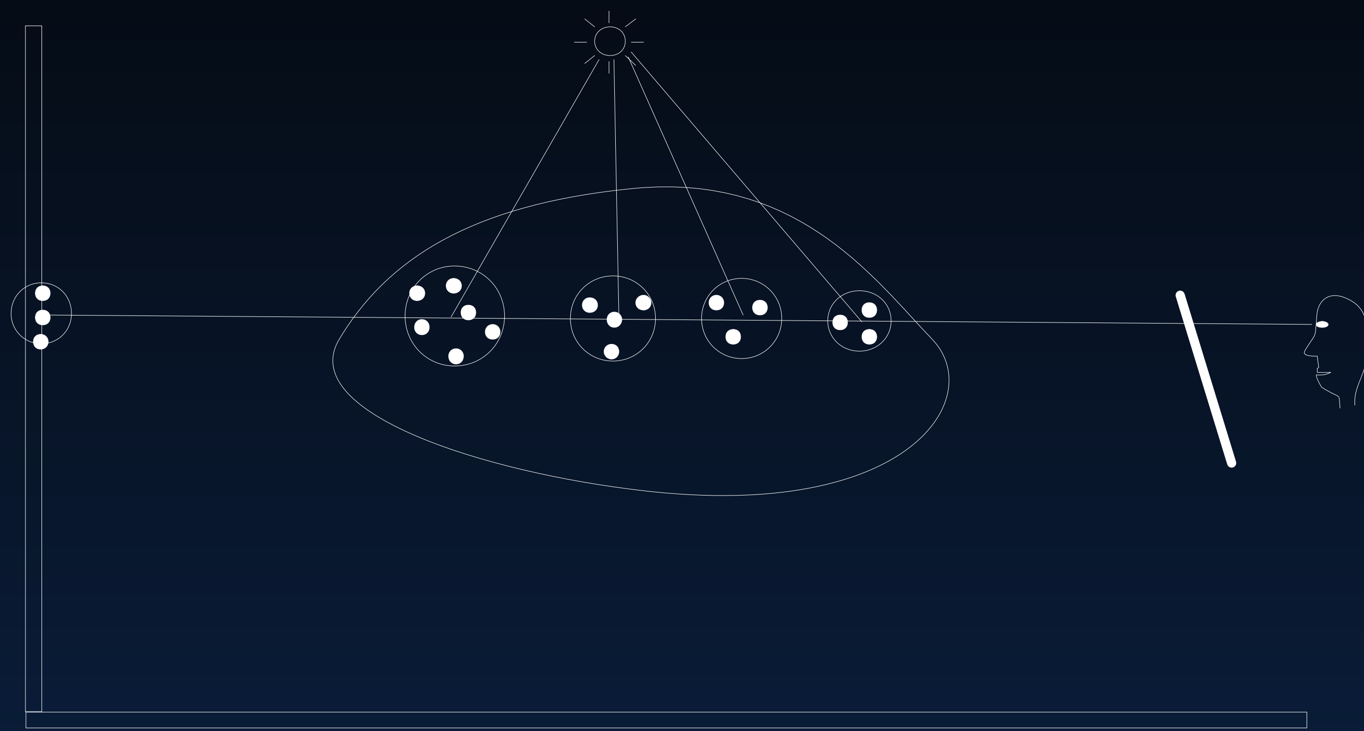


The volume radiance estimate



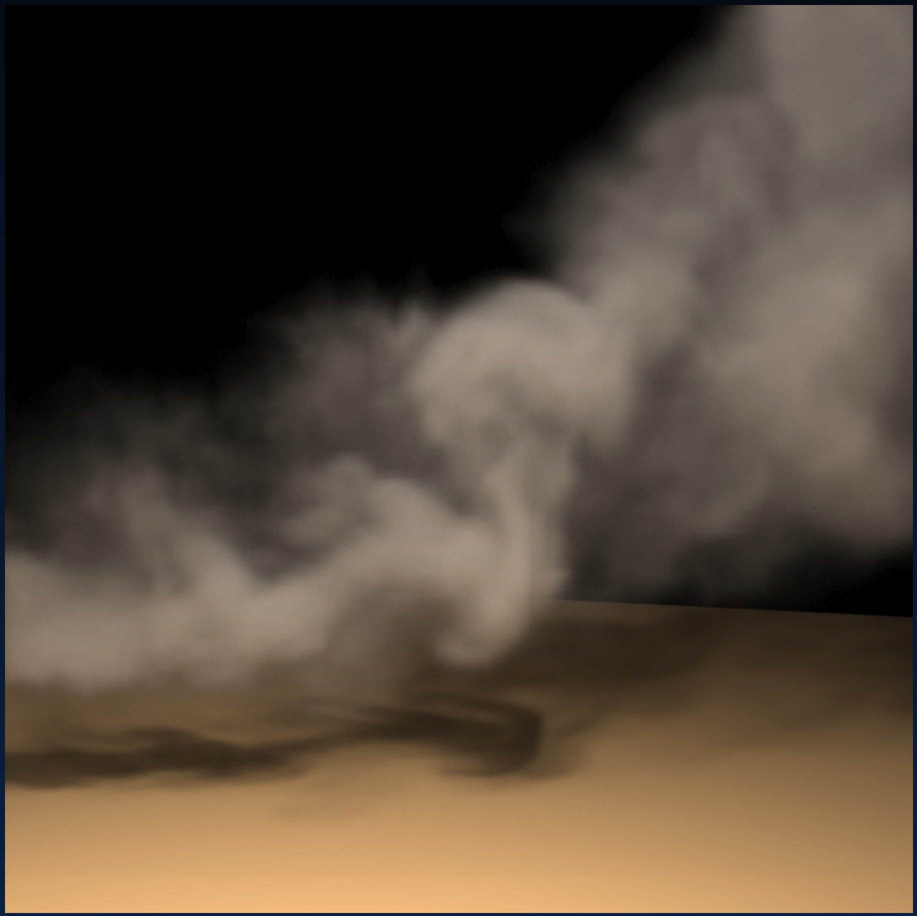
$$(\vec{\omega} \cdot \nabla) L(x, \vec{\omega}) = \sum_{p=1}^{n} p(x, \vec{\omega}'_p, \vec{\omega}) \frac{\Delta \Phi_p(x, \vec{\omega}'_p)}{\frac{4}{3}\pi r^3}$$

Rendering participating media

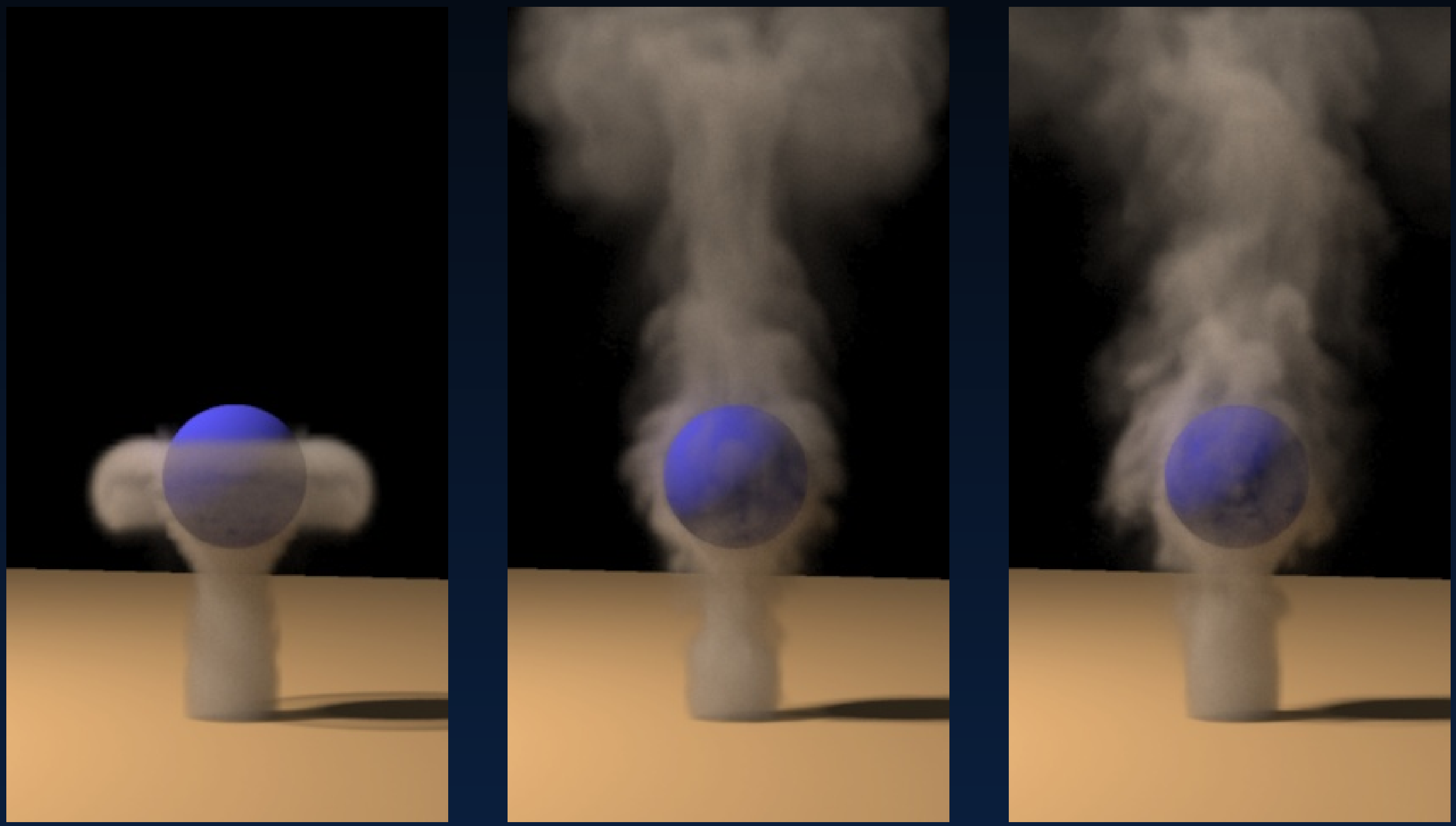


Volume caustic

Rising smoke



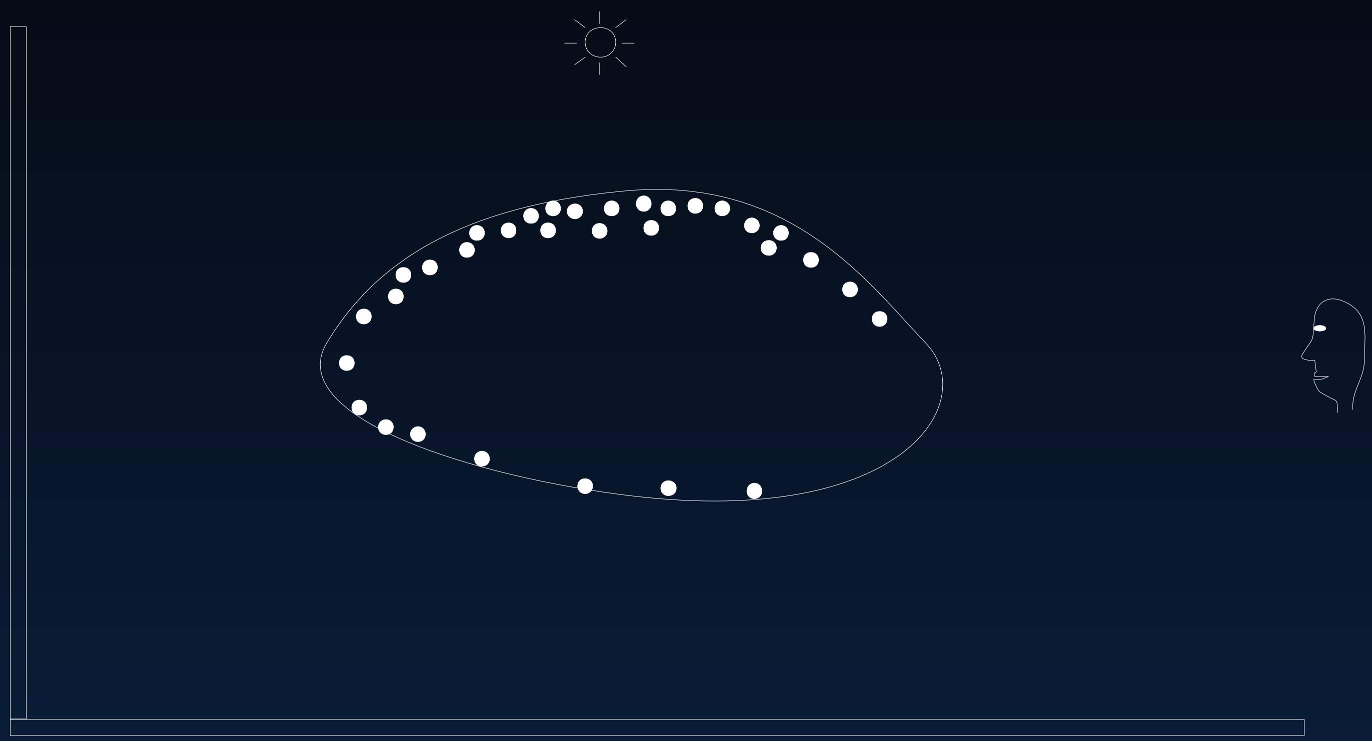
Rising smoke



Subsurface scattering

- Skin
- Marble
- Actually most materials

Subsurface scattering



David (subsurface scattering)

David (subsurface scattering)

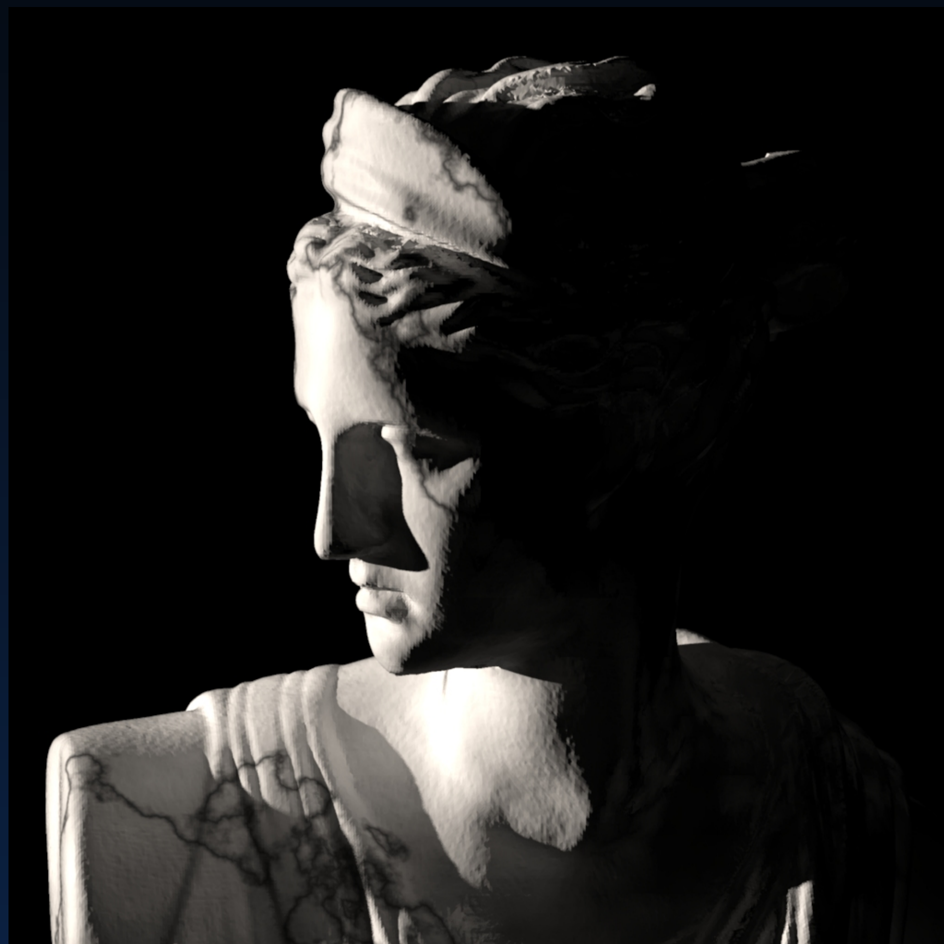
Diana the Huntress

Diana the Huntress

Diana the Huntress

Diana the Huntress

Diana the Huntress: no subsurface scattering



Diana the Huntress: subsurface scattering

More information

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