Radiosity

Classic radiosity = finite element method

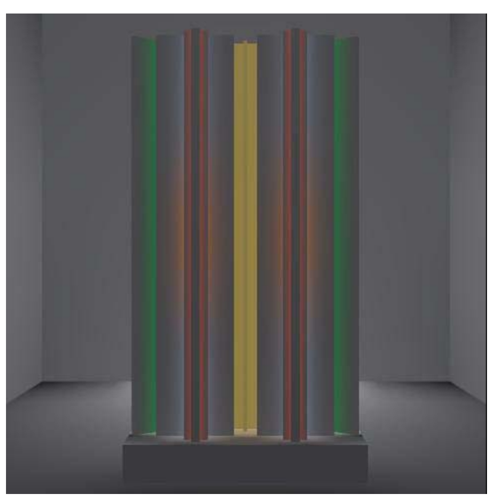
Assumptions

- **Diffuse reflectance**
- **Usually polygonal surfaces**

Advantages

- **Soft shadows and indirect lighting**
- **View independent solution**
- **Precompute for a set of light sources**
- **Useful for walkthroughs**

Early Radiosity

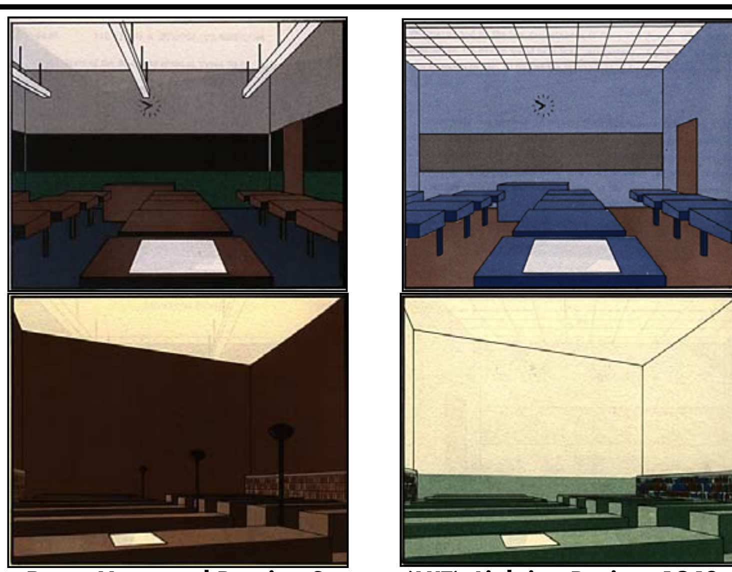


From Goral, Torrance, Greenberg, Battaile 1984

Early Radiosity

From Cohen, Chen, Wallace and Greenberg 1988

First Radiosity Pictures ...



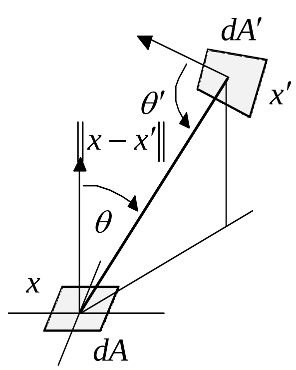
Parry Moon and Domina Spencer (MIT), Lighting Design, 1948

Finite Element Method

The Radiosity Equation

Assume diffuse reflection only

Solve for radiosity (2D function)



$$B(x) = B_e(x) + \rho(x) E(x)$$

$$B(x) = B_e(x) + \rho(x) \int_{M^2} F(x, x') B(x')\, dA'$$

$$F(x, x') = \frac{\cos\theta \cos\theta'}{\pi \|x - x'\|^2} V(x, x')$$

Classic Radiosity Algorithm

1. Mesh Surfaces into Elements
2. Compute Form Factors Between Elements
3. Solve Linear System for Radiosities
4. Reconstruct and Display Solution

Simple Room Scene

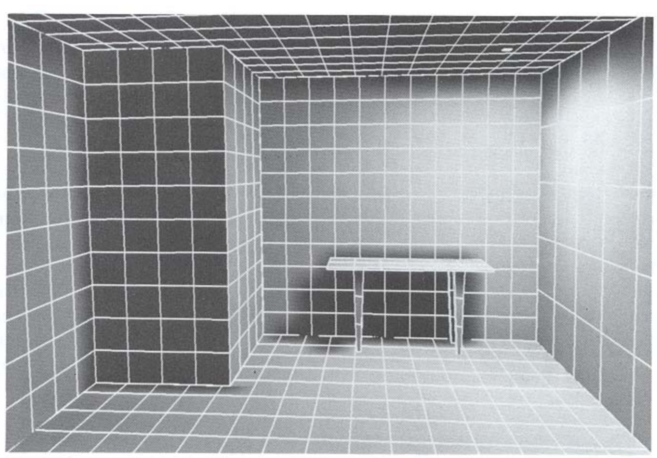
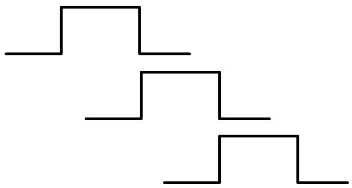


Table in room sequence from Cohen and Wallace

Basic Functions

Piecewise constant basis functions



$N_i(x)$

Express radiosity as sum of basis functions

$$B(x) = \sum_i B_i N_i(x)$$

Constant radiosity assumption

$$B_e(x) = \sum_i E_i N_i(x)$$

$$\rho(x) = \sum_i \rho_i N_i(x)$$

Derivation

Convert integral equation to matrix equation

$$B(x) = B_e(x) + \rho(x) \int_{M^2} F(x,x') \sum B_j N_j(x')\, dA'$$

$$\sum_i B_i N_i(x) = \sum_i B_i N_i(x) + \sum_i \rho_i N_i(x) \left[\sum_j B_j \int_{A_j} F(x,x') N_j(x')\, dA' \right]$$

$$\int \left(\sum_i B_i N_i(x) = \sum_i B_i N_i(x) + \sum_i \rho_i N_i(x) \left[\sum_j B_j \int_{A_j} F(x,x') N_j(x')\, dA' \right] \right) dA$$

$$B_i A_i = E_i A_i + \rho_i \sum_j B_j \int_{A_i} \int_{A_j} F(x,x') N_i(x) N_j(x')\, dA\, dA'$$

Form Factor

Form Factor

$$A_i F_{ij} = A_j F_{ji} = \int_{A_i}\int_{A_j} \frac{\cos\theta'_o \cos\theta_i}{\pi \left\| x - x' \right\|^2} V(x, x')\, dA\, dA'$$

Summation

$$\sum_j F_{ij} = 1$$

Form factor is the percentage of light leaving i ***that makes it to*** j

Classic Radiosity

Power balance

$$B_i A_i = E_i A_i + \rho_i \sum_j B_j A_i F_{ij}$$

$$B_i = E_i + \rho_i \sum_j F_{ij} B_j$$

Linear system of equations

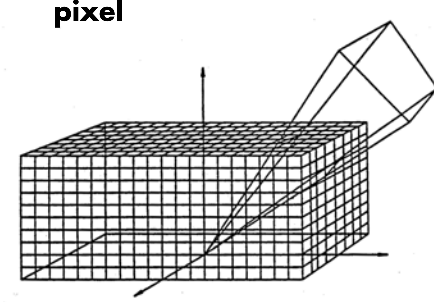
$$\begin{pmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \cdots & -\rho_2 F_{21} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1-\rho_n F_{nn} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix} = \begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix}$$

Form Factors

Hemicube Algorithm

First radiosity algorithm to deal with occlusion

1. Render scene from the point of view of each vertex/element
2. Compute delta form factors – contribution from each pixel



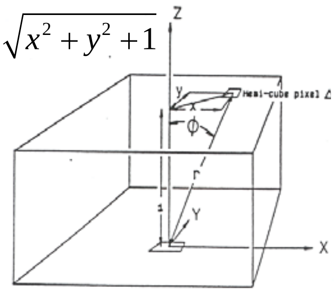
Render source elements from POV of receiving element

$$F_{dA_i,A_j} = \sum_{p \in A_j} \Delta F_p$$

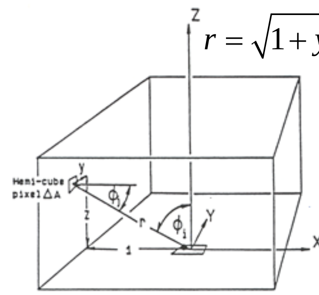
Typical resolution: 32x32

Hemicube Delta Form Factors

$$r = \sqrt{x^2 + y^2 + 1}$$



$$r = \sqrt{1 + y^2 + z^2}$$



$$\cos\phi = \frac{1}{\sqrt{x^2 + y^2 + 1}} \qquad \cos\phi = \frac{1}{\sqrt{1 + y^2 + z^2}}$$

$$\Delta F = \frac{\Delta A}{\pi (x^2 + y^2 + 1)^2} \qquad \Delta F = \frac{\Delta A}{\pi (1 + y^2 + z^2)^2}$$

Hemicube Algorithms

Advantages

- \+ First practical method -> Patent!
- \+ Use existing rendering systems; Hardware
- \+ Computes row of form factors in $O(n)$

Disadvantages

- \- Computes differential-finite form factor
- \- Aliasing errors due to sampling
  - Randomly rotate/shear hemicube
- \- Proximity errors
- \- Visibility errors
- \- Expensive to compute a single form factor

Solving

Solve [F][B] = [E]

Direct methods: $O(n^3)$

- **Gaussian elimination**

  Goral, Torrance, Greenberg, Battaile, 1984

Iterative methods: $O(n^2)$

Energy conservation

$\rightarrow$ ***diagonally dominant*** $\rightarrow$ ***iteration converges***

- **Gauss-Seidel, Jacobi: Gathering**

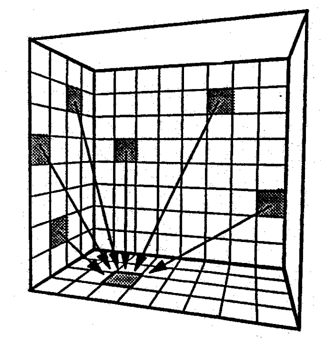
  Nishita, Nakamae, 1985

  Cohen, Greenberg, 1985

- **Southwell: Shooting**

  Cohen, Chen, Wallace, Greenberg, 1988

Gathering



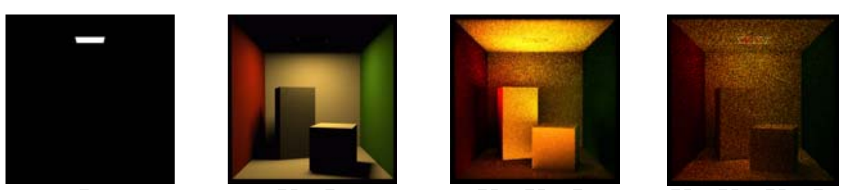
```
for(i=0; i<n; i++)
  B[i] = Be[i];

while( !converged ) {
  for(i=0; i<n; i++) {
    E[i] = 0;
    for(j=0; j<n; j++)
      E[i] += F[i][j]*B[j];
    B[i] = Be[i]+rho[i]*E[i];
  }
}
```

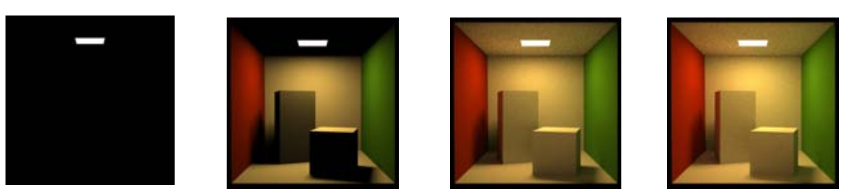
Row of F times B

Calculate one row of F and discard

Successive Approximation

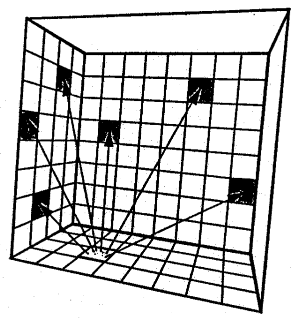


L_e | $K \circ L_e$ | $K \circ K \circ L_e$ | $K \circ K \circ K \circ L_e$



L_e | $L_e + K \circ L_e$ | $L_e + \cdots K^2 \circ L_e$ | $L_e + \cdots K^3 \circ L_e$

Shooting

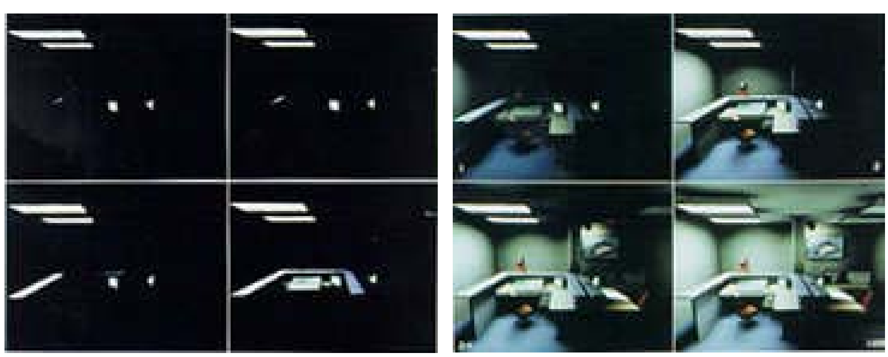


```
for(i=0; i<n; i++) {
  B[i] = dB[i] = Be[i];
  while( !converged ) {
    set i st dB[i] is the largest;
    for(j=0;j<n;j++)
      if(i!=j) {
        db =rho[j]*F[j][i]*dB[i];
        dB[j] += db;
        B[j] += db;
      }
    dB[i]=0;
  }
}
```

Brightness order

Column of F times B

Progressive Radiosity



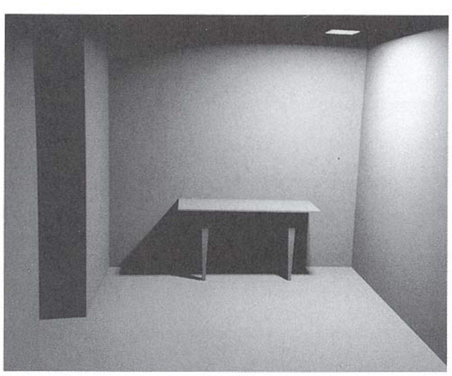
(a) (b)

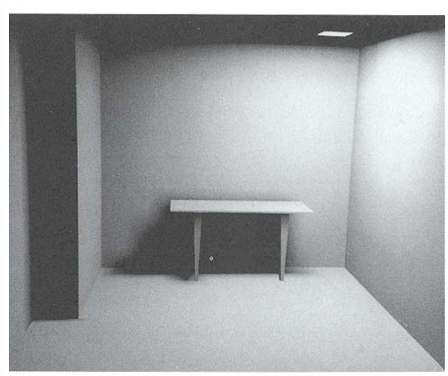
(a) Traditional Gauss-Seidel iteration of 1, 2, 24 and 100.
(b) Progressive Refinement (PR) iteration of 1, 2, 24 and 100.

From Cohen, Chen, Wallace, Greenberg 1988

Meshing

Accuracy

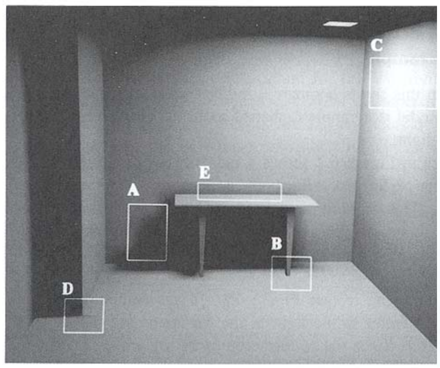


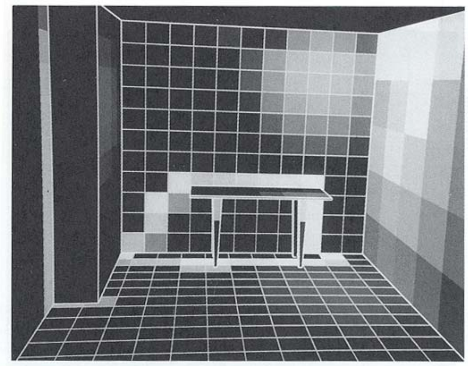


Reference Solution **Uniform Mesh**

Table in room sequence from Cohen and Wallace

Artifacts

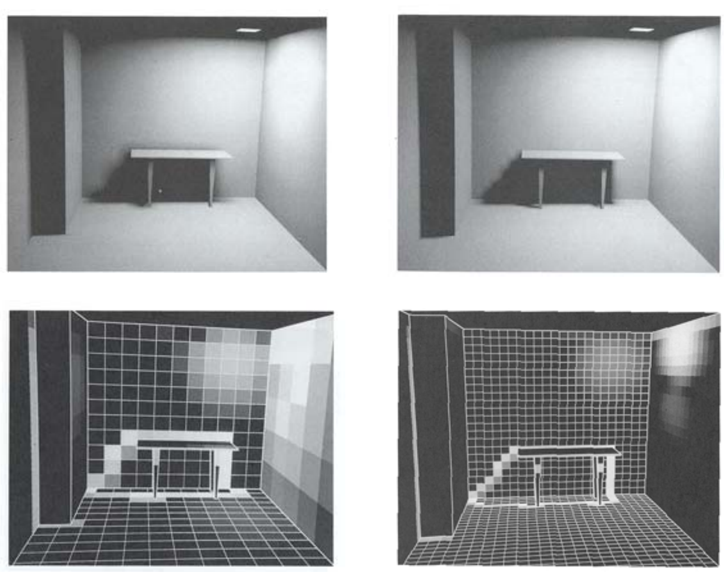




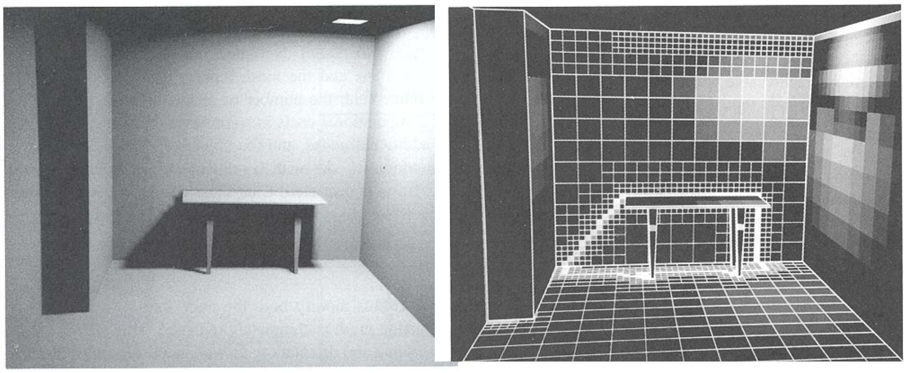
Error Image

A. Blocky shadows
B. Missing features
C. Mach bands
D. Inappropriate shading discontinuities
E. Unresolved discontinuities

Increasing Resolution



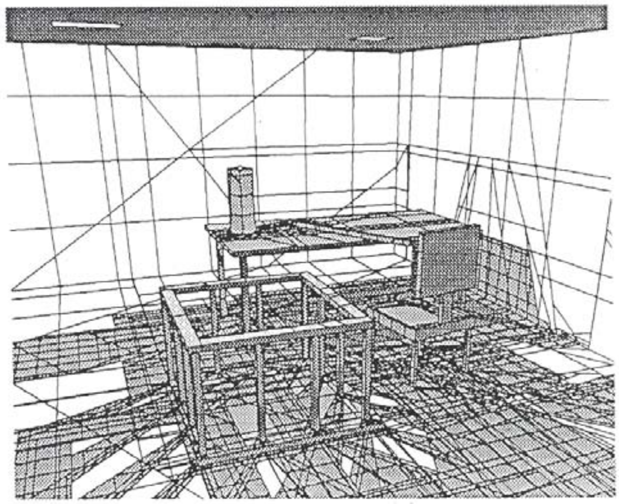
Adaptive Meshing



Discontinuity Mesh

From Baum et al.

Discontinuity Mesh

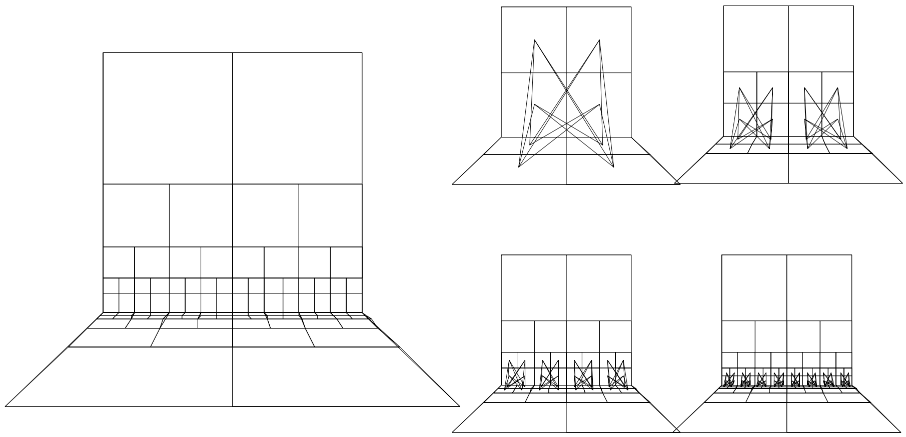


From Campbell et al.

Discontinuity Meshing

From Lischinski, Tampieri, Greenberg 1992

Hierarchical Radiosity



Summary

Remember assumptions

- **Diffuse reflectance**
- **Polygons**

Difficult to relax assumptions

Computation challenges

- **Meshing**
  - **Complex input geometry**
  - **Complexity due to shadows**
- **Dense coupling**
  - $O(n^2)$ **matrix elements**
  - **HR leads to** $O(n)$ **algorithm (ignoring discontinuities)**