Monte Carlo II: Direct Illumination

cs348b
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Review: Basic MC Estimator

$$E\left[\frac{1}{N}\sum_{i=1}^{N} f(x_i)\right] = \frac{1}{N}\sum_{i=1}^{N} E[f(x_i)] \qquad x_i \sim p(x)$$

$$= \frac{1}{N}\sum_{i=1}^{N} \int_0^1 f(x)p(x)dx$$

$$= \frac{1}{N}\sum_{i=1}^{N} \int_0^1 f(x)dx$$ if p(x) is uniform

$$= \int_0^1 f(x)dx$$

Increasing number of samples N reduces variance (noise)

Overview

- The direct lighting problem
- Improving efficiency
  - Splitting and Russian roulette
  - Importance sampling
  - Stratified sampling
- Building blocks
  - Sampling BRDFs
  - Sampling light sources
- Multiple importance sampling

Applications: Imaging

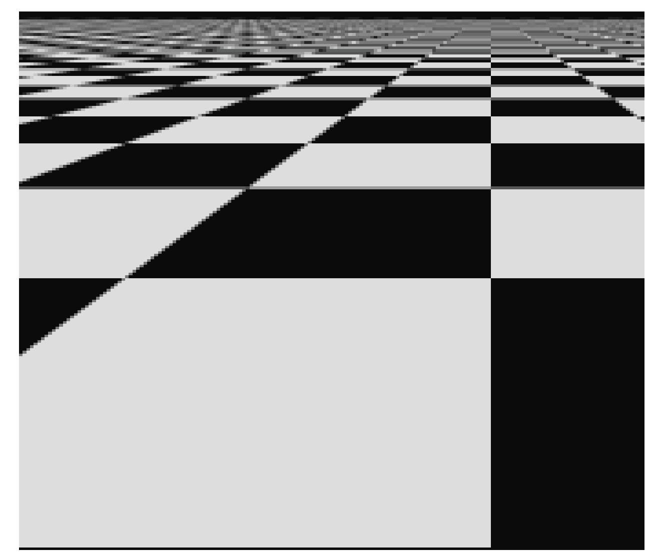
$$\int\int\int\int\int L(x, y, t, u, v)\,dx\,dy\,dt\,du\,dv$$

Motion Blur

Cook, Porter, Carpenter, 1984

Depth of Field

Mitchell, 1991



AntiAliasing

Applications: Direct Lighting

$$L(x,\omega_o) = \int_\Omega f(\omega \to \omega_o) L_i(x,\omega) \cos\theta d\omega$$

$$= \int_A f(\omega \to \omega_o) L_e(x_l,\omega_l) \frac{\cos\theta \cos\theta_l}{|x - x_l|^2} V(x,x_l) dA(x_l)$$

Sample light uniformly by area

$$L(x,\omega_o) \approx \frac{1}{N} \sum_{i=1}^{N} f(\omega \to \omega_o) L_e(x_{l[i]},\omega_{l[i]}) \frac{\cos\theta \cos\theta_{l[i]}}{|x - x_{l[i]}|^2} V(x,x_{l[i]})$$

Improving Efficiency: Splitting

- Take additional samples in some dimensions

$$\int_0^1 \int_0^1 f(x)g(y)dxdy \approx \frac{1}{N} \sum f(x_i)g(y_i)$$

$$\approx \frac{1}{N_f} \sum_{i=1}^{N_f} f(x_i) \left(\frac{1}{N_g} \sum_{j=1}^{N_g} g(y_j) \right)$$

- e.g. multiple shadow rays

Improving Efficiency: Russian Roulette

- Randomly skip computations with low expected value...in an unbiased manner
- Algorithm:
  - Determine probability of skipping p_s
  - Choose uniform random number ξ
  - Skip if $\xi < p_s$
  - Otherwise estimate is $\frac{f(x_i)}{1-p_s}$
- Expected value

$$p_s \cdot 0 + (1-p_s) \cdot E[f/(1-p_s)] = E[f]$$

Importance Sampling

$$E\left[\frac{1}{N}\sum_{i=1}^{N}\frac{f(x_i)}{p(x_i)}\right] = \frac{1}{N}\sum_{i=1}^{N}E\left[\frac{f(x_i)}{p(x_i)}\right] \qquad x_i \sim p(x)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\int_0^1 \frac{f(x)}{p(x)}p(x)dx$$

$$= \frac{1}{N}\sum_{i=1}^{N}\int_0^1 f(x)dx$$

$$= \int_0^1 f(x)dx$$

Importance Sampling

- Very useful for reducing variance in rendering problems
- Must have p(x) > 0 for all x where f(x) > 0
- Application to generalizing domains:

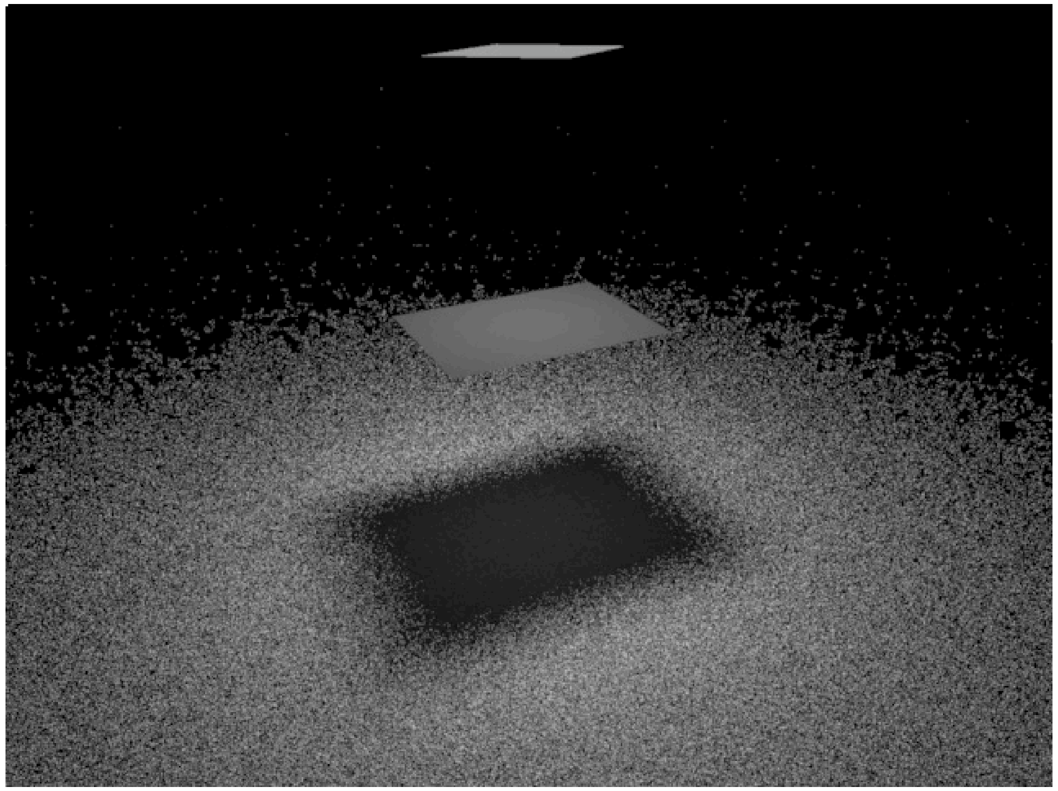
$$\int_a^b f(x)dx = (b-a)\int_0^1 f(a+x(b-a))dx$$

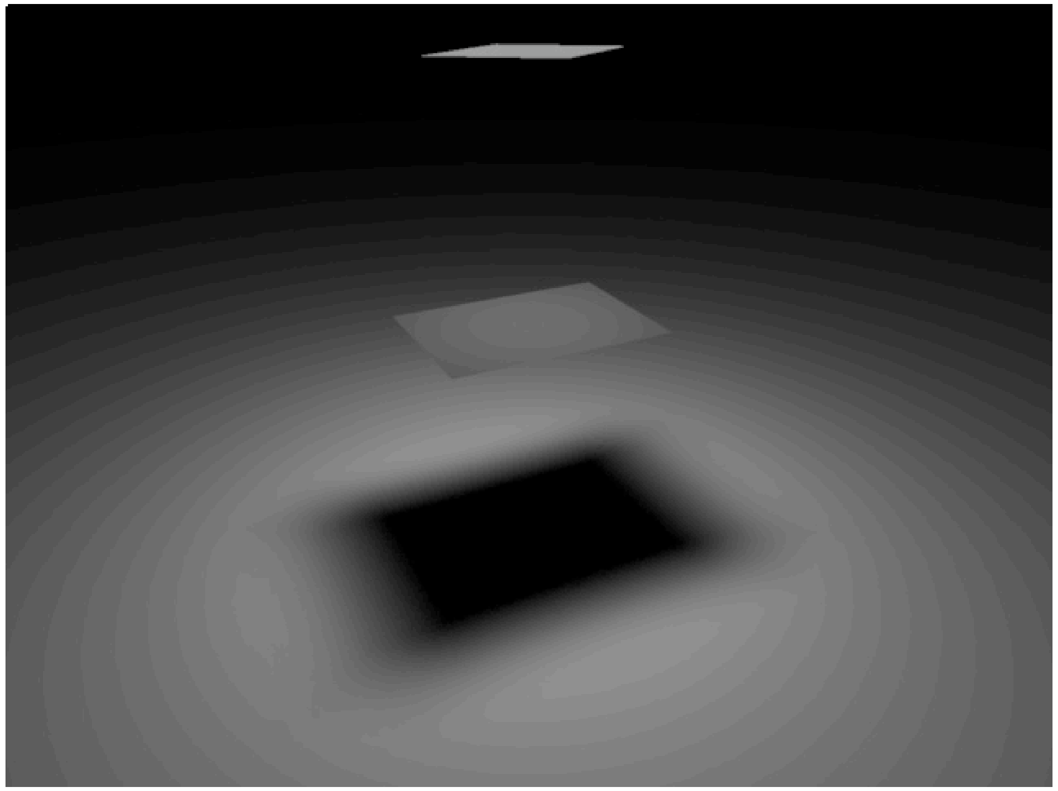
  - or

$$p(x) = \frac{1}{b-a}, a \leq x \leq b \qquad \text{(0 otherwise)}$$

$$\frac{1}{N}\sum^N f(x)/p(x) = (b-a)\frac{1}{N}\sum^N f(x)$$

Direct Lighting





Sampling the Hemisphere

- Recall that

$$\int_{\Omega} f(\omega) d\omega = \int_0^{\pi/2} \int_0^{2\pi} f(\theta, \phi) \sin\theta d\theta d\phi$$

- Sample

$$\theta \sim [0, \pi/2], \phi \sim [0, 2\pi]$$

- Estimate by

$$\pi^2 \frac{1}{N} \sum^{N} f(\theta_i, \phi_i) \sin\theta$$

- Inefficient but correct

Sampling the Hemisphere (Better)

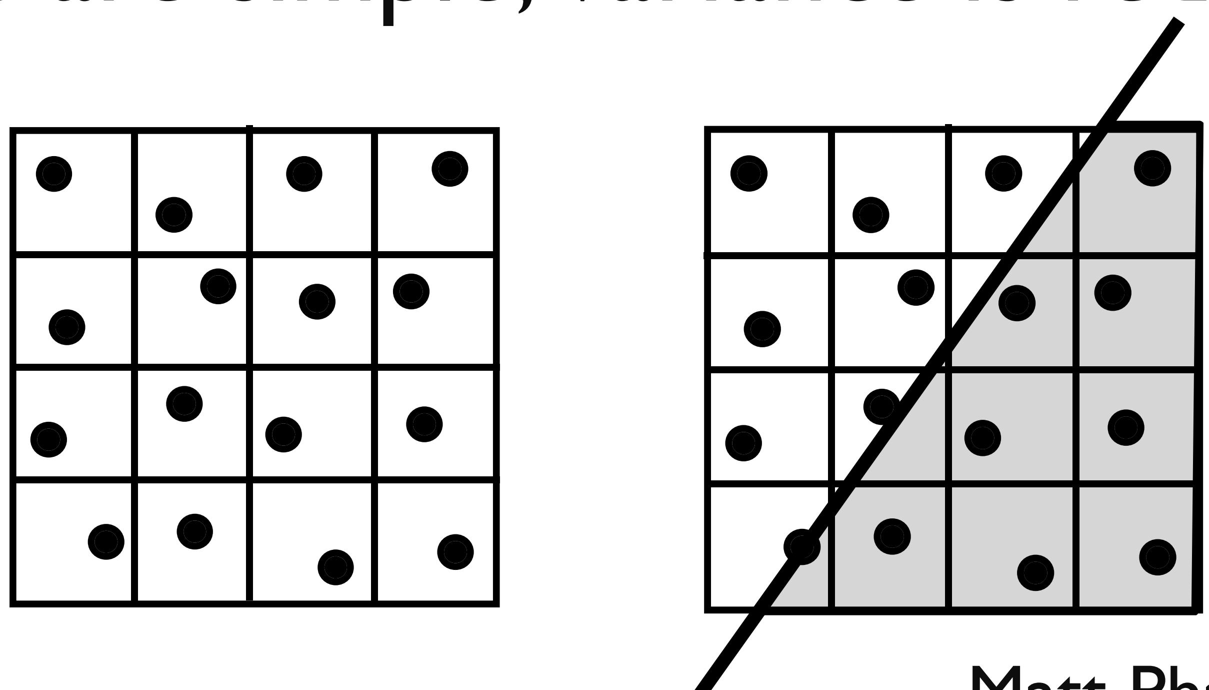
- Find pdf for uniform samples:

$$
\begin{aligned}
1 &= \int_\Omega p(\omega) d\omega \qquad\qquad = \int_\Omega c\, d\omega \\
&= \int_0^{2\pi} \int_0^{\pi/2} c\, \sin\theta\, d\theta\, d\phi \\
&= 2\pi c \\
&\qquad \longrightarrow p(x) = 1/2\pi
\end{aligned}
$$

- Sample phi uniformly $\phi = 2\pi\xi$
- Sample theta by $\theta = \arccos \xi$

Stratified Sampling

- Why not just use rejection sampling?
- Stratification: divide domain into regions, allocate samples to the regions.
- Variance is sum of region variances
- If some are simple, variance is reduced



Sampling a Cone

- Given cone about normal direction of spread angle θ_{max}
- Find normalization constant for pdf

$$1 = c \int_0^{\theta_{max}} 1 \sin\theta \, d\theta$$

- Invert to sample using ξ
- Change of basis for arbitrary center direction

Sampling a Phong Lobe

- Sample an offset from the reflected direction

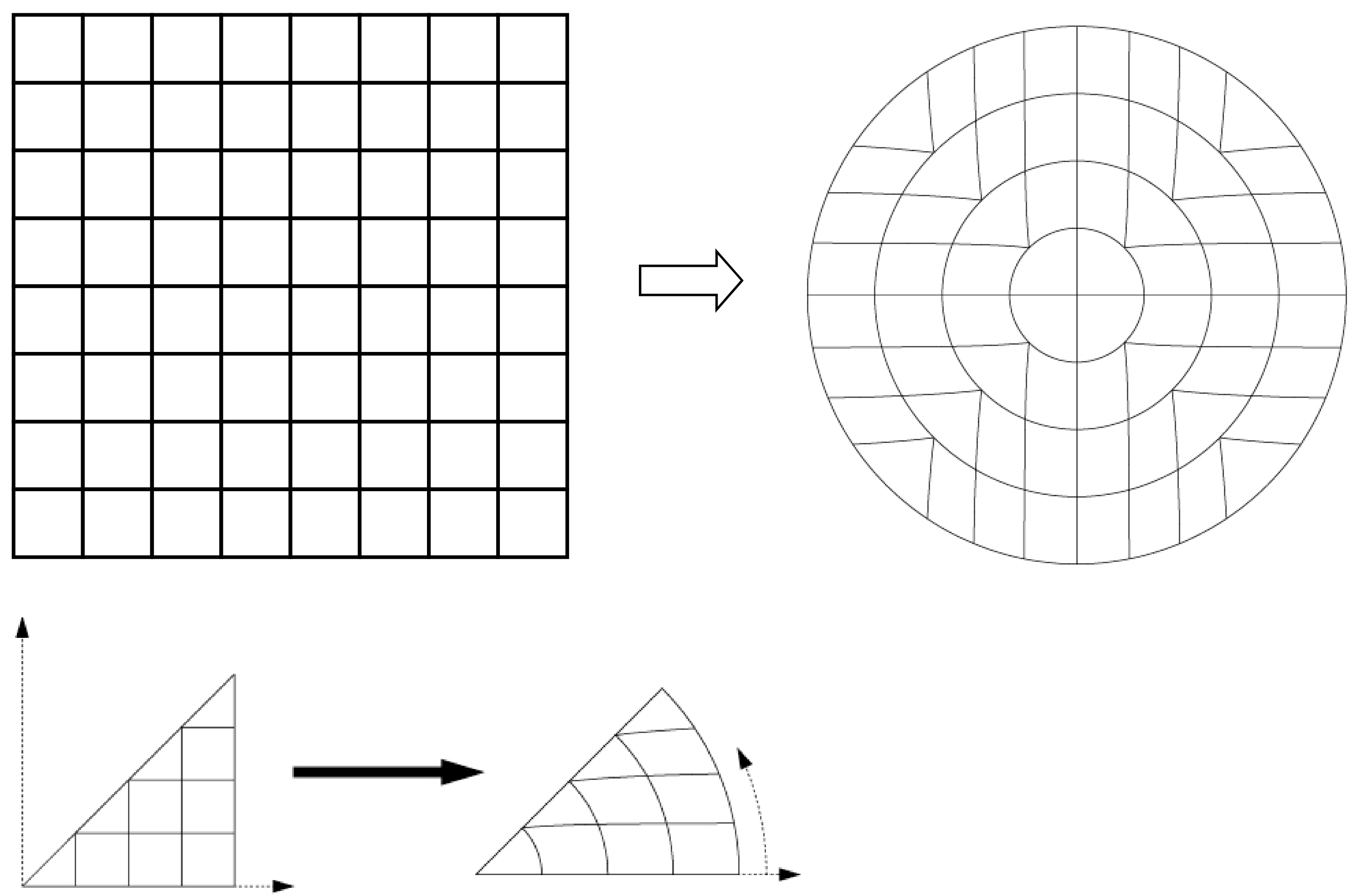
$$\int_{\Omega} (\cos\theta)^n d\omega \rightarrow$$

$$\int_0^{2\pi} \int_0^{\pi/2} (\cos\theta)^n \sin\theta d\theta d\phi$$

Direct Lighting

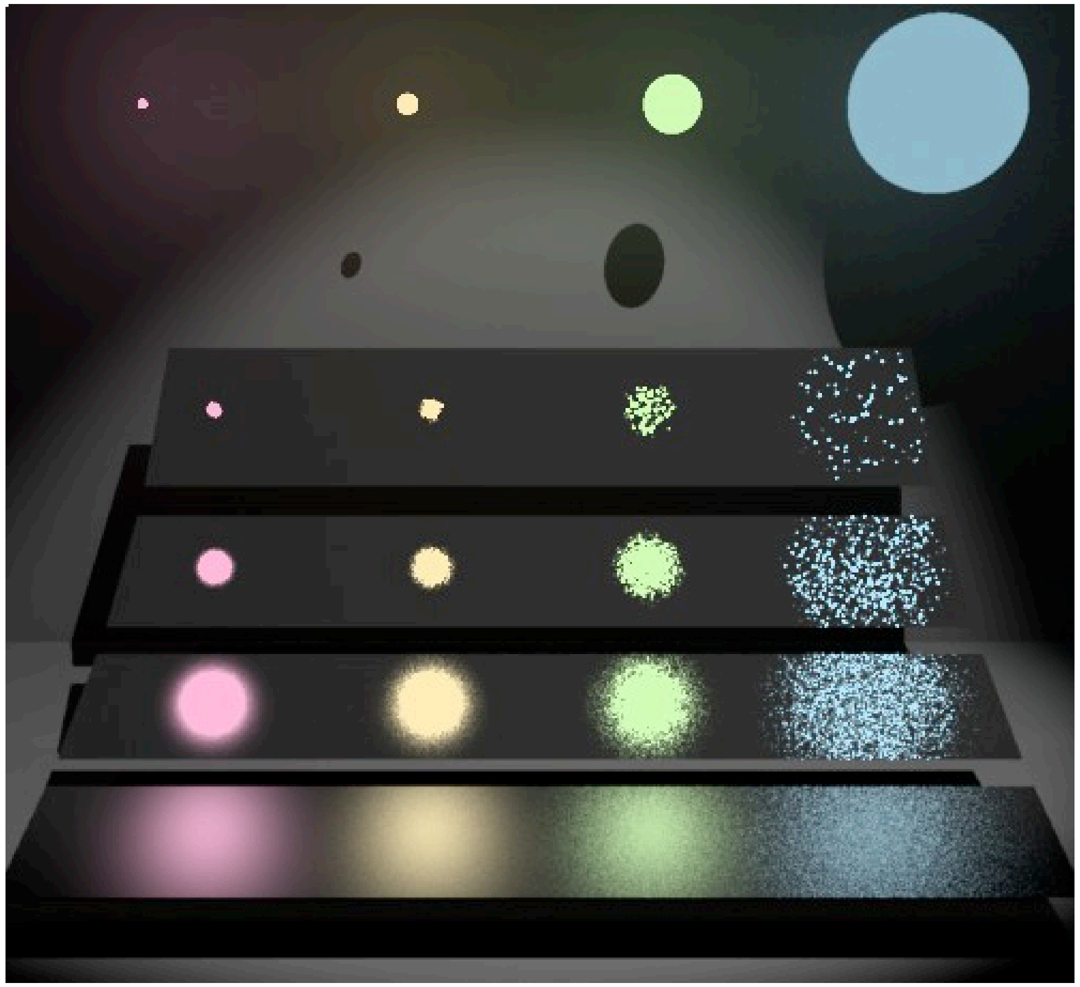
Sampling a Circle

- Shirley's Mapping

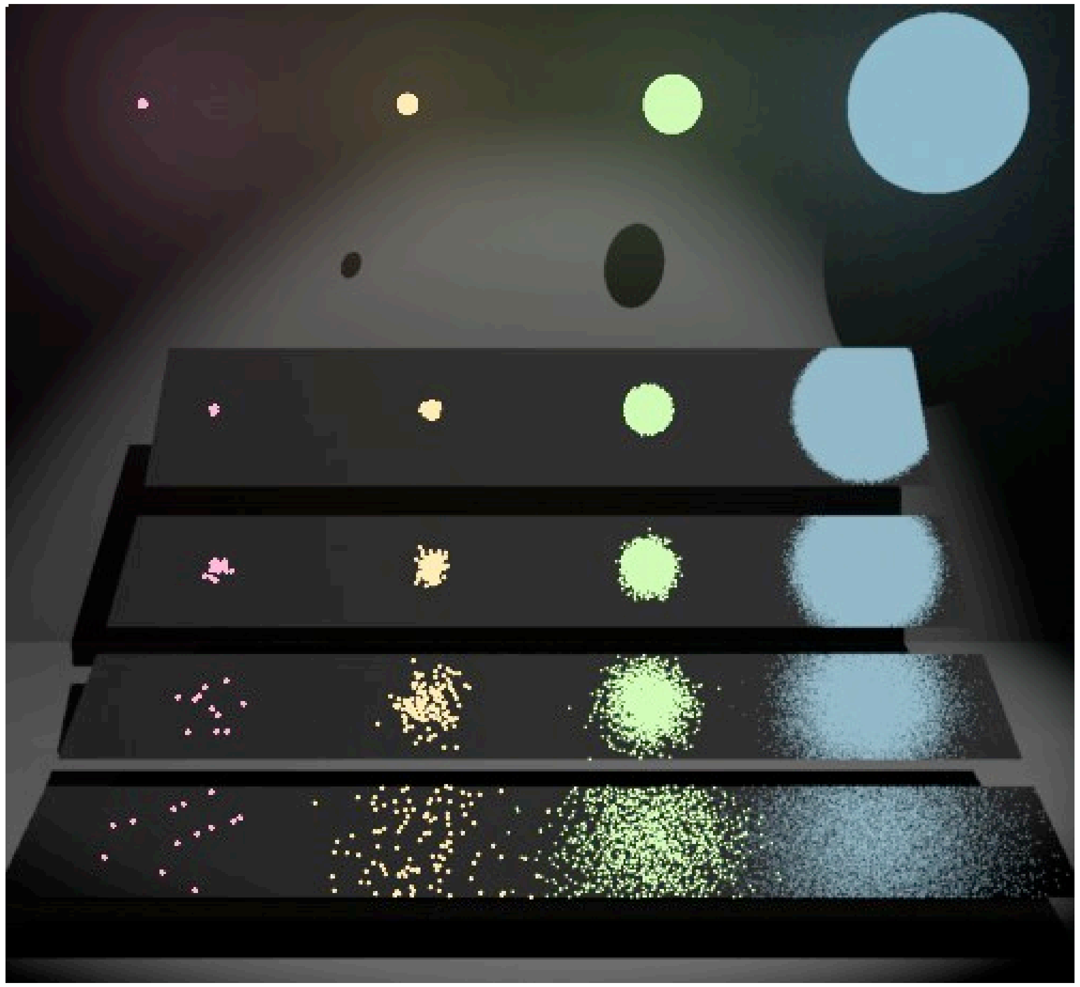


Sampling Strategies

Sample Light



Sample BRDF



Multiple Importance Sampling

- Have a family of sampling distributions $p_i(x)$
- Take N_i samples from each one, $N = \sum N_i$ total samples
- Balance heuristic: compute MC estimate

$$\int f(x)dx \approx \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{\sum_j (N_j/N) p_j(x_i)}$$

- Reduces spiky noise due to surprisingly large value of f(x)

Multiple Importance Sampling

Result: better than either of the two strategies alone