Reflection II

cs348b
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Administrivia

- Rendering competition updates
  - June 10, 3pm
- HW2 updates

Fresnel Reflection

No Fresnel Reflection

Overview

- Reflection setting
- Phong model
- Microfacet models
  - Torrance-Sparrow
  - Oren-Nayar
- Anisotropic reflection
- Hierarchies of scale
  - Simulation to generate BRDFs
- Models for measured data

Basic Quantities

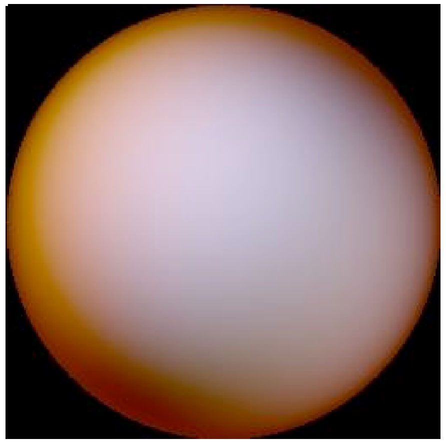
- N, ω_i, ω_o
- $\cos\theta_i = (N \cdot \omega_i)$
- reflection: $R(N, \omega)$
- half-angle: $H = (\omega_i + \omega_o)/|\omega_i + \omega_o|$
- Reflection coordinate system

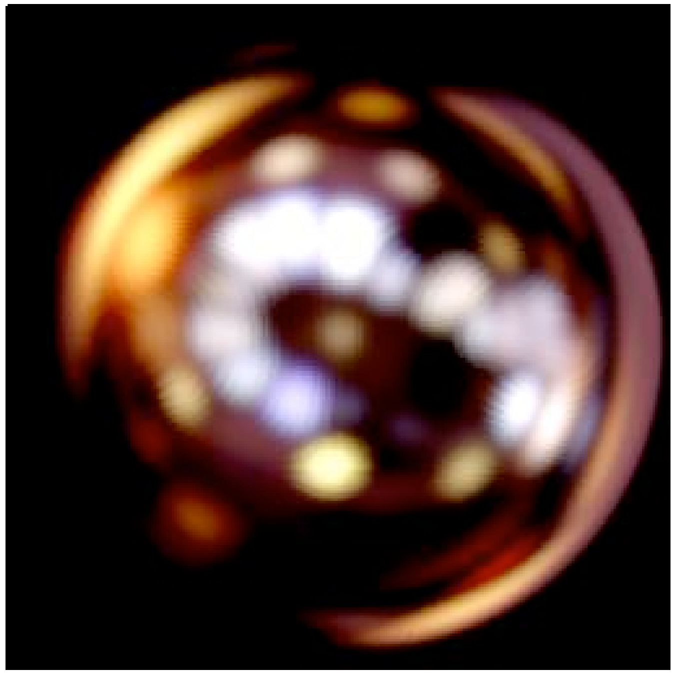
Phong Model

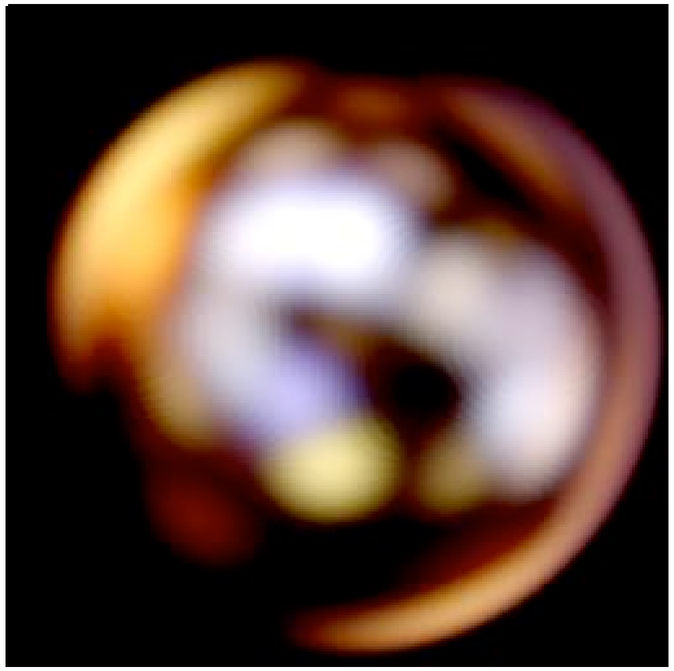
$$f_r(\omega_i \to \omega_o) = (\omega_i \cdot R(N, \omega_o))^n$$

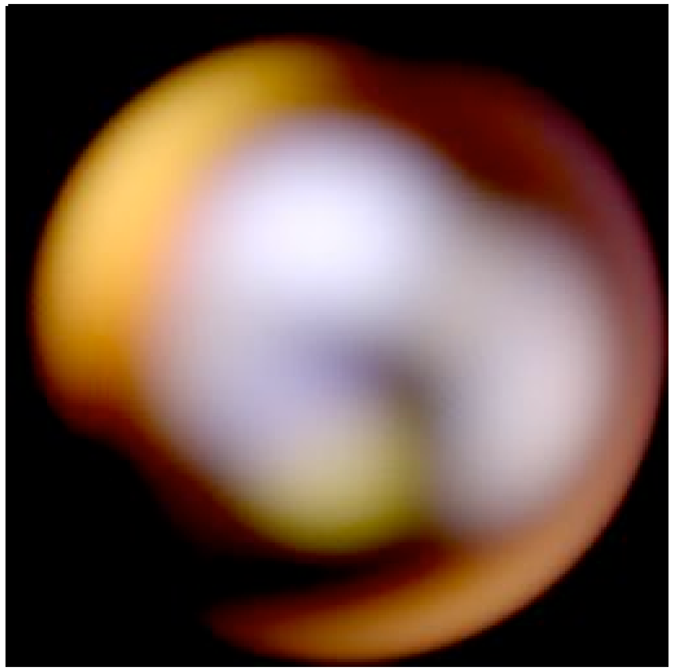
- Empirical blur of perfect specular reflection
- Versus perfect reflection of area light source

Phong Model









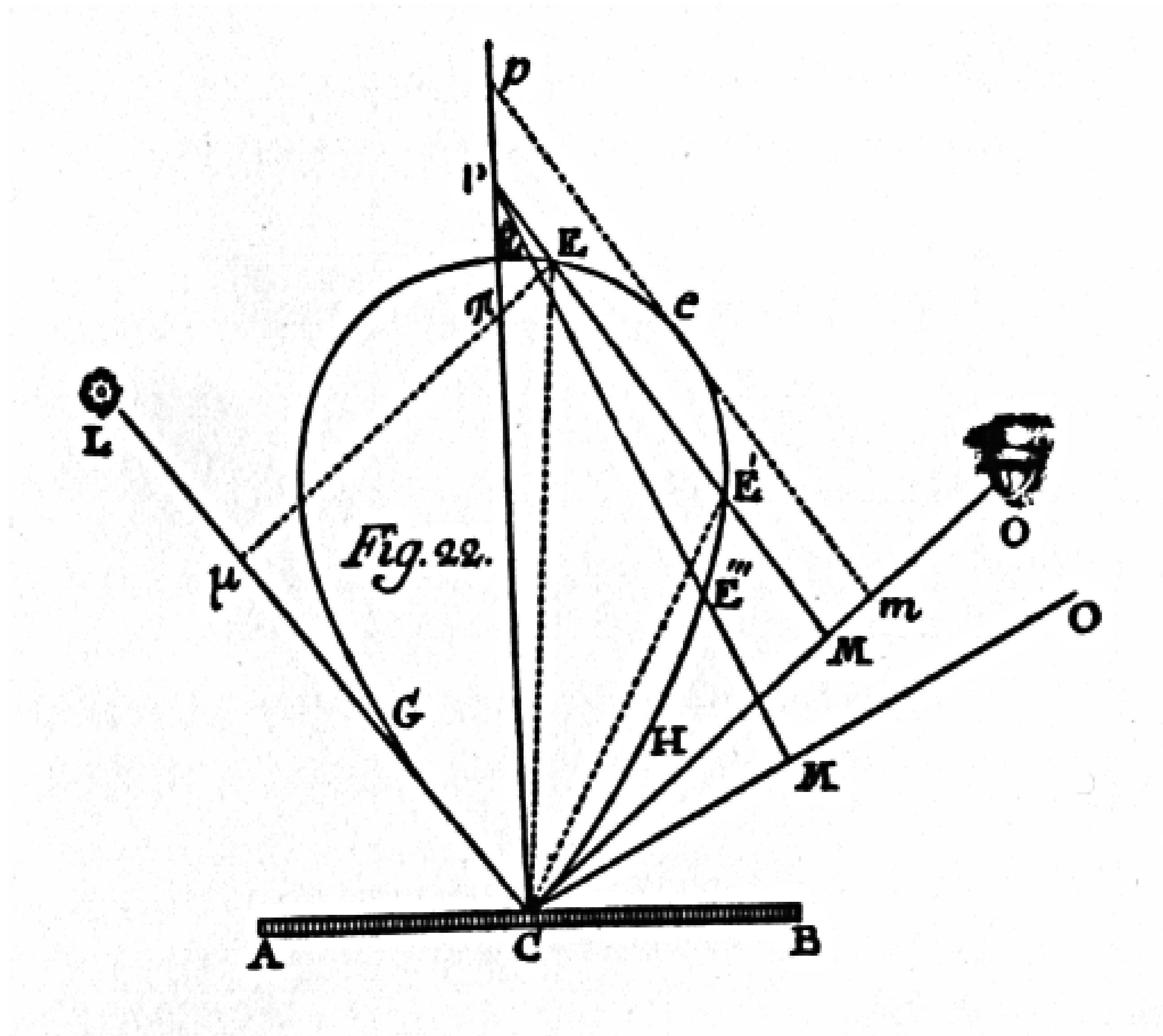
Energy-Conserving Phong

$$\rho(H^2 \to \omega_o) = \int_{H^2(N)} (\omega_o \cdot R(N, \omega)^n \cos\theta d\omega$$

$$\leq \int_{H^2(R)} (\omega_o \cdot R(N, \omega)^n d\omega$$

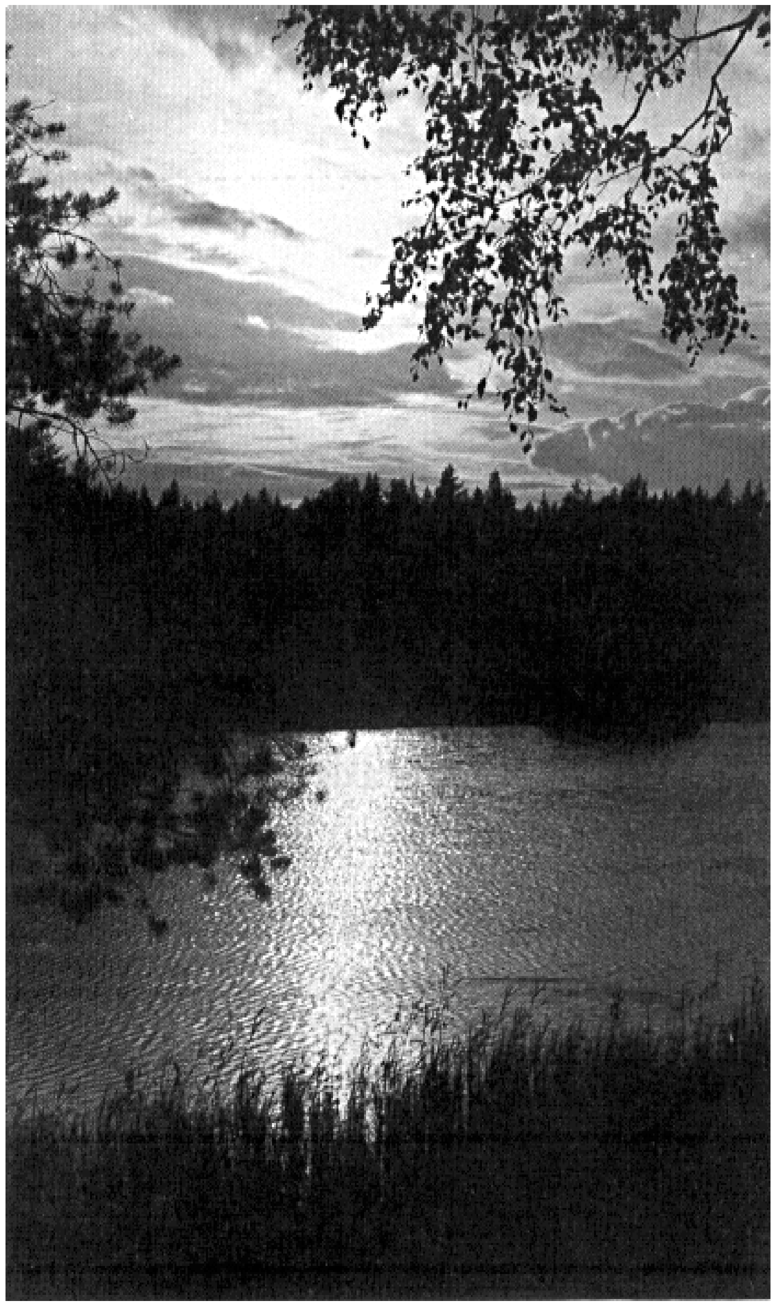
$$\leq \int_{H^2} \cos^n \theta d\omega = \frac{2\pi}{n+1}$$

Bouguer's "little faces"

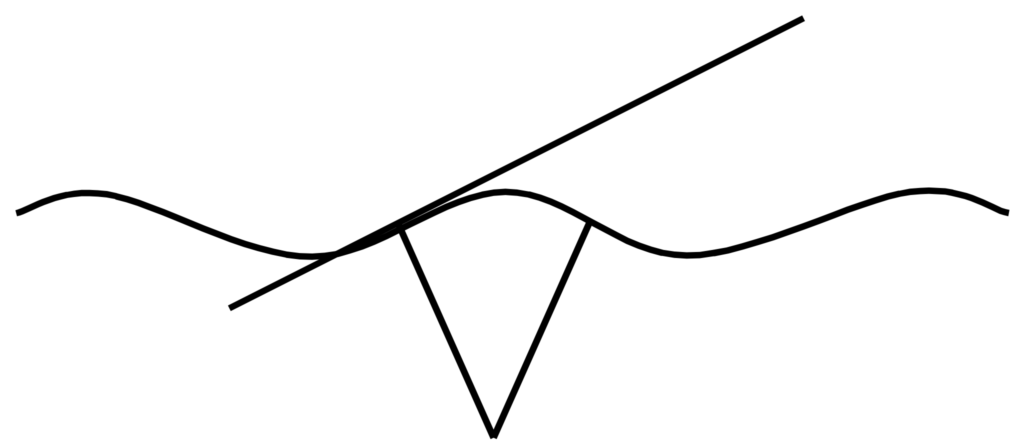


P. Bouguer, "Treatise on Optics", 1760

Reflection of the Sun



Minnaert, "Light and Color in the Outdoors"



Microfacet Distributions

- $D(\omega_h)$ gives probability density of facet oriented along ω_h
- Normalization:

$$\int_{H^2} \cos\theta_h dA(\omega_h) = dA$$

$$\int_{H^2} D(\omega_h) \cos\theta_h d\omega_h = 1$$

- Isotropic: $D(\omega_h) = D(\alpha)$

Microfacet Examples

- Blinn

$$D(\alpha) = \cos^{c_1} \alpha$$

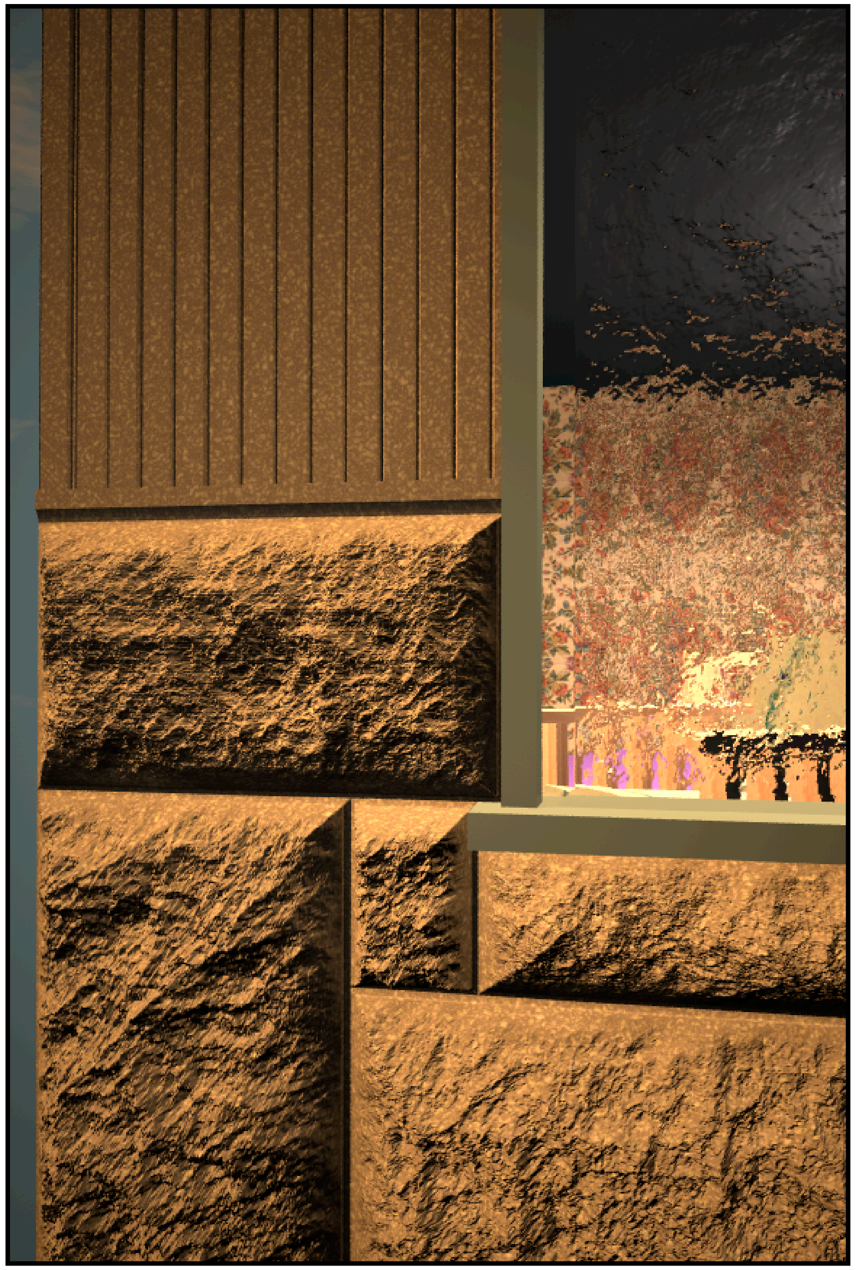
- Torrance-Sparrow

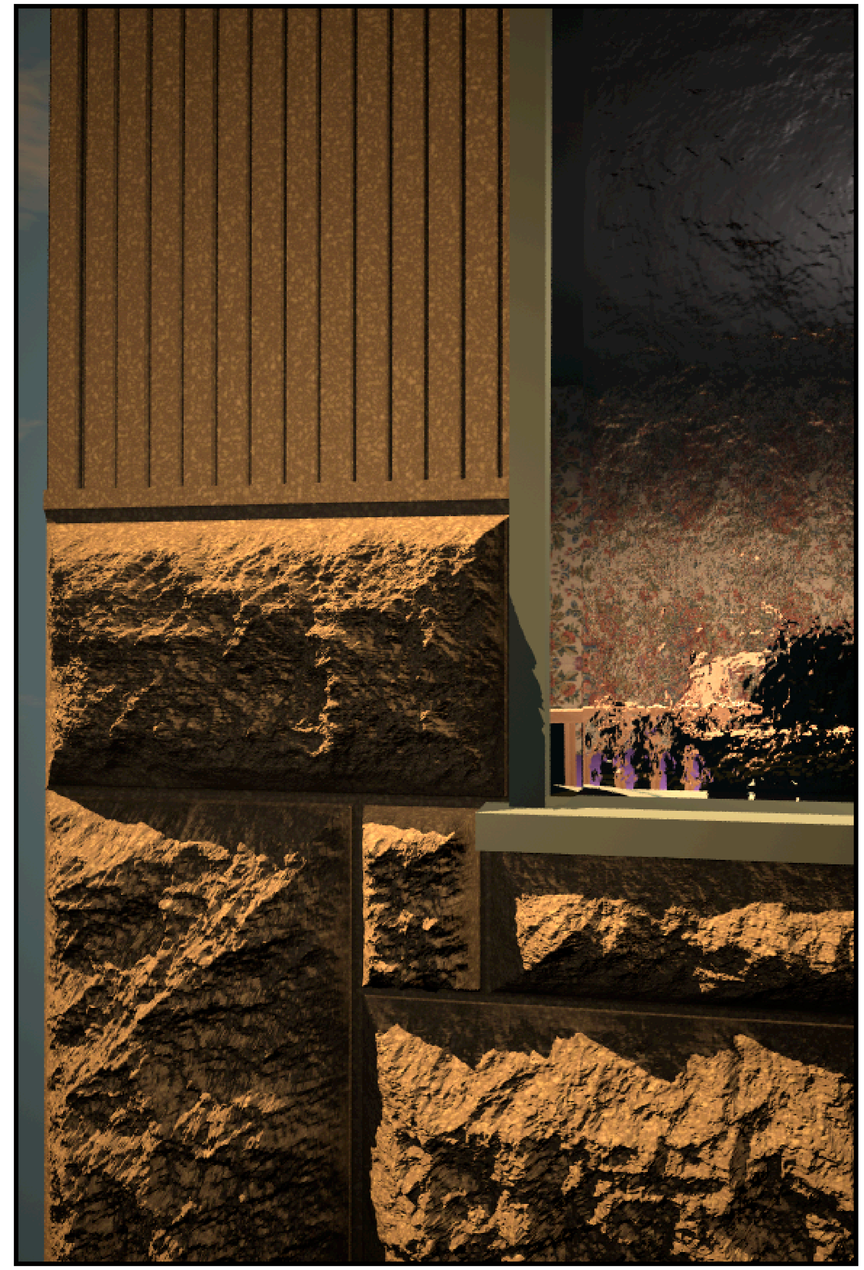
$$D(\alpha) = e^{(-c_2 \alpha)^2}$$

- Trowbridge-Reitz

$$D(\alpha) = \frac{c_3^2}{(1 - c_3^2) \cos^2 \alpha - 1}$$

Self Shadowing





Gaussian Surface

- Beckmann: Gaussian distribution of heights:

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}$$

- Gives Gaussian distribution of slopes:

$$D(\alpha) = \frac{1}{\sqrt{\pi} m^2 \cos^2 \alpha} e^{-\frac{\tan^2 \alpha}{m^2}} \qquad m = \frac{2\sigma}{\tau}$$

Gaussian Surface

- ...gives closed form solution to probability of shadowing

$$S(\theta) = \frac{1 - \frac{1}{2}\mathrm{erfc}(\frac{\sigma}{\sqrt{2}m})}{1 + \Lambda(\sigma)}$$

$$2\Lambda(\sigma) = \left(\sqrt{\frac{2}{\pi}}\right) \frac{m}{\sigma} e^{-\frac{\sigma^2}{2m^2} - \mathrm{erfc}\left(\frac{\sigma}{\sqrt{2}m}\right)}$$

Self-Shadowing Consistency Condition

- The sum of the areas of the illuminated surface projected onto the plane normal to the direction of incidence is independent of the roughness of the surface, and equal to the projected area of the underlying mean plane.

$$\int S(\theta) D(\alpha) \cos\theta_a d\omega_\alpha = \cos\theta$$

- Otherwise: energy conservation violated!

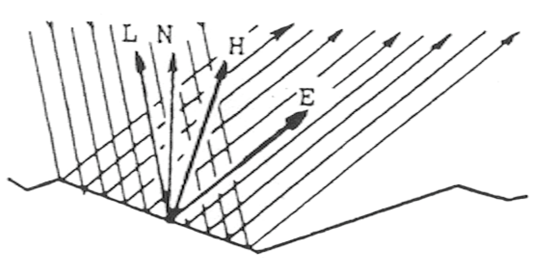
Torrance Sparrow Model

- Microfacet distribution D
- Fresnel reflection F
- Geometric attenuation G

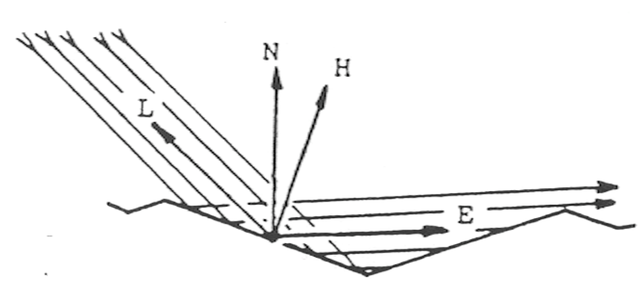
$$f_r(\omega_i \omega_o) = \frac{D(\omega_h)\, G(\omega_i, \omega_o)\, F(\omega_i, \omega_h)}{4 \cos\theta_i \cos\theta_o}$$

Self-Shadowing: V-Groove Model

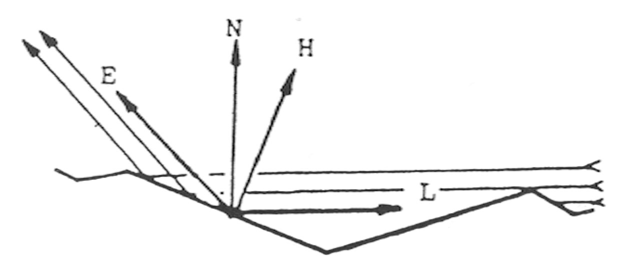
- Symmetric, longitudianal, infinitely-long V-grooves
  - Masking, shadowing
  - Ignores actual roughness, interreflection



$$G = 1$$

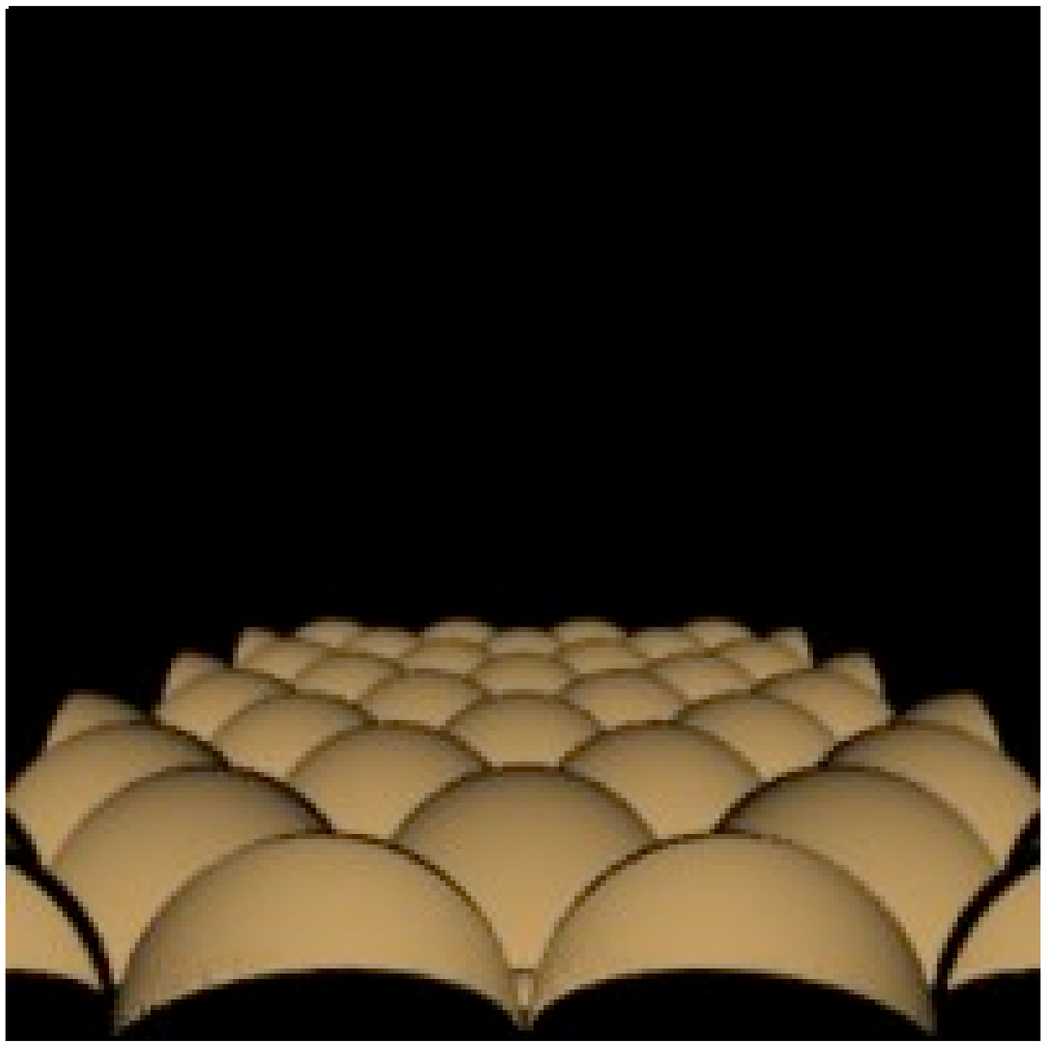


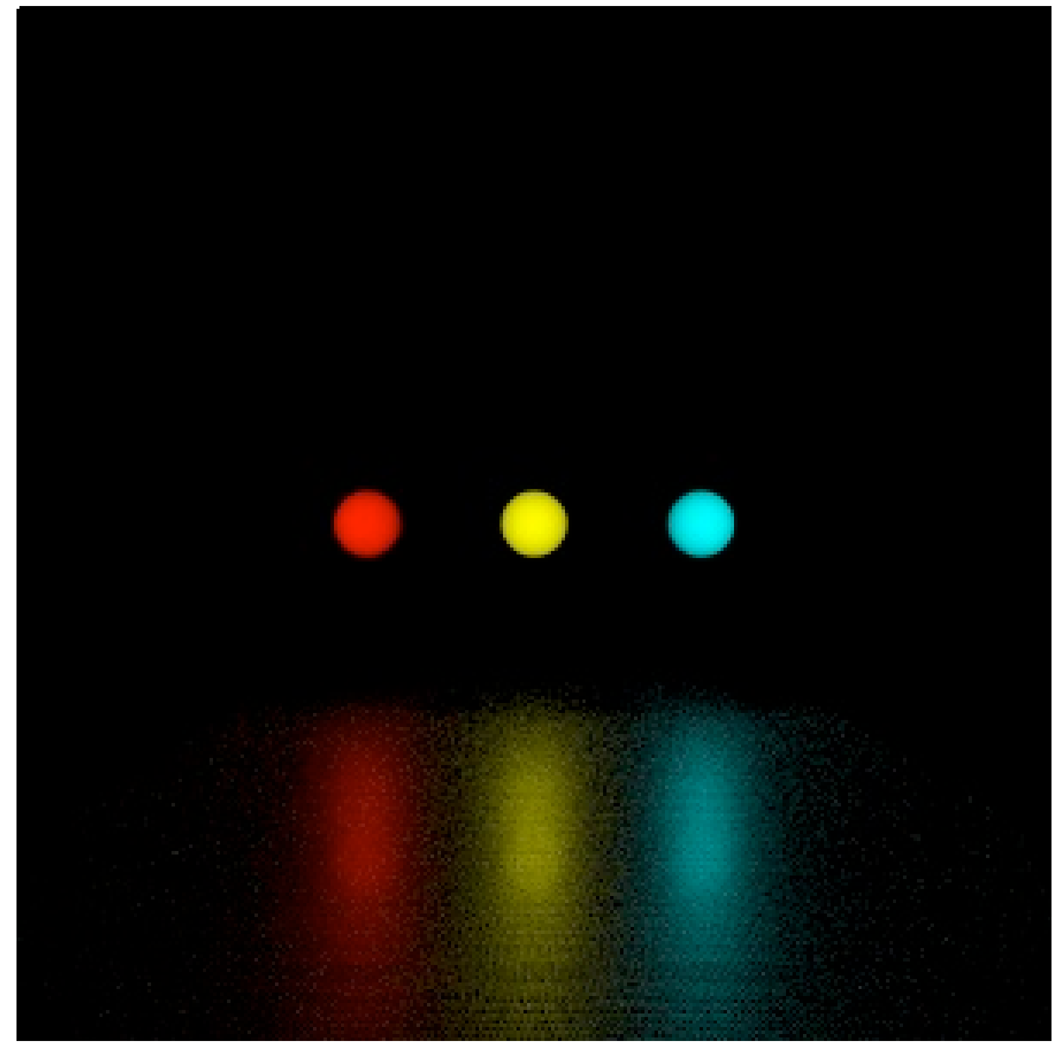
$$G = \frac{2(N \cdot H)(N \cdot \omega_i)}{(H \cdot \omega_i)}$$



$$G = \frac{2(N \cdot H)(N \cdot \omega_o)}{(H \cdot \omega_o)}$$

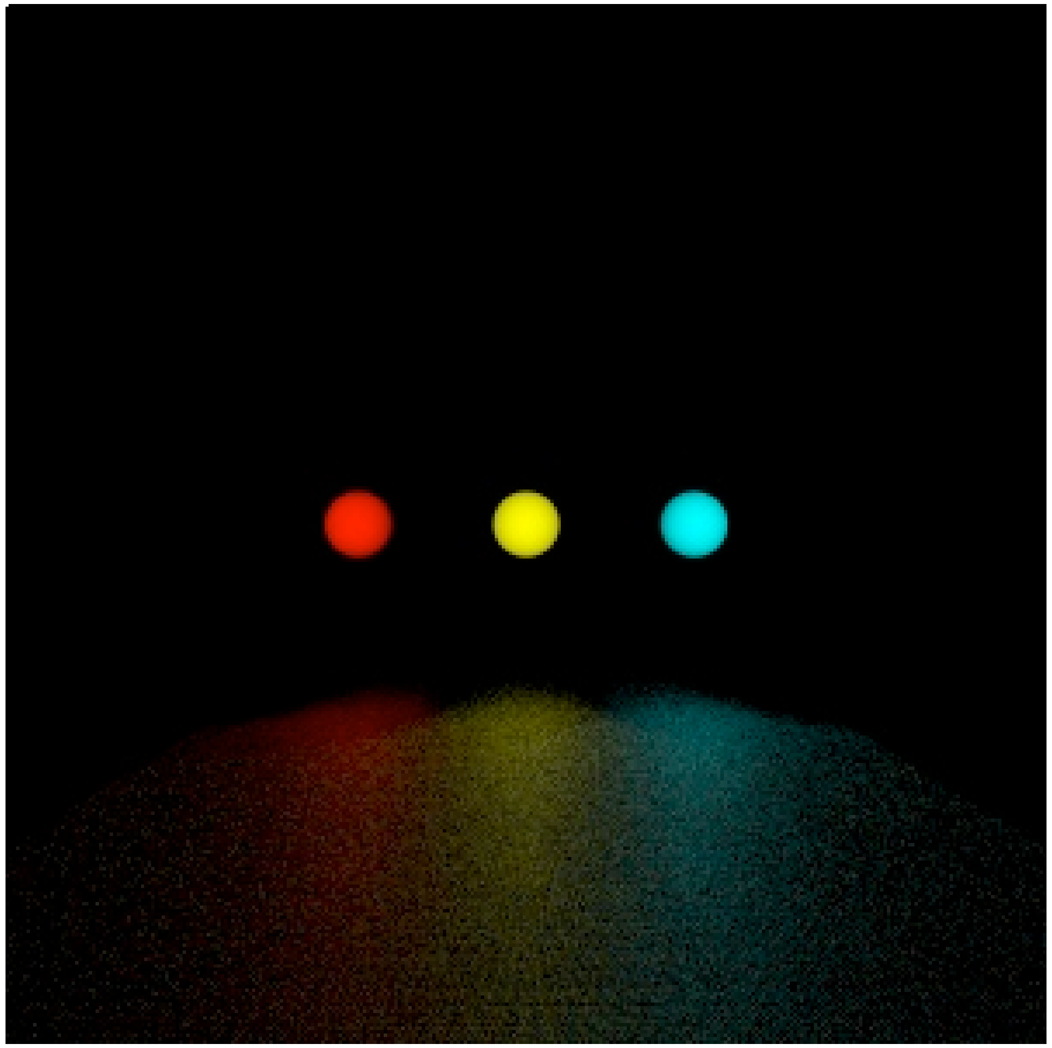
Brute Force Microfacet Simulation

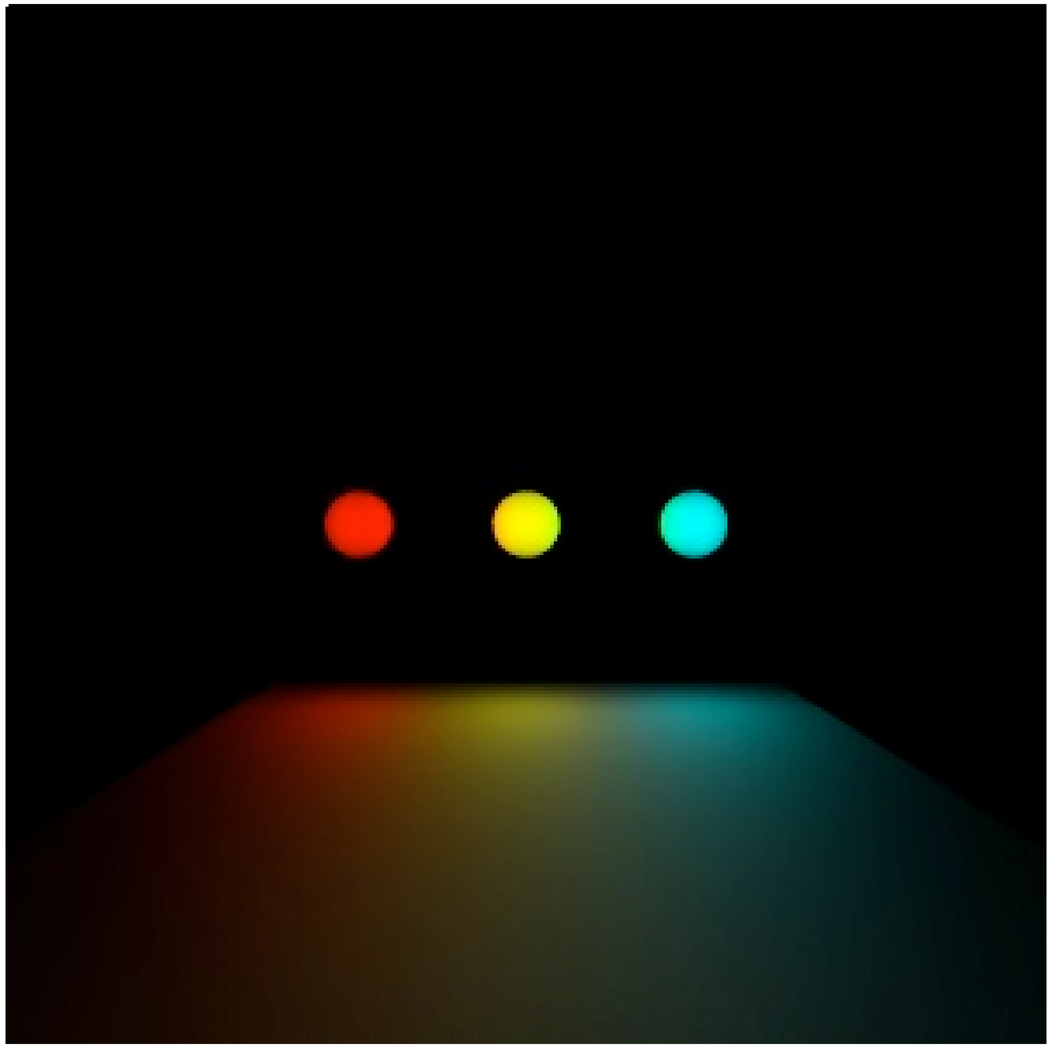




Kurt Akeley

Brute Force vs. Microfacet Distribution





Kurt Akeley

Generalized Diffuse Reflection

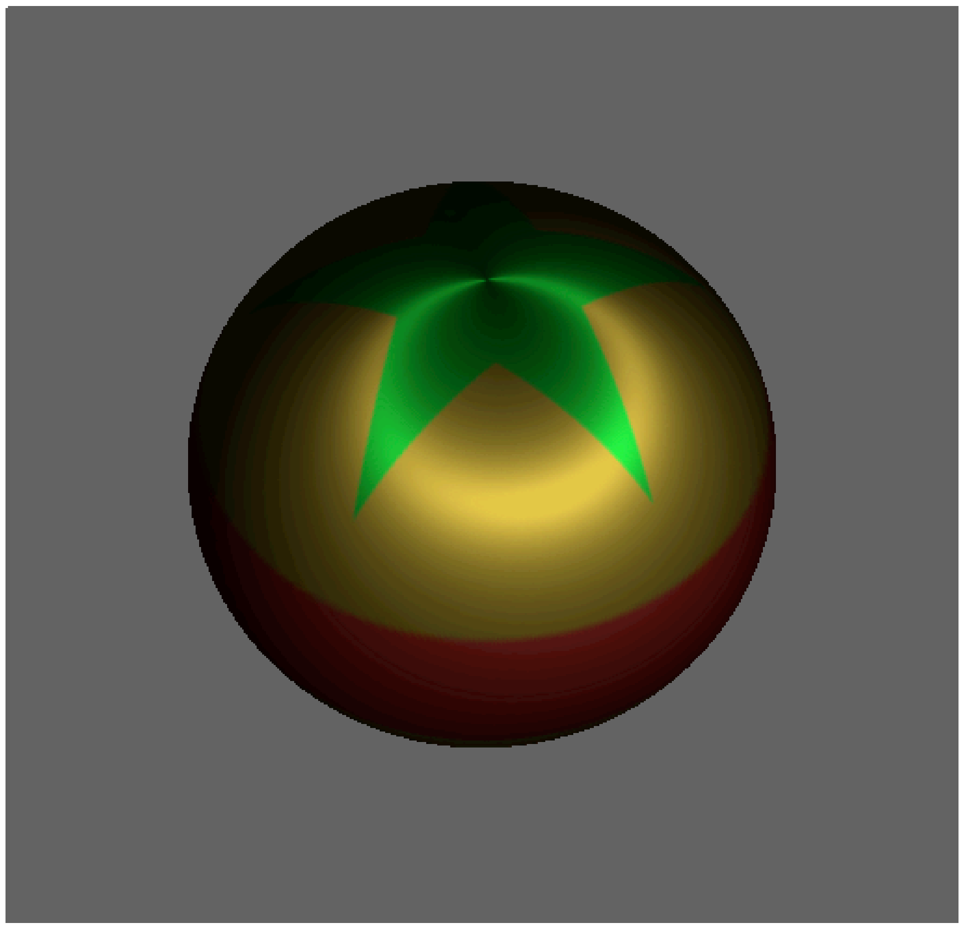
- Oren-Nayar addressed shortcomings in Lambertian model
  - Rough surfaces get brighter as viewing direction approaches light direction
- Rough microfacets, individually Lambertian
- Functional approximation to resulting reflection

Standard Diffuse Model

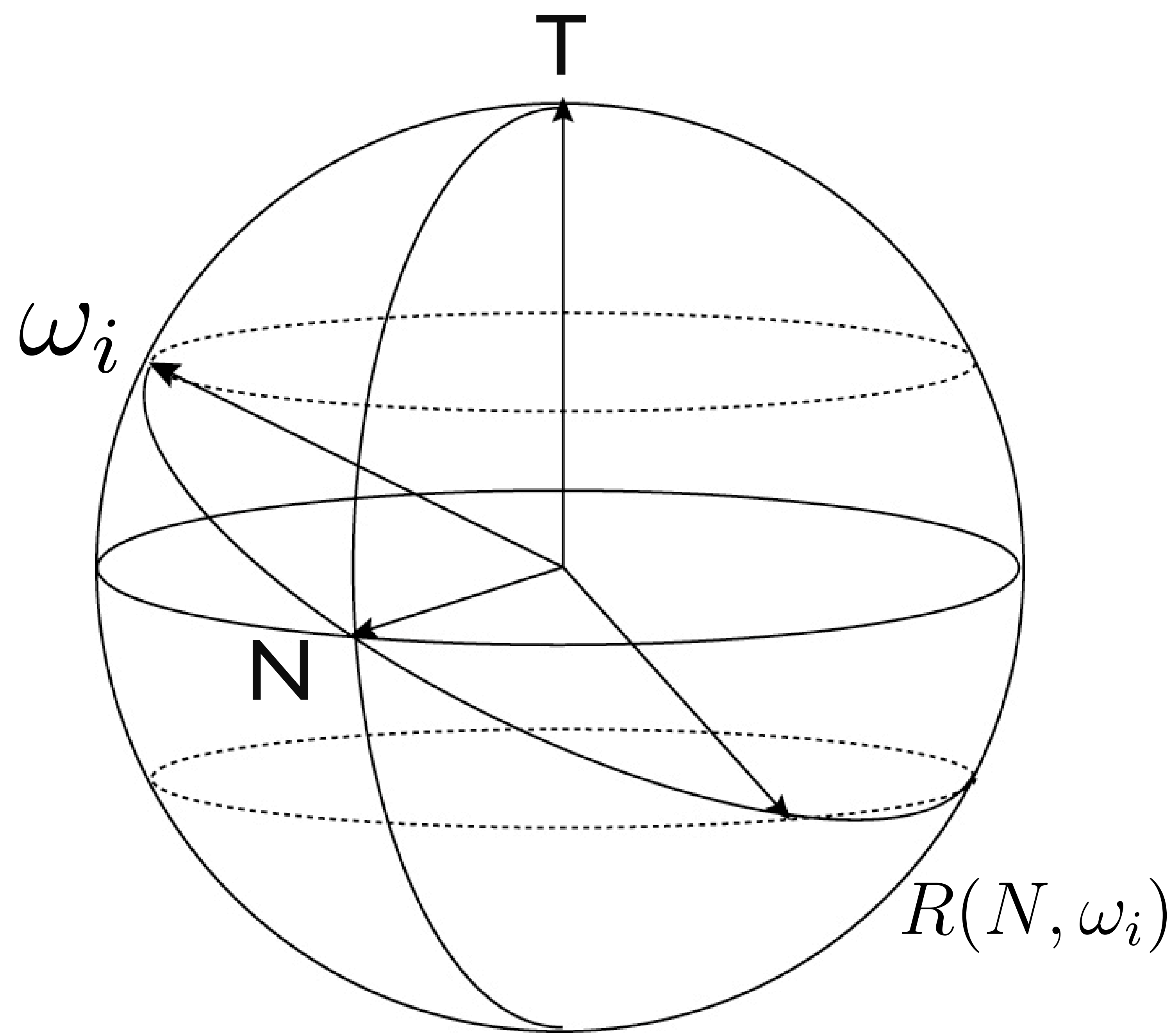
Oren-Nayar (Low Roughness)

Oren-Nayar (High Roughness)

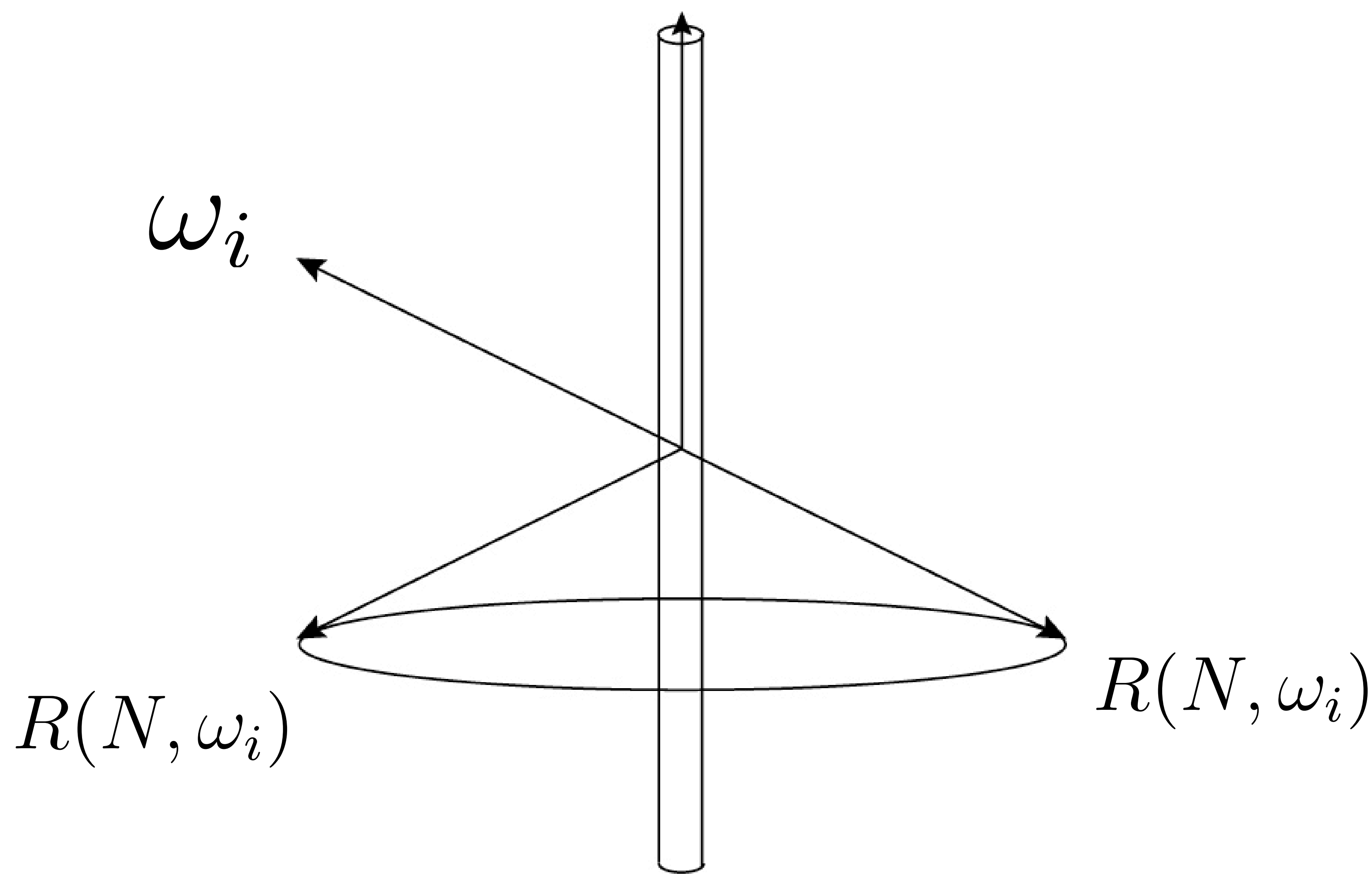
Anisotropic Reflection



Reflection from a Cylinder



Reflection from a Cylinder



Kay-Kajiya Model

- Integrate Lambertian reflection over directions of cylinder for diffuse:

$$\sin(t, \omega_i) = \sqrt{1 - (t \cdot \omega_i)^2}$$

- Specular ad-hoc model:

$$\cos^n(\theta_i - \theta_o) = (\cos\theta_i \cos\theta_o + \sin\theta_i + \sin\theta_o)^n$$

Kay-Kajiya Model

Hierarchies of Scale

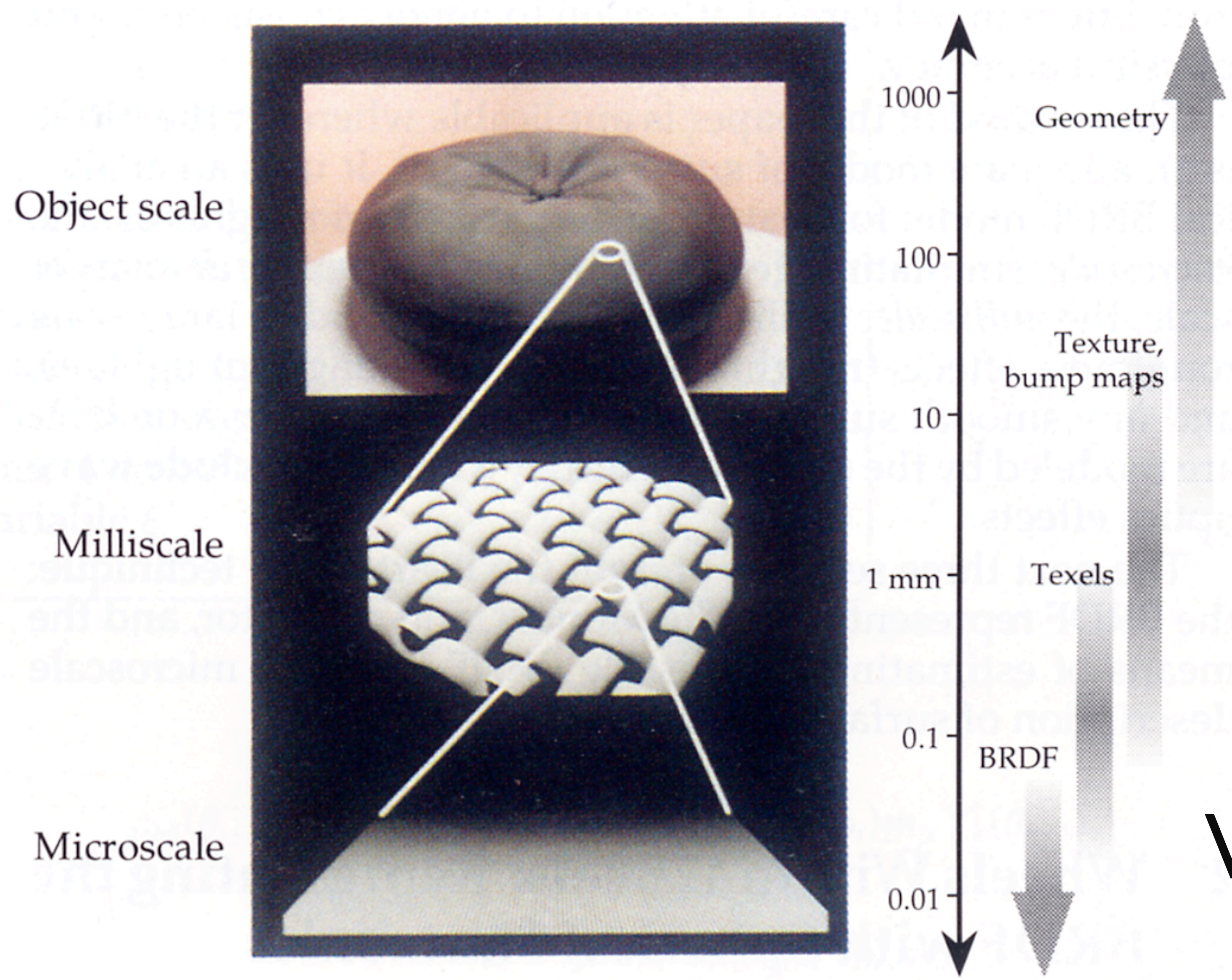


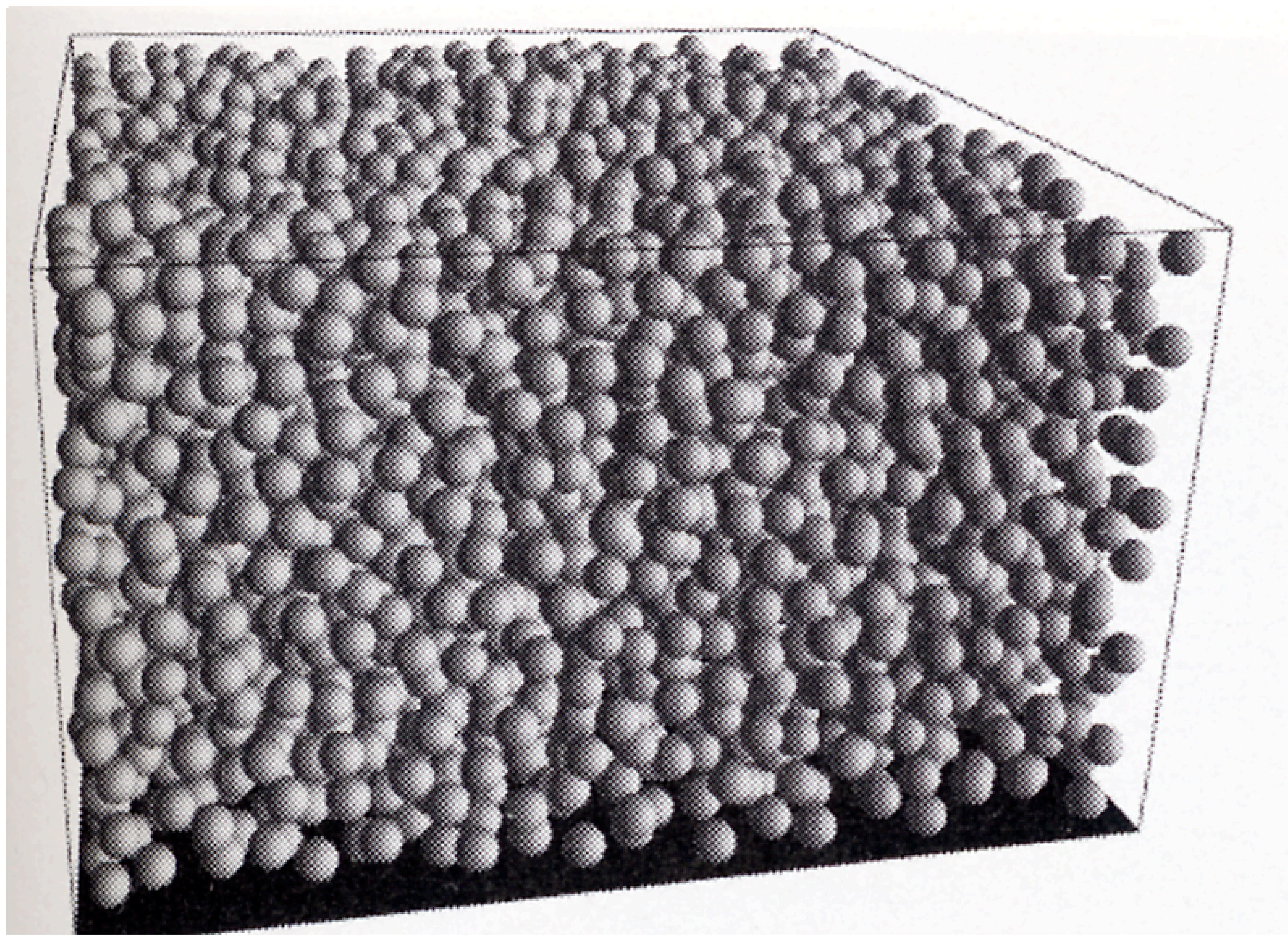
Figure 1: Applicability of Techniques

Westin et al

BRDFs from Hierarchies of Scale

- Simulate light interactions at microscopic level
  - Large number of simulations gives statistical sample of light distribution
- Basis functions to represent BRDF
  - Spherical harmonics (Westin et al)
  - Spherical wavelets
  - ...
- Tabularize data and interpolate
- Just do simulation at render-time

Paint Microgeometry



Gondek et al

Velvet Microstructure

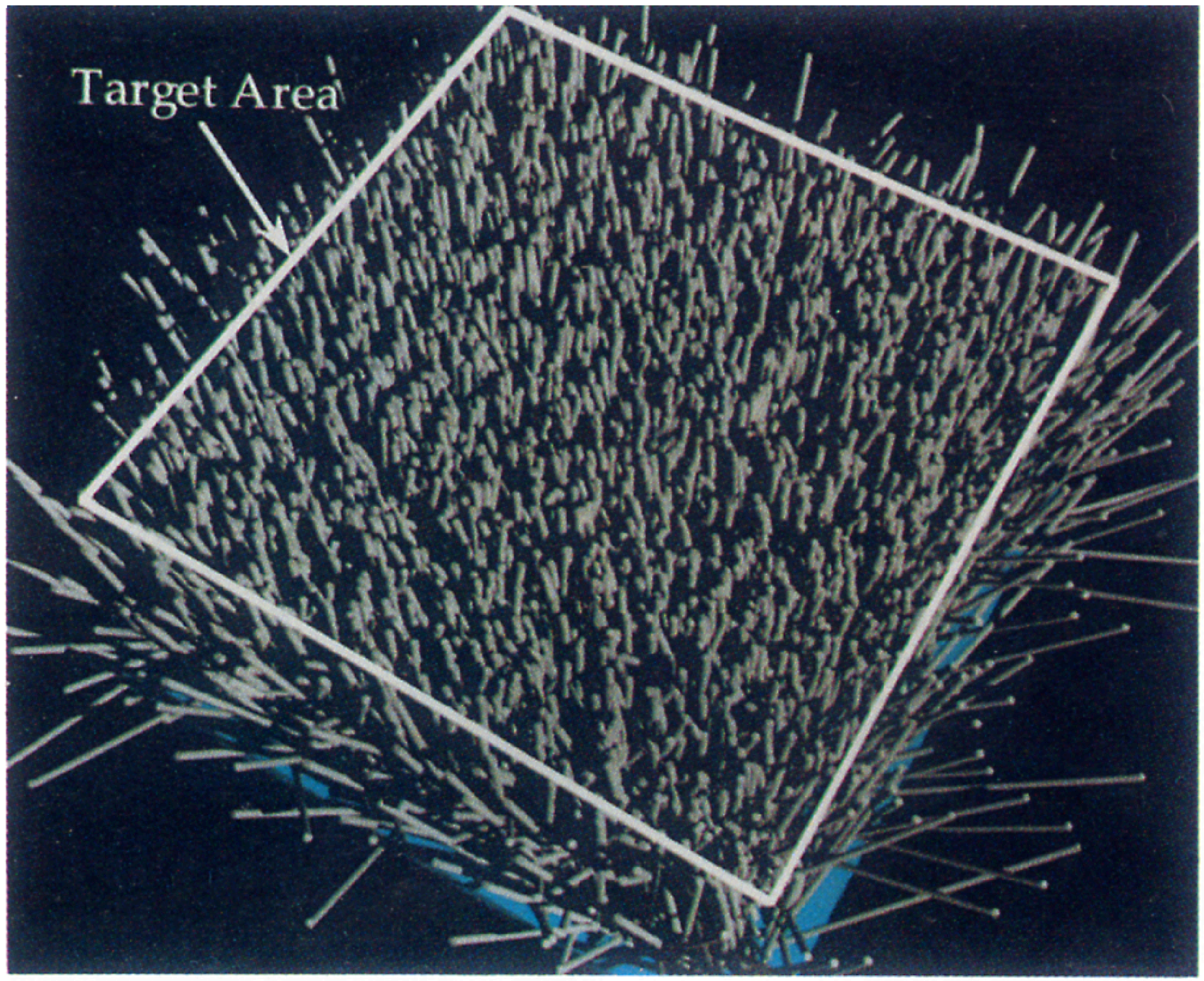


Figure 11: Microscale Geometry for Velvet

Westin et al

Velvet Doughnut

Figure 12: Velvet Doughnut

Westin et al

Cloth Microstructure

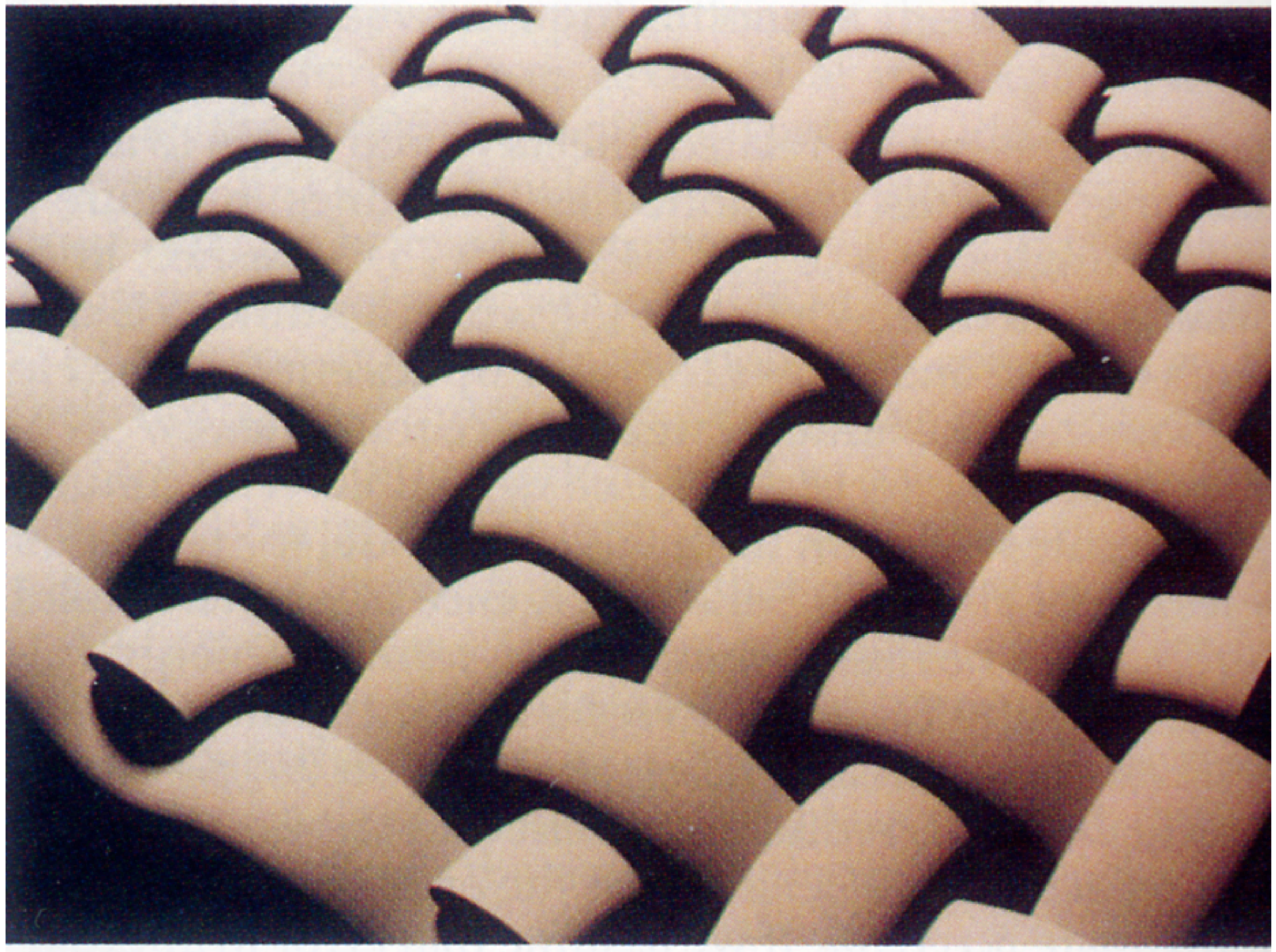


Figure 13: Microscale Structure of Cloth Model

Westin et al

Lafortune et al Model

- Generalize Phong model
- Make reflection lobes orientable
- In BRDF coordinate system,

$$R(N, \omega) \equiv (-1, -1, 1) \times \omega$$

$$f_r(\omega_i \to \omega_o) = \sum_i ((x_i, y_i, z_i) \times \omega_i) \cdot \omega_o)^{n_i}$$