The Light Field

Light field = radiance function on rays

Surface and field radiance

Conservation of radiance

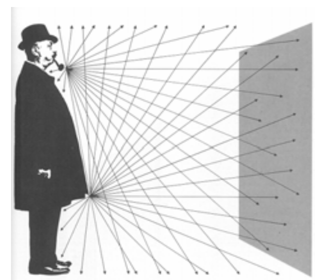
Measurement

Irradiance from area sources

Measuring rays

Form factors and throughput

Conservation of throughput

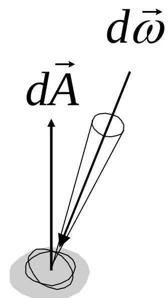


From London and Upton

Light Field = Radiance(Ray)

Incident Surface Radiance

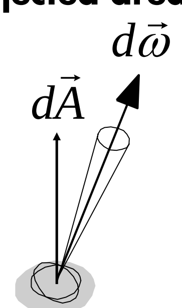
<u>Definition</u>: The incoming surface *radiance* (*luminance*) is the power per unit solid angle per unit projected area arriving at a receiving surface



$$L_i(x,\omega) \equiv \frac{d^2\Phi_i(x,\omega)}{d\vec{\omega} \bullet d\vec{A}}$$

Exitant Surface Radiance

<u>Definition</u>: The outgoing surface radiance (luminance) is the power per unit solid angle per unit projected area leaving at surface

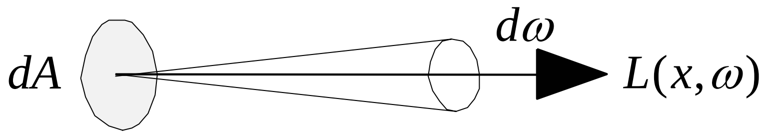


$$L_o(x,\omega) \equiv \frac{d^2\Phi_o(x,\omega)}{d\vec{\omega} \bullet d\vec{A}}$$

Alternatively: the intensity per unit projected area leaving a surface

Field Radiance

Definition: The field *radiance* (*luminance*) at a point in space in a given direction is the power per unit solid angle per unit area perpendicular to the direction

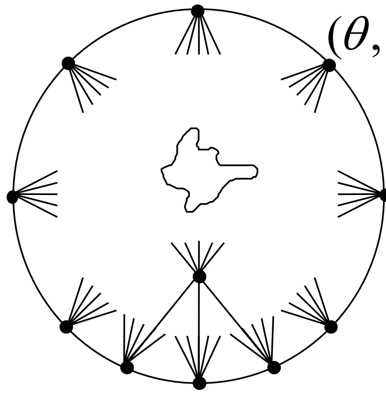


Environment Maps

$L(\theta,\varphi)$

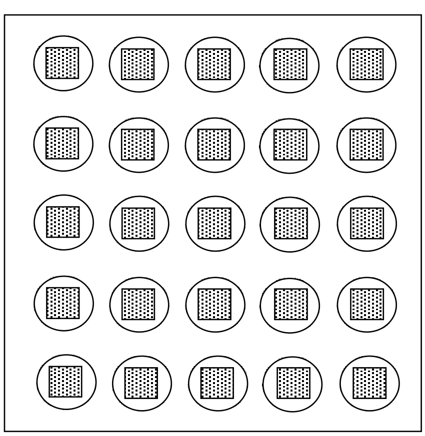
Miller and Hoffman, 1984

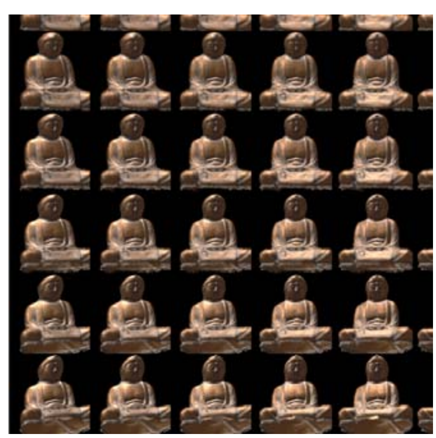
Spherical Gantry ⇒ Light Field



Capture all the light leaving an object - like a hologram

Two-Plane Light Field

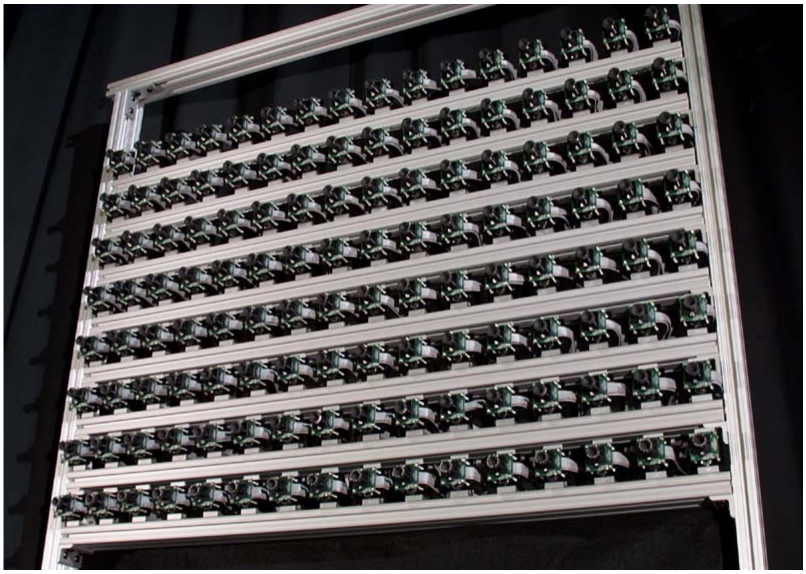




2D Array of Cameras **2D Array of Images**

$$L(u, v, s, t)$$

Multi-Camera Array ⇒ Light Field



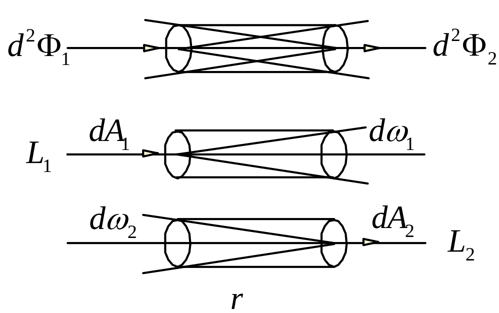
Properties of Radiance

Properties of Radiance

1. Fundamental field quantity that characterizes the distribution of light in an environment.
   - ∴ Radiance is a function on rays
   - ∴ All other field quantities are derived from it
2. Radiance invariant along a ray.
   - ∴ 5D ray space reduces to 4D
3. Response of a sensor proportional to radiance.

1st Law: Conversation of Radiance

The radiance in the direction of a light ray remains constant as the ray propagates



$$d^2\Phi_1 = d^2\Phi_2$$

$$d^2\Phi_1 = L_1 d\omega_1 dA_1$$

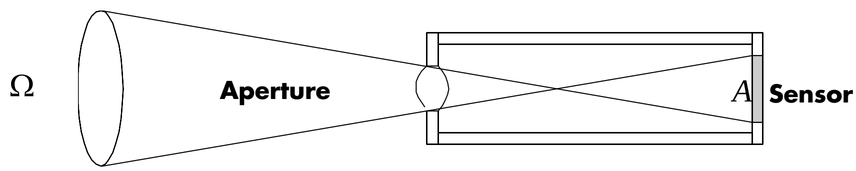
$$d^2\Phi_2 = L_2 d\omega_2 dA_2$$

$$d\omega_1 dA_1 = \frac{dA_1 dA_2}{r^2} = d\omega_2 dA_2$$

$$\therefore L_1 = L_2$$

Radiance: 2nd Law

The response of a sensor is proportional to the radiance of the surface visible to the sensor.



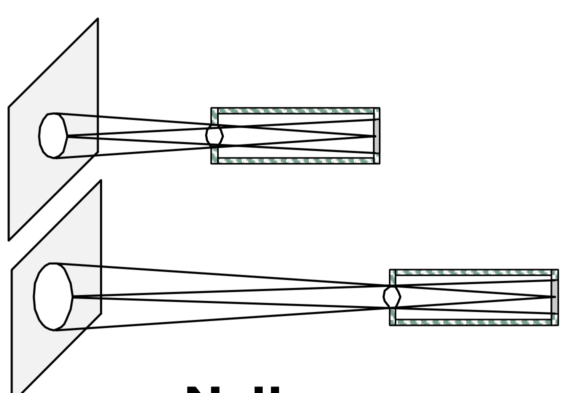
$$R = \int_A \int_\Omega L \, d\omega \, dA = \bar{L} T \qquad T = \int_A \int_\Omega d\omega \, dA$$

L **is what should be computed and displayed.**

T **quantifies the gathering power of the device.**

Quiz

Does the brightness that a wall appears to the sensor depend on the distance?



No!!

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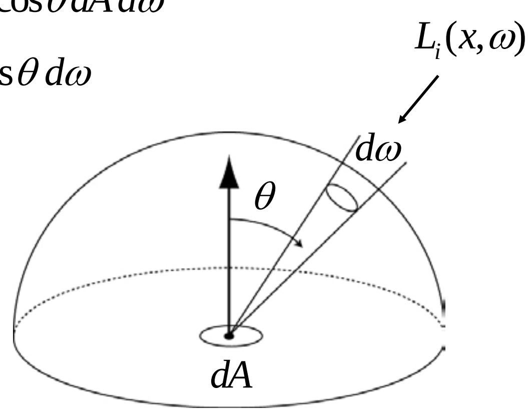
Irradiance from a Uniform Surface Source

Irradiance from the Environment

$$d^2\Phi_i(x,\omega) = L_i(x,\omega)\cos\theta\, dA\, d\omega$$

$$dE(x,\omega) = L_i(x,\omega)\cos\theta\, d\omega$$



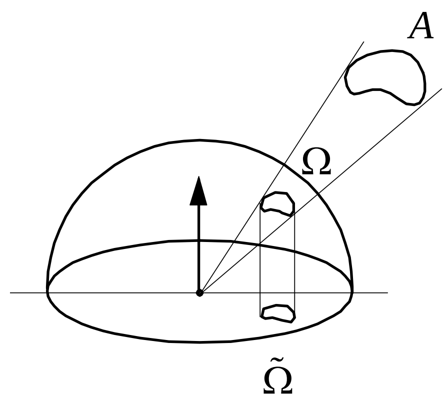
$$E(x) = \int_{H^2} L_i(x,\omega)\cos\theta\, d\omega$$

Irradiance from an Area Source

$$E(x) = \int_{H^2} L \cos\theta \, d\omega$$

$$= L \int_{\Omega} \cos\theta \, d\omega$$

$$= L\tilde{\Omega}$$



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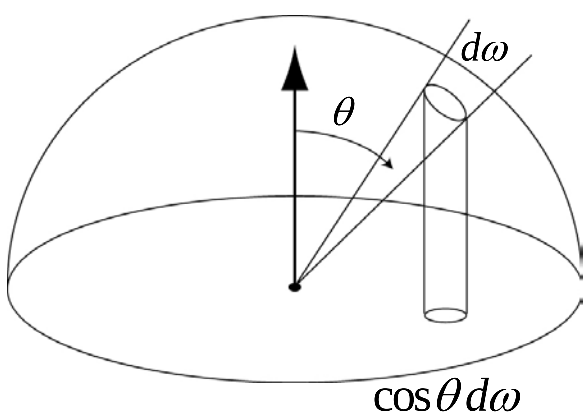
Projected Solid Angle



$$\int_{H^2} \cos\theta \, d\omega = \pi$$

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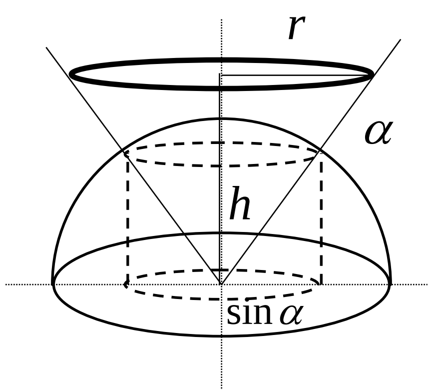
Uniform Disk Source

Geometric Derivation



$$\tilde{\Omega} = \pi \sin^2 \alpha$$

Algebraic Derivation

$$\tilde{\Omega} = \int_1^{\cos\alpha} \int_0^{2\pi} \cos\theta \, d\phi \, d\cos\theta$$

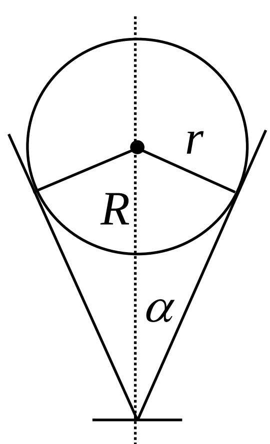
$$= 2\pi \left. \frac{\cos^2 \theta}{2} \right|_1^{\cos\alpha}$$

$$= \pi \sin^2 \alpha$$

$$= \pi \frac{r^2}{r^2 + h^2}$$

Spherical Source

Geometric Derivation



$$\tilde{\Omega} = \pi \sin^2 \alpha$$

Algebraic Derivation

$$\tilde{\Omega} = \int \cos\theta \, d\omega$$

$$= \pi \sin^2 \alpha$$

$$= \pi \frac{r^2}{R^2}$$

The Sun

Solar constant (normal incidence at zenith)

Irradiance 1353 W/m²

Illuminance 127,500 lm/m² = 127.5 kilolux

Solar angle

α **= .25 degrees = .004 radians (half angle)**

$\tilde{\Omega} = \pi \sin^2 \alpha \approx \pi\alpha^2$ **= 6 x 10^{-5} steradians**

Solar radiance

$$L = \frac{E}{\tilde{\Omega}} = \frac{1.353 \times 10^3 \, W/m^2}{6 \times 10^{-5} \, sr} = 2.25 \times 10^7 \frac{\mathrm{W}}{m^2 \cdot sr}$$

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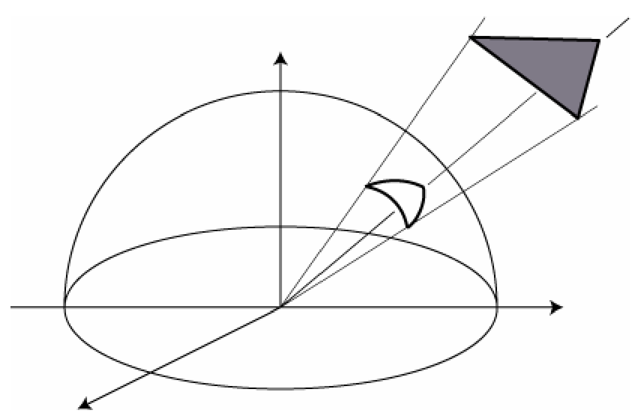
Polygonal Source



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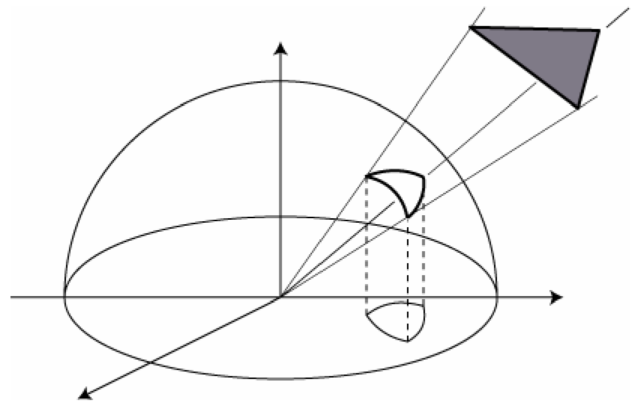
Polygonal Source



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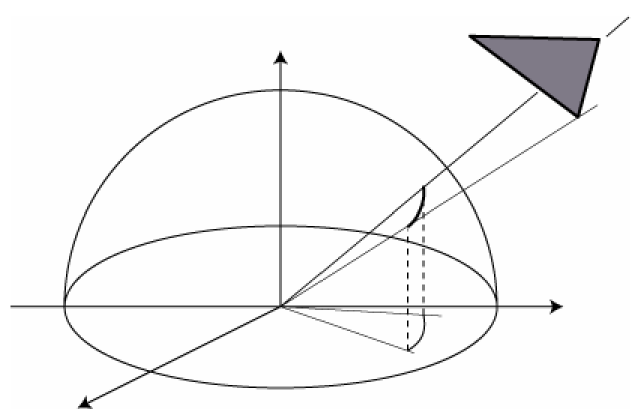
Polygonal Source



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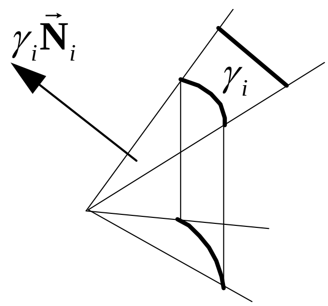
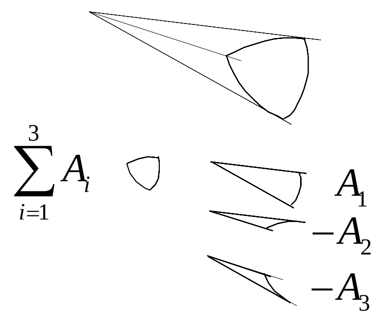
Lambert's Formula

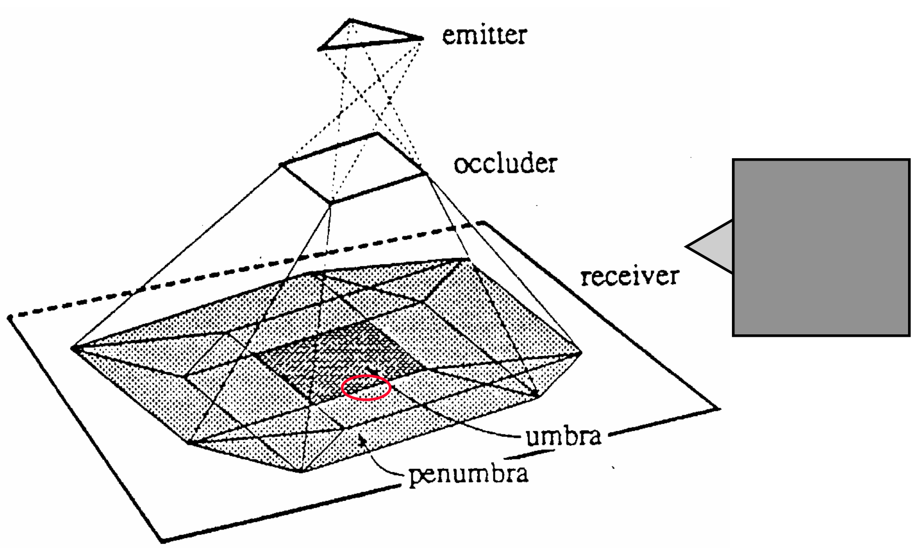


$$A_i = \gamma_i \vec{\mathbf{N}}_i \bullet \vec{\mathbf{N}}$$



$$\sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \gamma_i \vec{\mathbf{N}}_i \bullet \vec{\mathbf{N}}$$

Penumbras and Umbras

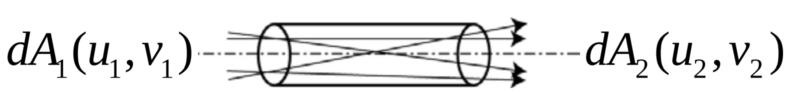


Measuring Rays = Throughput

Throughput Counts Rays

Define an infinitesimal beam as the set of rays intersecting two infinitesimal surface elements

$$r(u_1, v_1, u_2, v_2)$$



$$d^2T = \frac{dA_1 dA_2}{\left| x_1 - x_2 \right|^2}$$

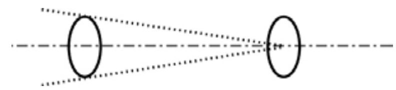
Measure the number of rays in the beam

This quantity is called the *throughput*

Parameterizing Rays

Parameterize rays wrt to receiver $r(u_2, v_2, \theta_2, \phi_2)$

$d\omega_2(\theta_2, \phi_2)$



$dA_2(u_2, v_2)$

$$d^2T = \frac{dA_1}{|x_1 - x_2|^2} dA_2 = d\omega_2 dA_2$$

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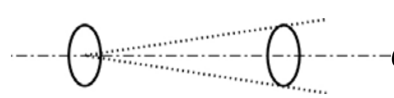
Parameterizing Rays

Parameterize rays wrt to source $r(u_1, v_1, \theta_1, \phi_1)$

$dA_1(u_1, v_1)$



$d\omega_1(\theta_1, \phi_1)$

$$d^2T = dA_1 \frac{dA_2}{|x_1 - x_2|^2} = dA_1 d\omega_1$$

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Parameterizing Rays

Tilting the surfaces reparameterizes the rays

$$r(u_1, v_1, u_2, v_2)$$

$d\vec{A}_1(u_1, v_1)$  $d\vec{A}_2(u_2, v_2)$

$$d^2T = \frac{\cos\theta_1 \cos\theta_2}{\left|x_1 - x_2\right|^2} dA_1\, dA_2$$

$$= d\vec{\omega}_1 \bullet d\vec{A}_1$$

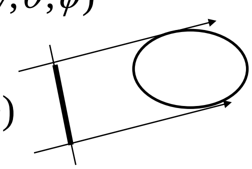
$$= d\vec{\omega}_2 \bullet d\vec{A}_2$$

All these throughputs must be equal.

Parameterizing Rays: $S^2 \times R^2$

Parameterize rays by $r(x, y, \theta, \phi)$

Projected area $\tilde{A}(\vec{\omega})$



$\vec{\omega}$

Measuring the number or rays that hit a shape

$$T = \int_{S^2} d\omega(\theta,\varphi) \int_{R^2} dA(x,y)$$

$$= \int_{S^2} \tilde{A}(\theta,\varphi)\, d\omega(\theta,\varphi)$$

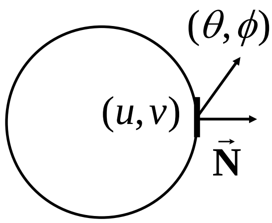
$$= 4\pi \bar{\tilde{A}}$$

Sphere:

$$T = 4\pi \bar{\tilde{A}} = 4\pi^2 R^2$$

Parameterizing Rays: $M^2 \times S^2$

Parameterize rays by $r(u,v,\theta,\phi)$



$$T = \underbrace{\left[\int_{M^2} dA(u,v)\right]}_{S} \underbrace{\left[\int_{H^2(\vec{\mathbf{N}})} \cos\theta \, d\omega(\theta,\varphi)\right]}_{\pi}$$

Sphere: $T = \pi S = 4\pi^2 R^2$

Crofton's Theorem: $4\pi \bar{\tilde{A}} = \pi S \Rightarrow \bar{\tilde{A}} = \dfrac{S}{4}$

Form Factors

Types of Throughput

1. Infinitesimal beam of rays

$$d^2T(dA, dA') \equiv \frac{\cos\theta\cos\theta'}{|x-x'|^2} dA(x)\, dA(x')$$

2. Infinitesimal-finite beam

$$dT(dA, A')\, dA \equiv \left[\int_{A'} \frac{\cos\theta\cos\theta'}{|x-x'|^2} dA(x')\right] dA(x)$$

3. Finite-finite beam

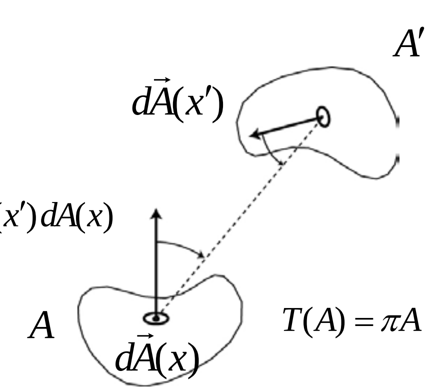
$$T(A, A') \equiv \int_A \int_{A'} \frac{\cos\theta\cos\theta'}{|x-x'|^2} dA(x')\, dA(x)$$

Probability of Ray Intersection

Probability of a ray hitting *A′* given it hits *A*

$$\Pr(A' \mid A) = \frac{T(A', A)}{T(A)} = \frac{T(A', A)}{\pi A}$$

$$T(A', A) = \int_A \int_{A'} \frac{\cos\theta\cos\theta'}{|x-x'|^2} dA(x')\, dA(x)$$



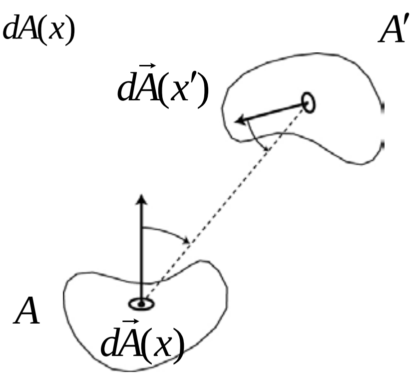
Probability of Ray Intersection

$$T(A',A) = \int_A \int_{A'} \frac{\cos\theta \cos\theta'}{|x - x'|^2} \, dA(x') \, dA(x)$$

$$= \pi \int_A \int_{A'} G(x,x') \, dA(x') \, dA(x)$$

$$G(x,x') \equiv \frac{\cos\theta \cos\theta'}{\pi |x - x'|^2} V(x,x')$$

$$V(x,x') = \begin{cases} 0 & \neg visible \\ 1 & visible \end{cases}$$



Form Factor

Probability of a ray hitting *A′* given it hits *A*

$$\Pr(A' \mid A)\ T(A) = \Pr(A \mid A')\ T(A') = T(A',A)$$

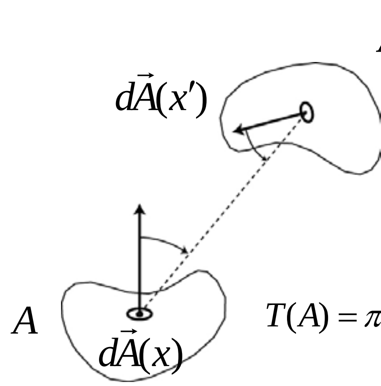
Form factor definition

$$F(A',A) = \Pr(A' \mid A)$$

$$F(A,A') = \Pr(A \mid A')$$

Form factor reciprocity

$$F(A',A)A = F(A,A')A'$$

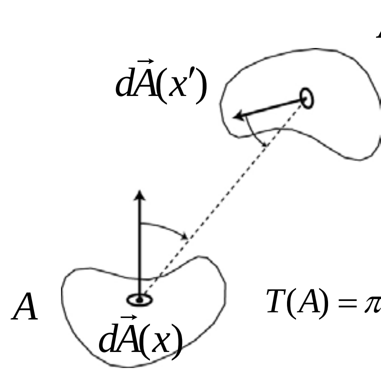


$$T(A) = \pi A$$

Form Factor

Power transfer from a constant source

$$\Phi(A, A') = LT(A, A')$$
$$= LT(A')\frac{T(A, A')}{T(A')}$$
$$= \Phi(A')F(A, A')$$

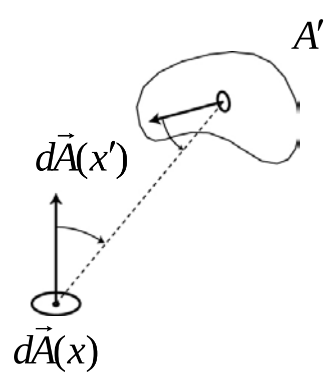


$$T(A) = \pi A$$

Differential Form Factor

Probability of a ray leaving *dA(x)* hitting *A′*

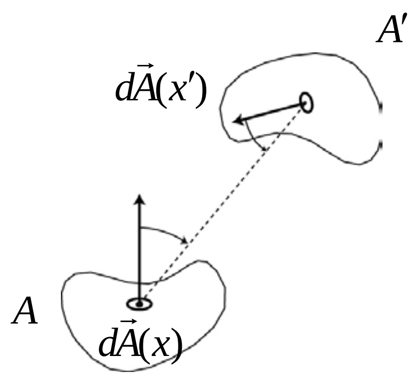
$$\Pr(A' \mid dA) = \frac{T(dA, A')dA}{T(dA)} = \frac{T(dA, A')dA}{\pi dA} = \frac{T(dA, A')}{\pi}$$
$$= \int_{A'} G(x, x')\, dA(x')$$



Form Factor

Power transfer from a constant source

$$
\begin{aligned}
d\Phi(dA, A') &= LT(dA, A') \\
&= LT(A')\frac{T(dA, A')}{T(A')} \\
&= \Phi(A')F(dA, A')
\end{aligned}
$$



Radiant Exitance

<u>Definition</u>: The *radiant* (*luminous*) *exitance* is the energy per unit area leaving a surface.

$$M(x) \equiv \frac{d\Phi_o}{dA}$$

$$\left[\frac{W}{m^2}\right] \left[\frac{lm}{m^2} = lux\right]$$

In computer graphics, this quantity is often referred to as the *radiosity (B)*

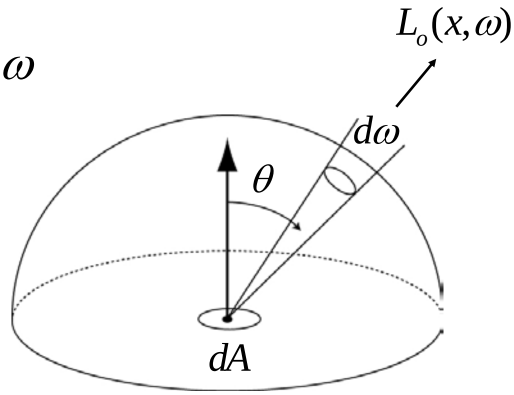
Uniform Diffuse Emitter

$$M = \int_{H^2} L_o \cos\theta \, d\omega$$

$$= L_o \int_{H^2} \cos\theta \, d\omega$$

$$= \pi L_o$$

$$L_o = \frac{M}{\pi}$$



Form Factor

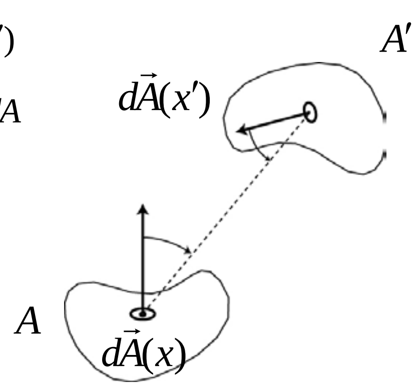
Irradiance from a constant source

$$d\Phi(dA, A') = \Phi(A')F(dA, A')$$

$$= \frac{\Phi(A')}{A'} A' F(dA, A')$$

$$= M(A')F(A', dA)dA$$

$$E(dA) = M(A')F(A', dA)$$



Form Factors and Throughput

Throughput measures the number of rays in a set of rays

Form factors represent the probability of ray leaving a surface intersecting another surface

- Only a function of surface geometry

Differential form factor

- Irradiance calculations

Form factors

- Radiosity calculations (energy balance)

Conservation of Throughput

- Throughput conserved during propagation
  - Number of rays conserved
  - Assuming no attenuation or scattering
- n^2 (index of refraction) times throughput invariant under the laws of geometric optics
  - Reflection at an interface
  - Refraction at an interface
    - Causes rays to bend (kink)
  - Continuously varying index of refraction
    - Causes rays to curve; mirages

Conservation of Radiance

Radiance is the ratio of two quantities:

1. Power

2. Throughput

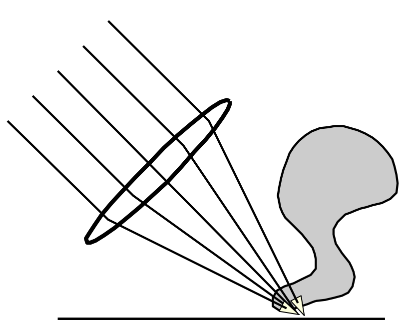
$$L(r) = \lim_{\Delta T \to 0} \frac{\Delta\Phi(\Delta T)}{\Delta T} = \frac{d\Phi}{dT}$$

Since power and throughput are conserved,

$\therefore$ **Radiance conserved**

Quiz

Does radiance increase under a magnifying glass?



No!!