Reflection Models

Last lecture

- Reflection models
- The reflection equation and the BRDF
- Ideal reflection, refraction and diffuse

Today

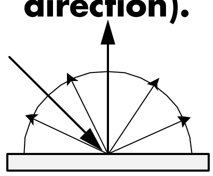
- Phong and microfacet models
- Gaussian height field on surface
- Self-shadowing
- Torrance-Sparrow model

Next

- Anisotropic: Grooves and hair
- Translucency: Skin

Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction (independent of input direction).



$$L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i) \cos\theta_i \, d\omega_i = f_{r,d} \int L_i(\omega_i) \cos\theta_i \, d\omega_i = f_{r,d} E$$

$$f_{r,d} = c$$

$$M = \int L_r(\omega_r) \cos\theta_r \, d\omega_r = L_r \int \cos\theta_r \, d\omega_r = \pi L_r$$

$$\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d} E}{E} = \pi f_{r,d} \quad \Rightarrow \quad f_{r,d} = \frac{\rho_d}{\pi}$$

Lambert's Cosine Law $M = \rho_d E = \rho_d E_s \cos\theta_s$

“Diffuse” Reflection

Theoretical

- **Bouguer - Special micro-facet distribution**
- **Seeliger - Subsurface reflection**
- **Multiple surface or subsurface reflections**

Experimental

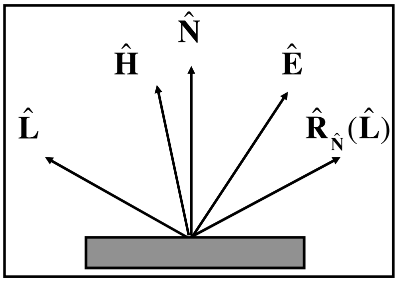
- **Pressed magnesium oxide powder**
- **Almost never valid at high angles of incidence**

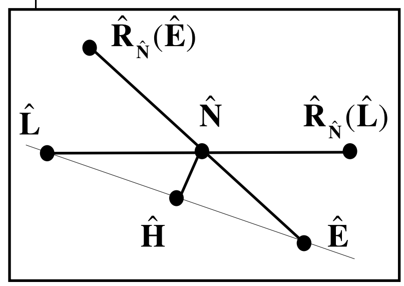
Paint manufactures attempt to create ideal diffuse

Phong Model

Reflection Geometry

$$\hat{\mathbf{H}} = \frac{\hat{\mathbf{L}} + \hat{\mathbf{E}}}{\left|\hat{\mathbf{L}} + \hat{\mathbf{E}}\right|}$$





$$\cos\theta_i = \hat{\mathbf{L}} \bullet \hat{\mathbf{N}}$$

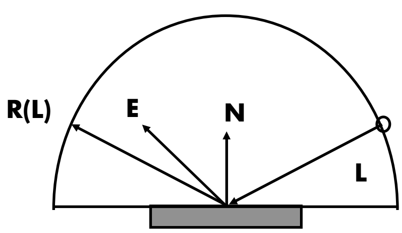
$$\cos\theta_r = \hat{\mathbf{E}} \bullet \hat{\mathbf{N}}$$

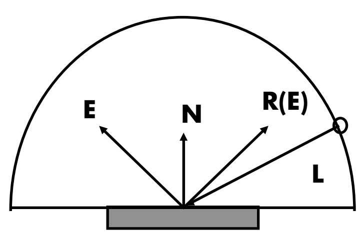
$$\cos\theta_s = \hat{\mathbf{E}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{L}}) = \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}}) \bullet \hat{\mathbf{L}}$$

$$\cos\theta_g = \hat{\mathbf{E}} \bullet \hat{\mathbf{L}}$$

$$\cos\theta_{s'} = \hat{\mathbf{H}} \bullet \hat{\mathbf{N}}$$

Phong Model





$$\left(\hat{\mathbf{E}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{L}})\right)^s \qquad \left(\hat{\mathbf{L}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}})\right)^s$$

Reciprocity: $\left(\hat{\mathbf{E}} \bullet \mathbf{R}(\hat{\mathbf{L}})\right)^s = \left(\hat{\mathbf{L}} \bullet \mathbf{R}(\hat{\mathbf{E}})\right)^s$

Distributed light source!

Phong Model

Mirror

Diffuse

$\longleftarrow$ s

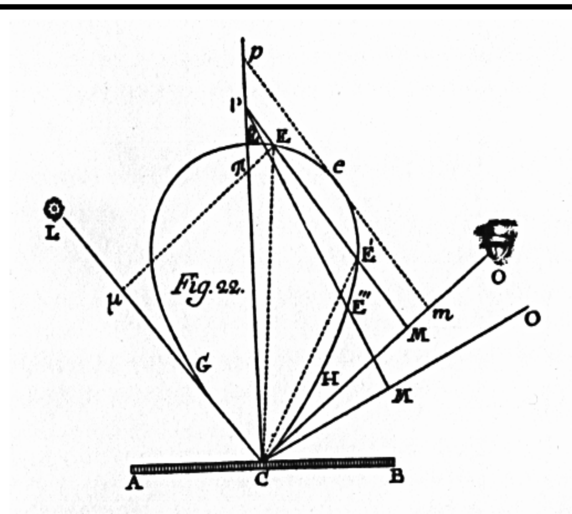
Energy Normalization

Energy normalize Phong Model

$$
\begin{aligned}
\rho(H^2 \to \omega_r) &= \int_{H^2(\hat{\mathbf{N}})} \left(\hat{\mathbf{L}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}})\right)^s \cos\theta_i \, d\omega_i \\
&\le \int_{H^2(\hat{\mathbf{N}})} \left(\hat{\mathbf{L}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}})\right)^s d\omega_i \\
&\le \int_{H^2(\mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}}))} \left(\hat{\mathbf{L}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}})\right)^s d\omega_R \\
&= \int_{H^2} \cos^s\theta \, d\omega = \frac{2\pi}{s+1}
\end{aligned}
$$

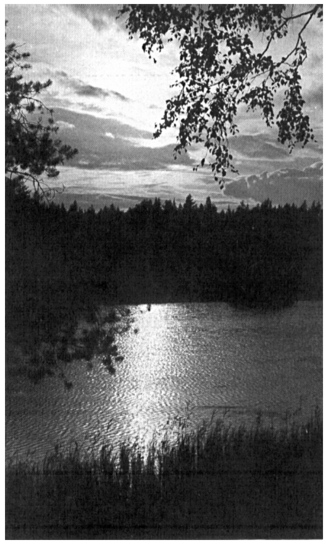
Microfacet Model

Bouguer's "little faces"

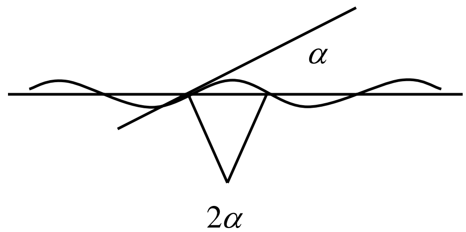


P. Bouguer, *Treatise on Optics*, 1760

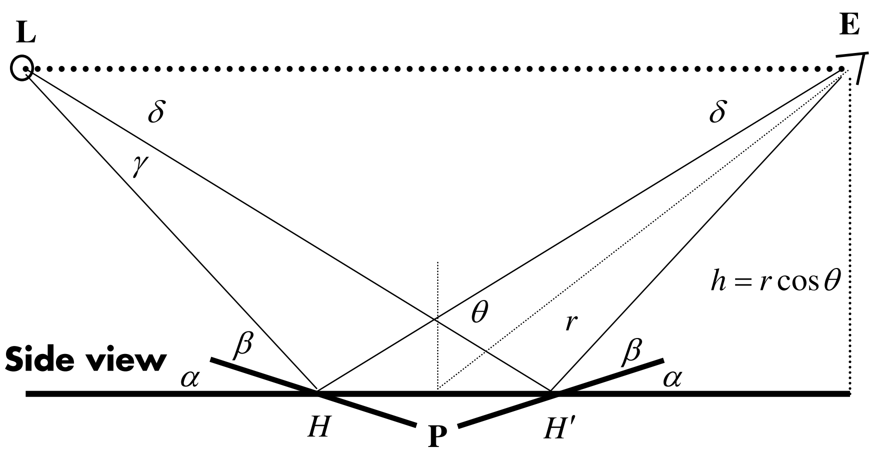
Reflection of the Sun from the Sea



***Minnaert, Light and Color in the Outdoors*, p. 28**



Reflection Angles

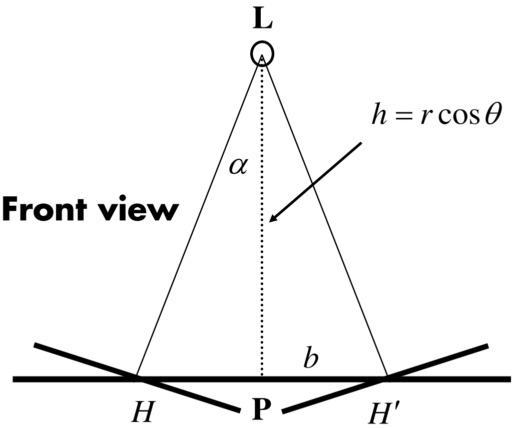


Assume L and E are at the same height h

$$\alpha + \beta = \gamma + \delta$$
$$\beta - \alpha = \delta$$
$$\Rightarrow \quad \gamma = 2\alpha$$

Reflection Angles

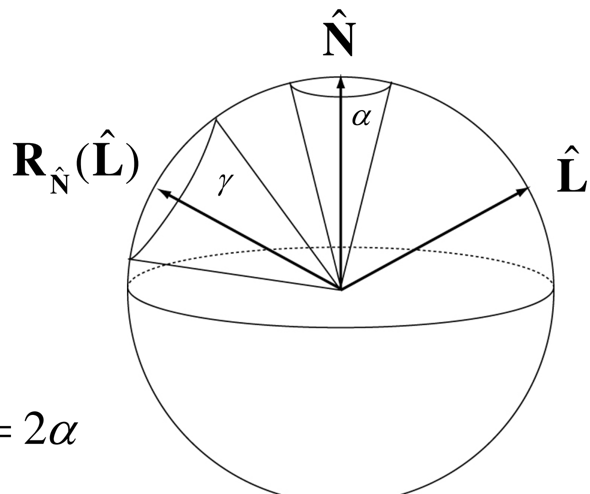
Front view



$$\tan\alpha = \frac{b}{h}$$

$$\tan\psi = \frac{b}{r} = \frac{h}{r}\tan\alpha = \tan\alpha\cos\theta$$

Analysis on the Sphere



$$\gamma = 2\alpha$$

$$\tan\psi = \tan\alpha\cos\theta$$

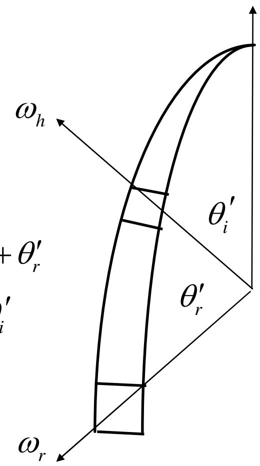
Solid Angle Distributions

$\theta_r' = \theta_i'$

$\theta_r = \theta_i' + \theta_r'$

$= 2\theta_i'$

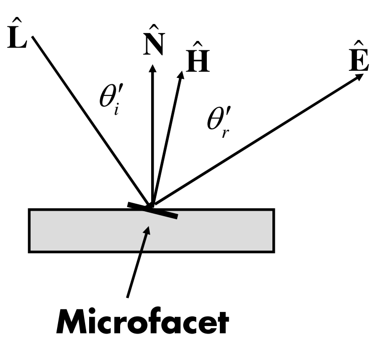
$\varphi_r = \varphi_i$



$$d\omega_r = \sin\theta_r \, d\theta_r \, d\varphi_r$$
$$= \left(\sin 2\theta_i'\right) 2 d\theta_i' \, d\varphi_i$$
$$= \left(2\sin\theta_i' \cos\theta_i'\right) 2 d\theta_i' \, d\varphi_i$$
$$= 4\cos\theta_i' \sin\theta_i' d\theta_i' \, d\varphi_i$$
$$= 4\cos\theta_i' \, d\omega_h$$

$$\frac{d\omega_h}{d\omega_r} = \frac{1}{4\cos\theta_i'}$$

Microfacet Distributions



Total projected area

$$\int_{H^2} dA(\omega_h) \cos\theta_h \, d\omega_h = dA$$

Probability distribution

$$\int_{H^2} D(\omega_h) \cos\theta_h \, d\omega_h = 1$$

Area distribution $dA(\omega_h)$

Microfacet distribution $D(\omega_h) \equiv dA(\omega_h) / dA$

Microfacet Distribution Functions

Isotropic distributions $D(\omega_h) \Rightarrow D(\alpha)$

Characterize by half-angle β $D(\beta) = \frac{1}{2}$

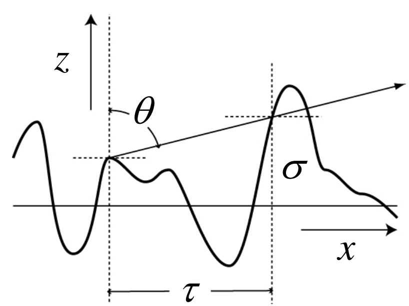
Examples:

- **Blinn** $D_1(\alpha) = \cos^{c_1}\alpha$ $c_1 = \frac{\ln 2}{\ln\cos\beta}$
- **Torrance-Sparrow** $D_2(\alpha) = e^{-(c_2\alpha)^2}$ $c_2 = \frac{\sqrt{2}}{\beta}$
- **Trowbridge-Reitz** $D_3(\alpha) = \frac{c_3^2}{(1-c_3^2)\cos^2\alpha - 1}$

$$c_3 = \left(\frac{\cos^2\beta - 1}{\cos^2\beta - \sqrt{2}}\right)^{1/2}$$

Gaussian Rough Surface

Gaussian distribution of heights



$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}$$

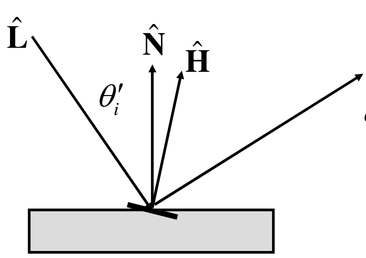
Gaussian distribution of slopes

$$D(\alpha) = \frac{1}{\sqrt{\pi}m^2\cos^2\alpha} e^{-\frac{\tan^2\alpha}{m^2}} \qquad m = \frac{2\sigma}{\tau}$$

Beckmann

Torrance-Sparrow Model

Torrance-Sparrow Model



$$\cos\theta_i = \hat{\mathbf{L}} \bullet \hat{\mathbf{N}}$$
$$\cos\theta_i' = \hat{\mathbf{L}} \bullet \hat{\mathbf{H}}$$

$$\begin{aligned} d\Phi_h &= L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, dA(\omega_h) d\omega_h \\ &= L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, D(\omega_h)\, dA\, d\omega_h \end{aligned}$$

$$dA(\omega_h) = D(\omega_h)\, dA$$

$$d\Phi_r = dL_r(\omega_i \to \omega_r)\cos\theta_r\, d\omega_r\, dA$$

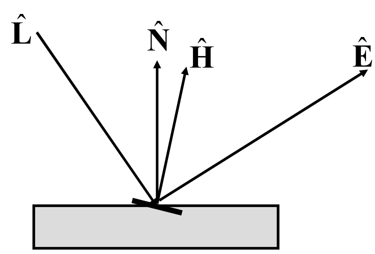
$$d\Phi_r = d\Phi_h$$

$$\begin{aligned} \therefore\, & dL_r(\omega_i \to \omega_r)\cos\theta_r\, d\omega_r\, dA \\ &= L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, D(\omega_h)\, d\omega_h\, dA \end{aligned}$$

Torrance-Sparrow Model

$$dL_r(\omega_i \to \omega_r)\cos\theta_r\, d\omega_r\, dA$$
$$= L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, D(\omega_h)\, d\omega_h\, dA$$

$$f_r(\omega_i \to \omega_r) \equiv \frac{dL_r(\omega_i \to \omega_r)}{dE_i(\omega_i)}$$



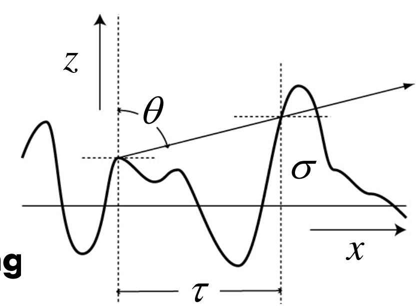
$$d\omega_i' = d\omega_i$$

$$= \frac{L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, D(\omega_h)\, d\omega_h\, dA}{\left(\cos\theta_r d\omega_r dA\right)\left(L_i(\omega_i)\cos\theta_i\, d\omega_i\right)}$$
$$= \frac{D(\omega_h)}{\cos\theta_i \cos\theta_r}\cos\theta_i' \frac{d\omega_h}{d\omega_r}$$
$$= \frac{D(\omega_h)}{4\cos\theta_i \cos\theta_r}$$

Self-Shadowing

Shadows on Rough Surfaces

Self-Shadowing Function

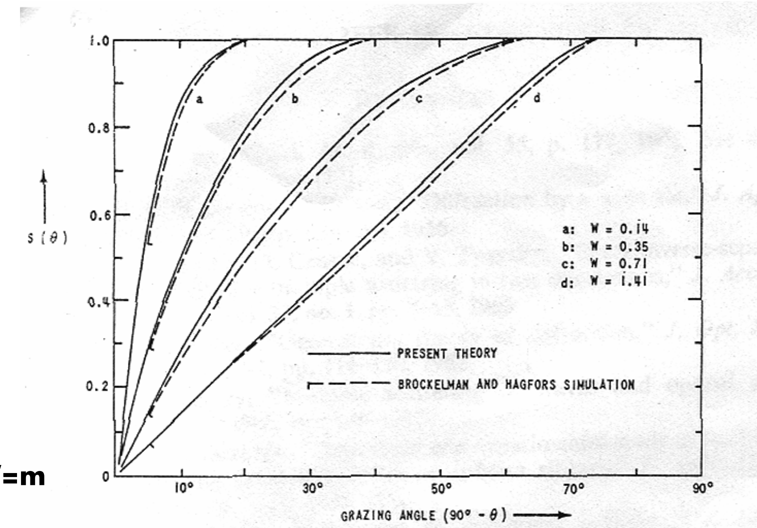


Probability of shadowing

$$S(\theta) = \frac{\left[1 - \frac{1}{2}\operatorname{erfc}\left(\mu/\sqrt{2}m\right)\right]}{1 + \Lambda(\mu)}$$

$$2\Lambda(\mu) = \left(\sqrt{\frac{2}{\pi}}\right)\frac{m}{\mu}e^{-\mu^2/2m^2} - \operatorname{erfc}\left(\mu/\sqrt{2}m\right)$$

Self-Shadowing Function



W=m

From Smith, 1967

Self-Consistency Condition

$$\int S(\theta)D(\alpha)\cos\theta' d\omega_\alpha = \cos\theta$$

The sum of the areas of the illuminated surface projected onto the plane normal to the direction of incidence is independent of the roughness of the surface, and equal to the projected area of the underlying mean plane.

Torrance-Sparrow Theory

$$f_r(\omega_i \rightarrow \omega_r) = \frac{F(\theta_i')S(\theta_i)S(\theta_r)D(\alpha)}{4\cos\theta_i\cos\theta_r}$$

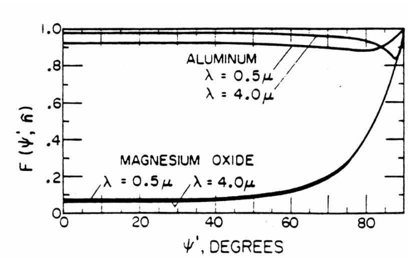


FIG. 6. Fresnel reflectance.

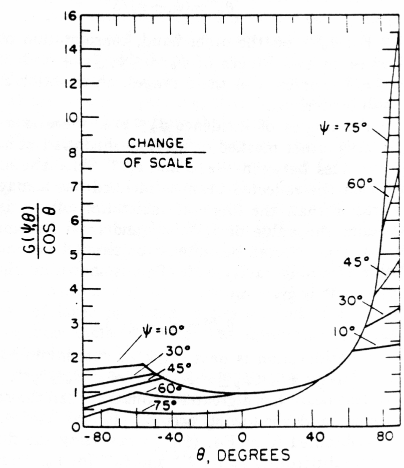
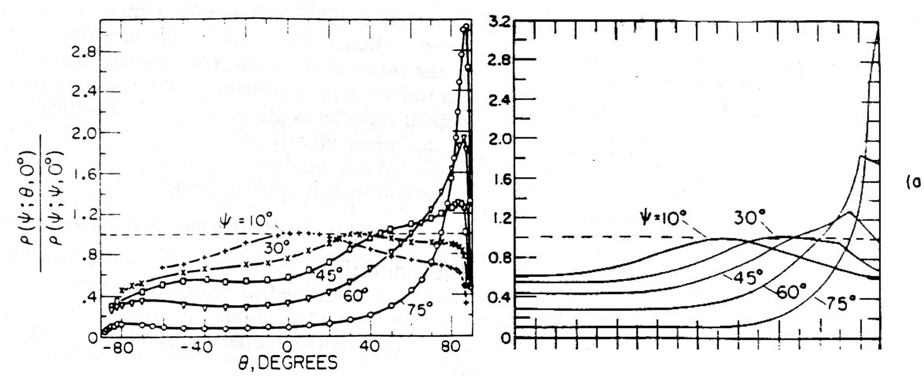


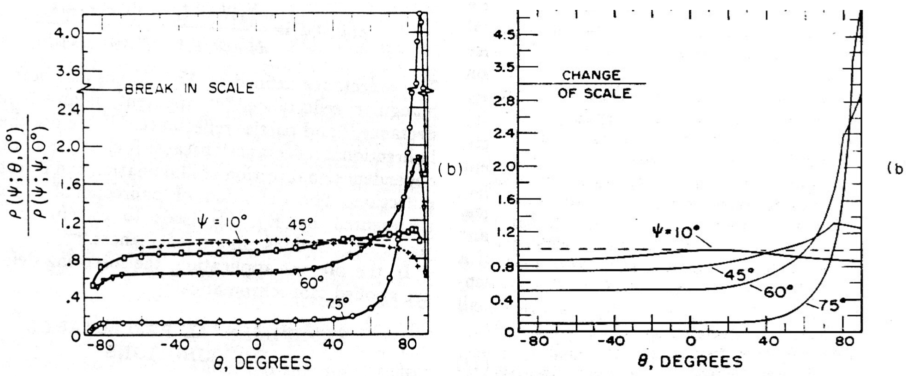
FIG. 7. The factor $G(\psi,\theta)/\cos\theta$ in the plane of incidence for various incidence angles ψ.

Torrance-Sparrow Comparison



Aluminum

Torrance-Sparrow Comparison



Magnesium Oxide

Limitations and Generalizations

Discussion

Limitations of microfacet models

- Do you think this is a good model of surface reflection?
- Why is it useful?
- What are its limitations?

Generalizations of microfacet models

- Remote sensing
  - Consider the BRDF of different parts of the earth from space. What determines the reflection function?