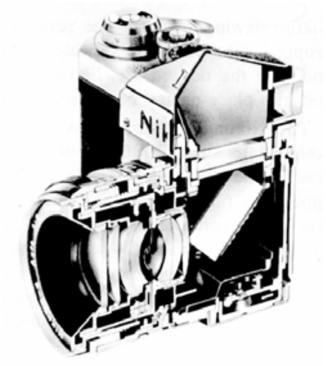
Camera Simulation



Effect	Cause
Field of view	Film size, stops and pupils
Depth of field	Aperture, focal length
Motion blur	Shutter
Exposure	Film speed, aperture, shutter

References

- Photography, B. London and J. Upton
- Optics in Photography, R. Kingslake
- The Camera, The Negative, The Print, A. Adams

Topics

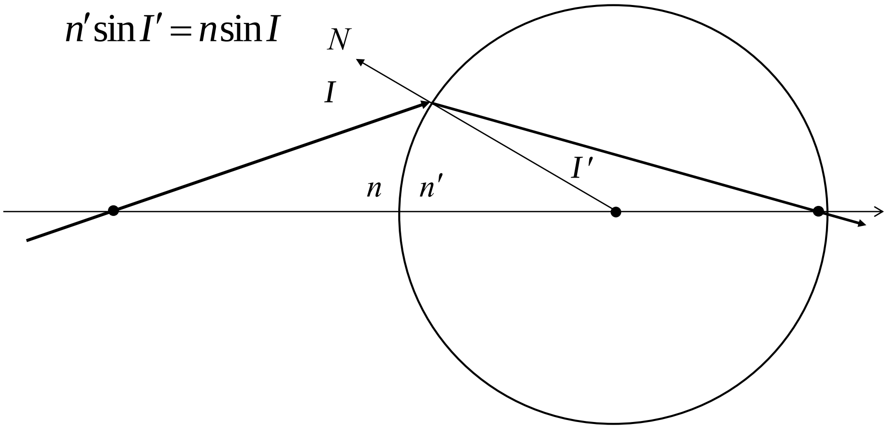
- Ray tracing lenses
- Focus
- Field of view
- Depth of focus / depth of field
- Exposure

Lenses

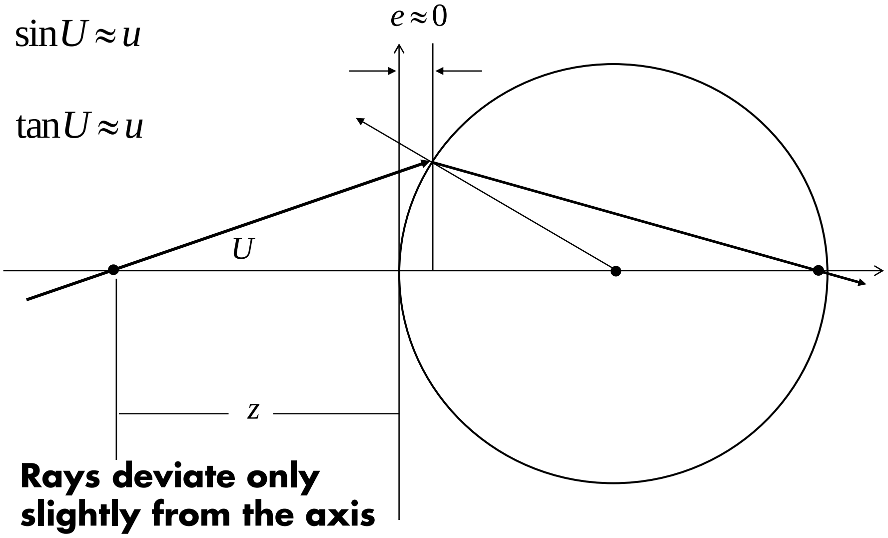
Refraction

Snell's Law

$$n' \sin I' = n \sin I$$



Paraxial Approximation



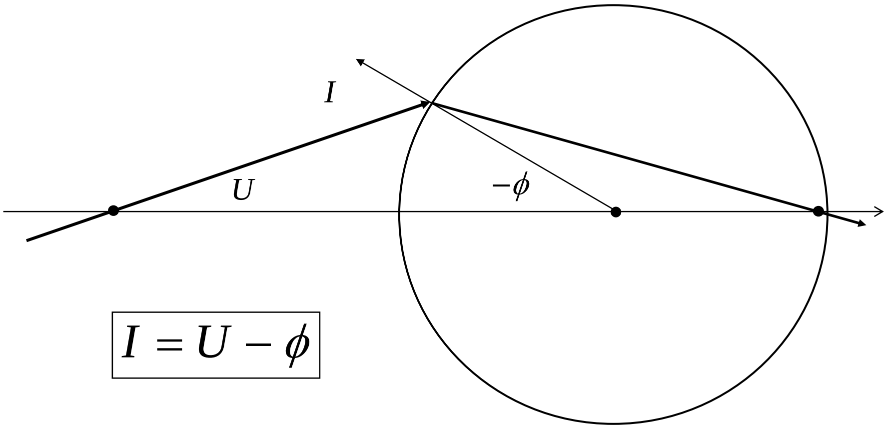
$$\sin U \approx u$$

$$\tan U \approx u$$

Rays deviate only slightly from the axis

Incident Ray

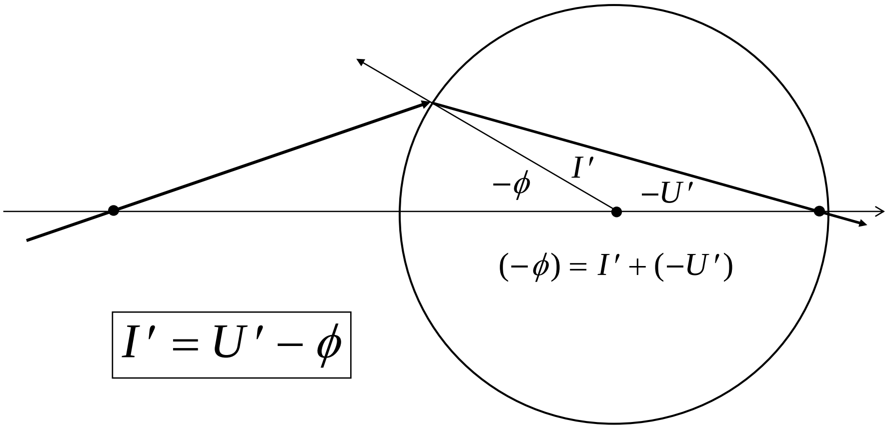
Angles: ccw is positive; cw is negative



$$\boxed{I = U - \phi}$$

The sum of the interior angles is equal to the exterior angle.

Refracted Ray



$$(-\phi) = I' + (-U')$$

$$\boxed{I' = U' - \phi}$$

Derivation

Paraxial approximation

$$I = U - \phi \quad \Rightarrow \quad i = u - \phi$$

$$I' = U' - \phi \quad \Rightarrow \quad i' = u' - \phi$$

Derivation

Paraxial approximation

$$I = U - \phi \quad \Rightarrow \quad i = u - \phi$$
$$I' = U' - \phi \quad \Rightarrow \quad i' = u' - \phi$$

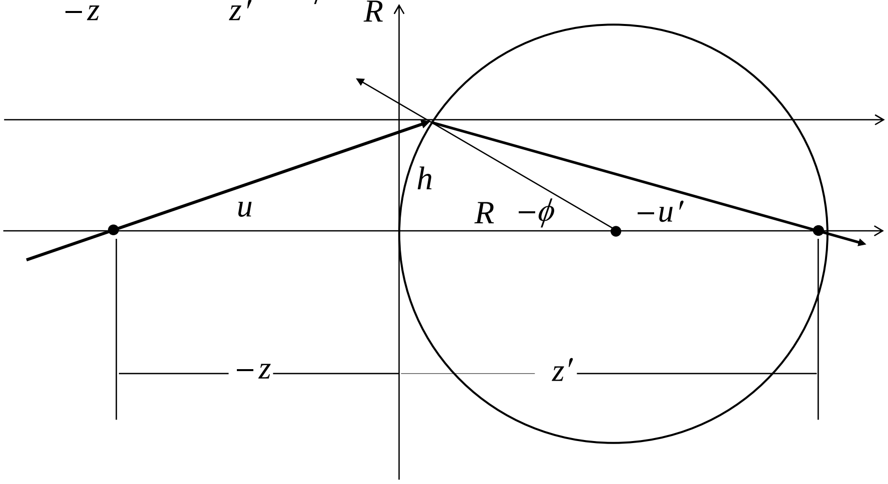
Snell's Law

$$n' \sin I' = n \sin I \quad \Rightarrow \quad n' i' = n i$$

$$\boxed{n'(u' - \phi) = n(u - \phi)}$$

Ray Coordinates

$$u = \frac{h}{-z} \qquad -u' = \frac{h}{z'} \qquad -\phi = \frac{h}{R}$$



Gauss' Formula

Paraxial approximation to Snell's Law

$$n'(u'-\phi)=n(u-\phi)$$

Ray coordinates

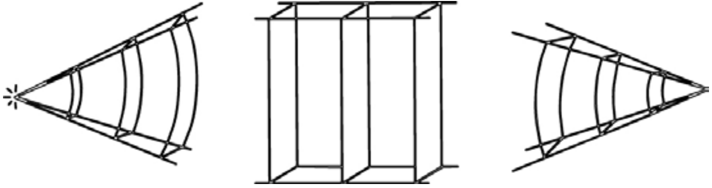
$$u'=-\frac{h}{z'} \qquad \phi=-\frac{h}{R} \qquad u=-\frac{h}{z}$$

Thin lens equation

$$n'\left(\frac{h}{z'}-\frac{h}{R}\right)=n\left(\frac{h}{z}-\frac{h}{R}\right)$$

$$\frac{n'}{z'}=\frac{n}{z}+\frac{(n'-n)}{R}$$ ⟸ **Holds for any height, any ray!**

Vergence



$V<0$ $V=0$ $V>0$

Vergence $V \equiv \frac{n}{r} \approx \frac{n}{z} \quad \left[\frac{1}{m}=diopters\right]$

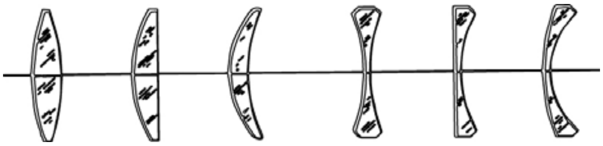
Thin lens equation $V'=V+P$

Surface Power equation $P \equiv (n'-n)\frac{1}{R}$

Lens-makers Formula

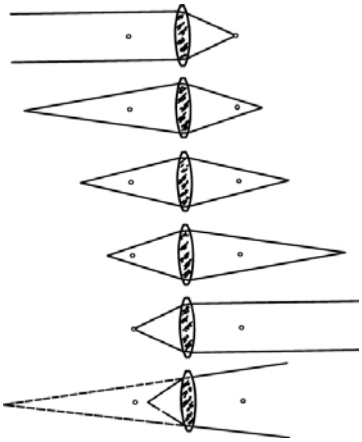
Refractive Power

$$P = (n' - n)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$



Converging **Diverging**

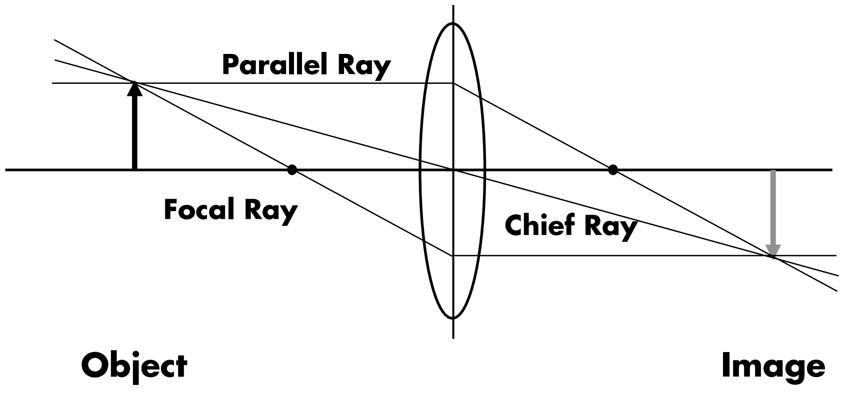
Conjugate Points



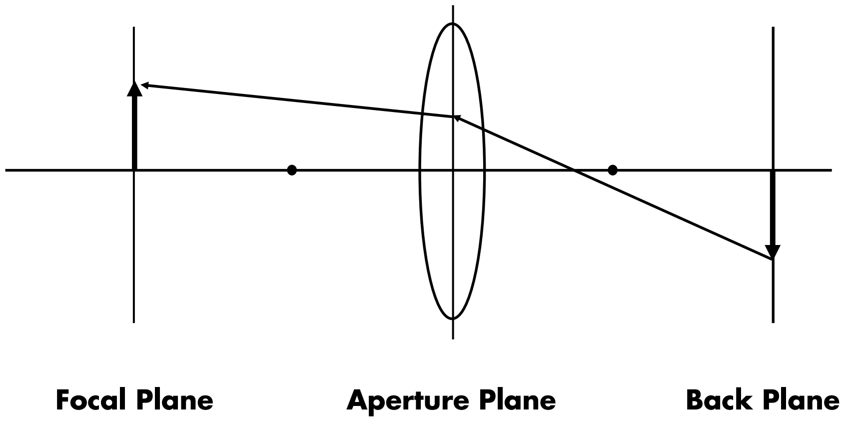
$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f}$$

To focus: move lens relative to backplane
Horizontal rays converge on focal point in the focal plane

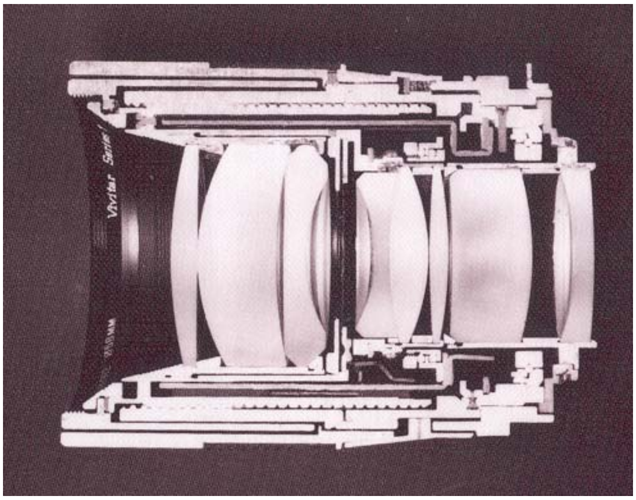
Gauss' Ray Tracing Construction



Ray Tracing: Finite Aperture



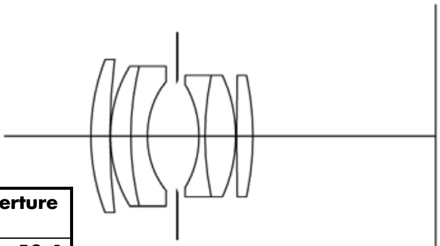
Real Lens



Cutaway section of a Vivitar Series 1 90mm f/2.5 lens
Cover photo, Kingslake, *Optics in Photography*

Double Gauss

Data from W. Smith,
Modern Lens Design, p 312

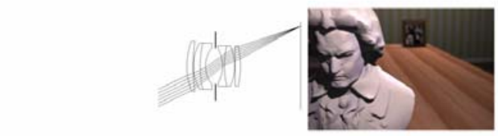


Radius (mm)	Thick (mm)	n_d	V-no	aperture
58.950	7.520	1.670	47.1	50.4
169.660	0.240			50.4
38.550	8.050	1.670	47.1	46.0
81.540	6.550	1.699	30.1	46.0
25.500	11.410			36.0
	9.000			34.2
-28.990	2.360	1.603	38.0	34.0
81.540	12.130	1.658	57.3	40.0
-40.770	0.380			40.0
874.130	6.440	1.717	48.0	40.0
-79.460	72.228			40.0

Ray Tracing Through Lenses

200 mm telephoto

35 mm wide-angle



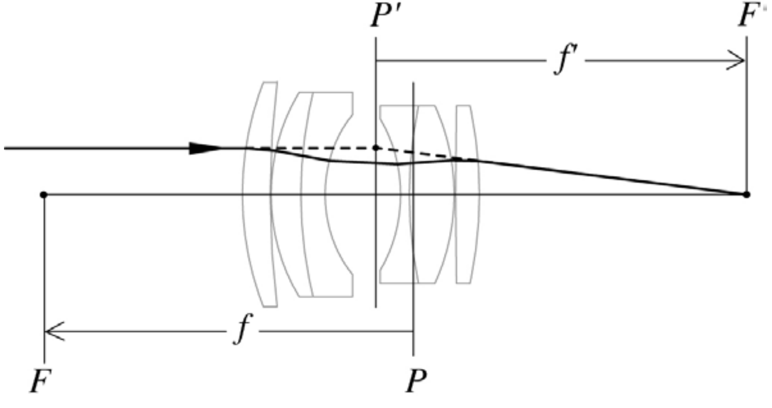
50 mm double-gauss

16 mm fisheye

From Kolb, Mitchell and Hanrahan (1995)

Thick Lenses

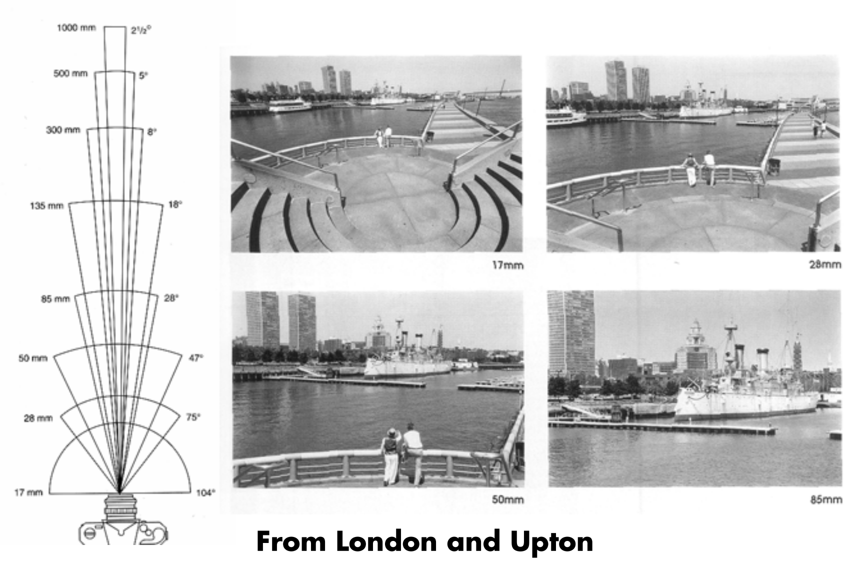
Equivalent Lens



Refraction occurs at the *principal planes*

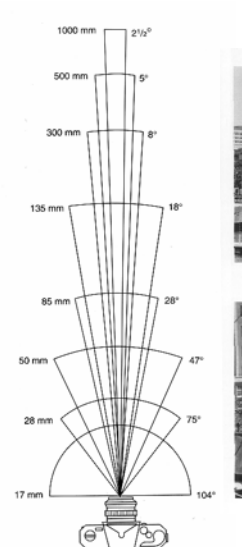
Field of View

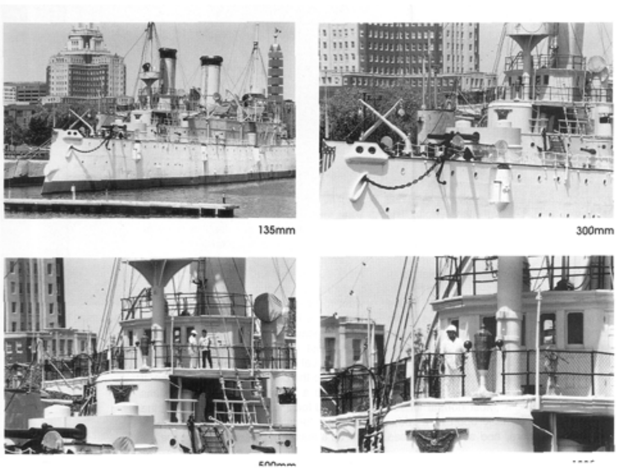
Field of View



From London and Upton

Field of View

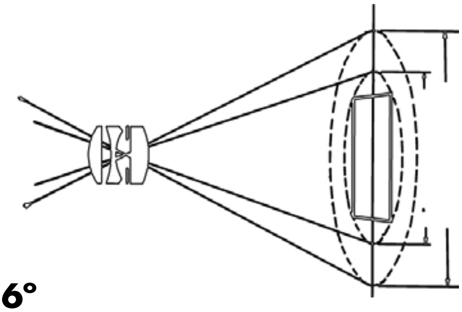




From London and Upton

Field of View

Redrawn from Kingslake, *Optics in Photography*



Field of view

$$\tan\frac{fov}{2} = \frac{filmsize}{f}$$

Types of lenses

- **Normal 26°**

  Film diagonal ~ focal length
- **Wide-angle 75-90°**
- **Narrow-angle 10°**

Perspective Transformation

Thin lens equation

$$\frac{1}{z'} = \frac{1}{z} + \frac{1}{f} \Rightarrow z' = \frac{fz}{z+f}$$

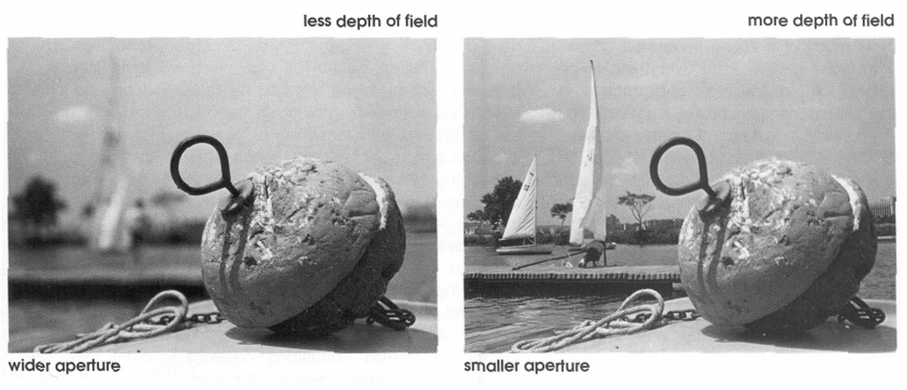
$$\Rightarrow x' = \frac{fx}{z+f}$$

$$\Rightarrow y' = \frac{fy}{z+f}$$

Represent transformation as a 4x4 matrix

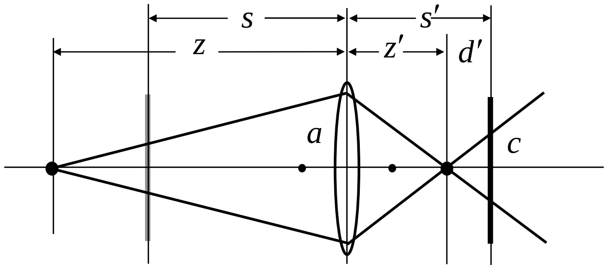
Depth of Field

Depth of Field



From London and Upton

Circle of Confusion



Focal Plane **Back Plane**

Circle of confusion proportional to the size of the aperture

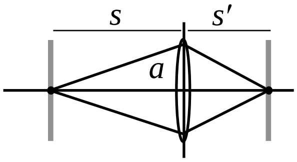
$$\frac{c}{a}=\frac{d'}{z'}=\frac{s'-z'}{z'}$$

Depth of Focus [Image Space]

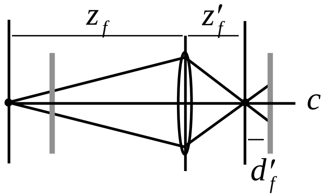
Depth of focus ≡

Equal circles of confusion

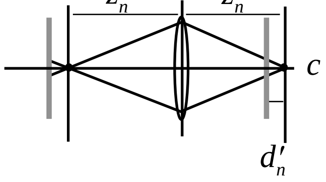
Two planes: near and far



$$\frac{c}{a}=\frac{d'_f}{z'_f}=\frac{s'-z'_f}{z'_f}$$



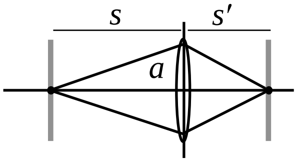
$$\frac{c}{a}=\frac{d'_n}{z'_n}=\frac{z'_n-s'}{z'_n}$$



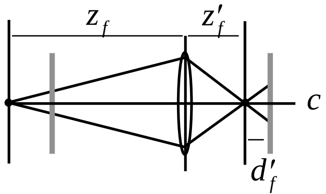
Depth of Focus [Image Space]

Depth of focus ≡

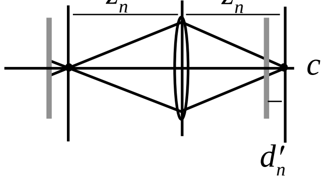
Equal circles of confusion



$$\frac{c}{a}=\frac{d'_f}{z'_f}=\frac{s'-z'_f}{z'_f} \quad \Rightarrow \quad \frac{1}{z'_f}=\frac{1}{s'}\left(1+\frac{c}{a}\right)$$



$$\frac{c}{a}=\frac{d'_n}{z'_n}=\frac{z'_n-s'}{z'_n} \quad \Rightarrow \quad \frac{1}{z'_n}=\frac{1}{s'}\left(1-\frac{c}{a}\right)$$

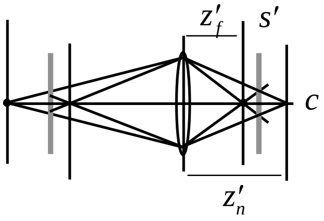


Depth of Focus [Image Space]

Depth of focus ≡

Equal circles of confusion

$$\frac{1}{z'_f}=\frac{1}{s'}\left(1+\frac{c}{a}\right) \qquad \frac{1}{z'_n}=\frac{1}{s'}\left(1-\frac{c}{a}\right)$$



$$\frac{1}{z'_f}+\frac{1}{z'_n}=2\frac{1}{s'}$$

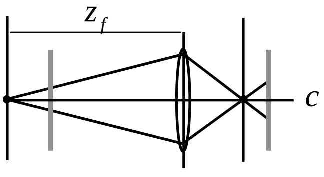
$$\frac{1}{z'_f}-\frac{1}{z'_n}=\frac{2c}{a}\frac{1}{s'}$$

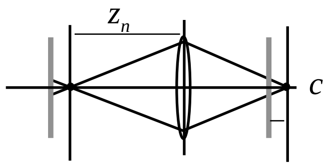
Depth of Field [Object Space]

Depth of field ≡

Equal circles of confusion

$$\frac{1}{s'}=\frac{1}{s}+\frac{1}{f} \qquad \frac{1}{z'_n}=\frac{1}{z_n}+\frac{1}{f} \qquad \frac{1}{z'_f}=\frac{1}{z_f}+\frac{1}{f}$$





$$\frac{1}{z_n}+\frac{1}{z_f}=2\frac{1}{s}$$

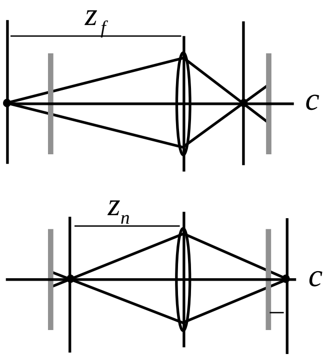
$$\frac{1}{z_n}-\frac{1}{z_f}=\frac{2c}{a}\left(\frac{1}{f}-\frac{1}{s}\right)\approx\frac{2c}{a}\frac{1}{f}$$

Hyperfocal Distance

$$\frac{1}{z_n}+\frac{1}{z_f}=2\frac{1}{s}$$

$$N \equiv \frac{a}{f}$$

$$\frac{1}{z_n}-\frac{1}{z_f}=\frac{2c}{a}\frac{1}{f}=2\frac{cN}{f^2}\equiv 2\frac{1}{H}$$

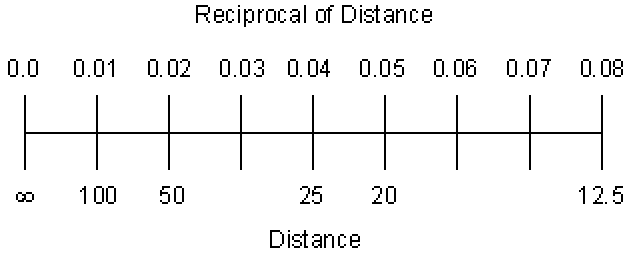


When

$$s \to H \Rightarrow z_n = \frac{H}{2},\ z_f = \infty$$

H is the hyperfocal distance

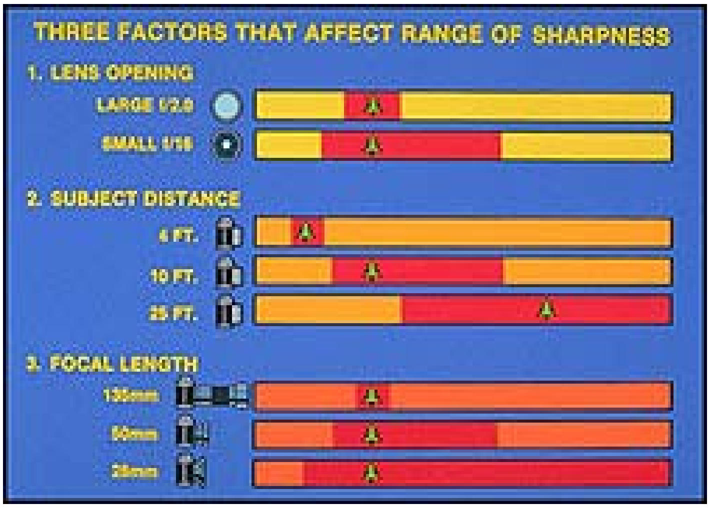
Depth of Field Scale



Factors Affecting DOF

From http://www.kodak.com/global/en/consumer/pictureTaking/cameraCare/cameCar6.shtml

$$\frac{1}{H} = \frac{cN}{f^2}$$



Resolving Power

- **Diffraction limit**

$$c = 1.22\frac{f}{a}\lambda \quad \left[=1.22 \times 64 \times .500\,\mu\text{m}=0.040\,\text{mm}\right]$$

- **35mm film (Leica standard)**

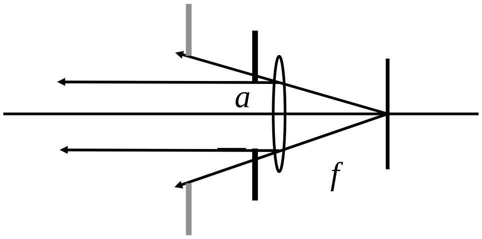
$$c = 0.025\,\text{mm}$$

- **CCD/CMOS pixel aperture**

$$c = 0.0116\,\text{mm (Nikon D1)}$$

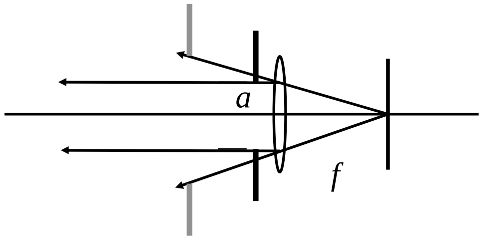
Exposure

Image Irradiance



$$E = \int_{\Omega} L \cos\theta \, d\omega = L \pi \sin^2 \theta = L \frac{\pi}{4} \left(\frac{a}{f} \right)^2$$

Relative Aperture or F-Stop



$$a = \frac{f}{N}$$

F-Number and exposure: $E = L\frac{\pi}{4}\frac{1}{N^2}$

Fstops: 1.4 2 2.8 4.0 5.6 8 11 16 22 32 45 64

- **1 stop doubles exposure**

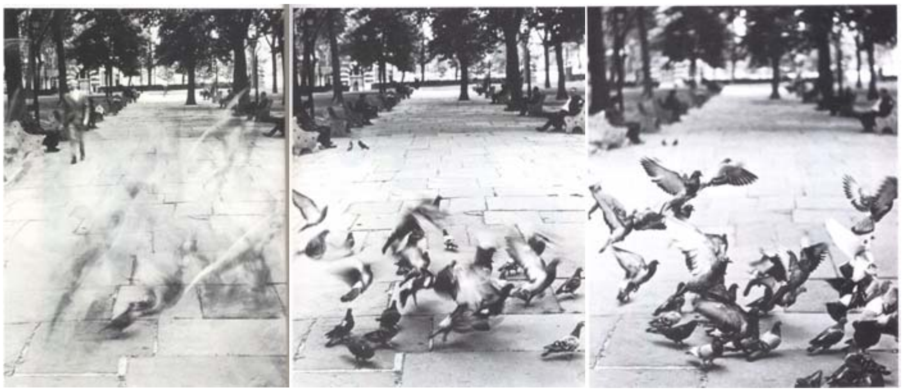
Camera Exposure

Exposure $H = E \times T$

Exposure overdetermined

- **Aperture: f-stop - 1 stop doubles *H***
  - **Decreases depth of field**
- **Shutter: Doubling the open time doubles *H***
  - **Increases motion blur**

Aperture vs Shutter



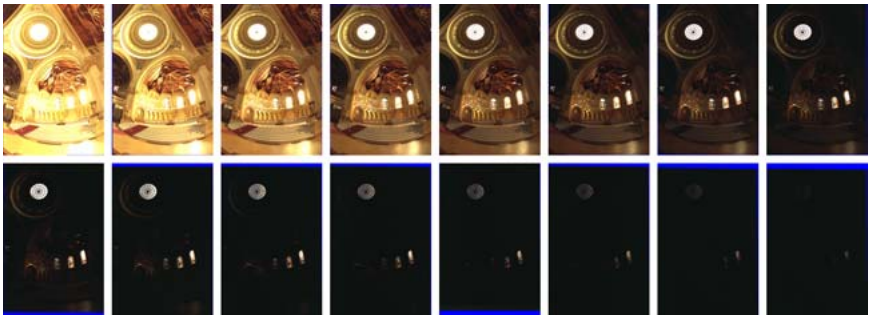
f/16
1/8s

f/4
1/125s

f/2
1/500s

From London and Upton

High Dynamic Range

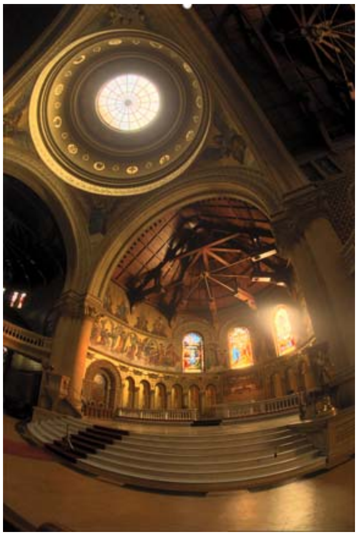


Sixteen photographs of the Stanford Memorial Church taken at 1-stop increments from 30s to 1/1000s.

From Debevec and Malik, High dynamic range photographs.

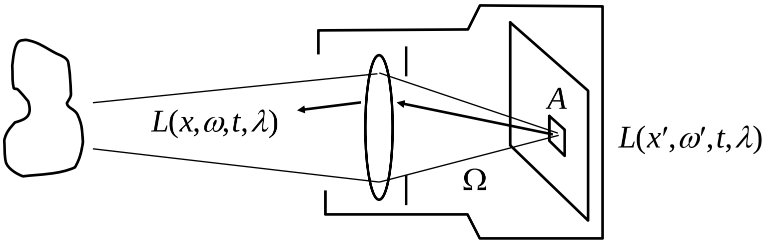
Simulated Photograph

Adaptive histogram



With glare, contrast, blur

Camera Simulation



$$R=\int_A\int_\Omega\int_T\int_\Lambda P(x',\lambda)S(x',\omega',t)L(T(x',\omega',\lambda),t,\lambda)\,d\vec{A}(x')\bullet d\vec{\omega}'\,dt\,d\lambda$$

Sensor response	$P(x',\lambda)$
Lens	$(x,\omega)=T(x',\omega',\lambda)$
Shutter	$S(x',\omega',t)$
Scene radiance	$L(x,\omega,t,\lambda)$