Trees and Graphs

Pat Hanrahan

Tree Drawing

Why Trees?

Hierarchies

- File systems and web sites
- Organization charts
- Categorical classifications
- Similiarity and clustering

Branching processes

- Genealogy and lineages
- Phylogenetic trees

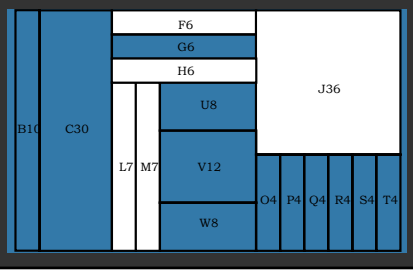
Decision processes

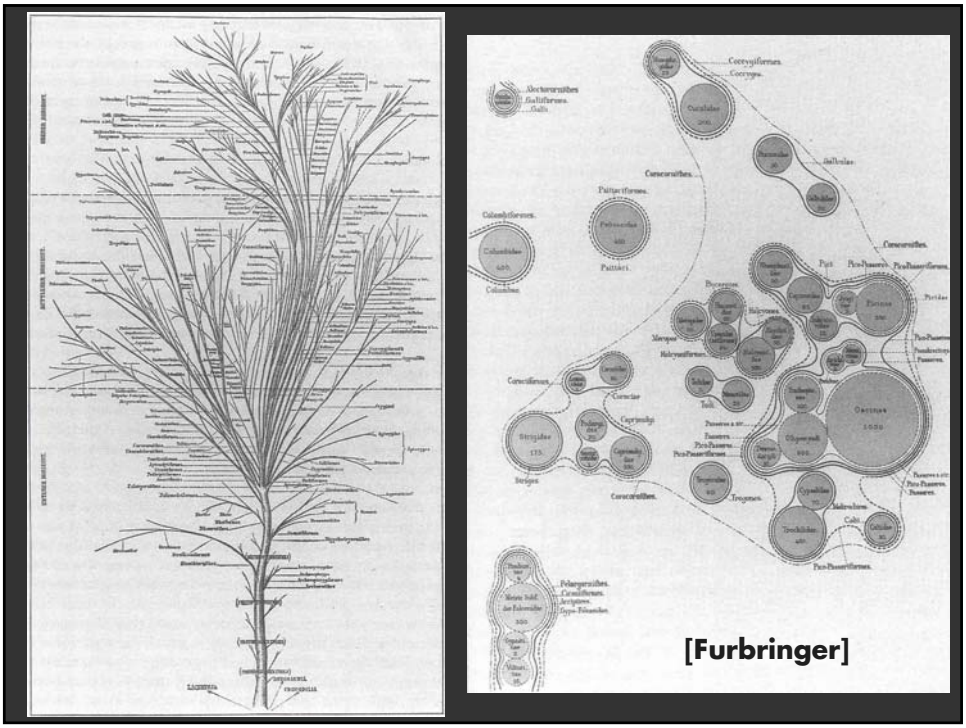
- Indices or search trees
- Decision trees
- Tournaments

Two Major Visual Representations

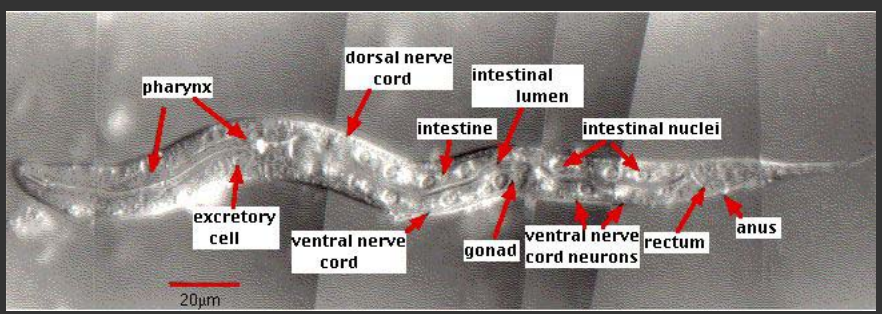
Connection: Node / Link Diagrams

Containment / Enclosure

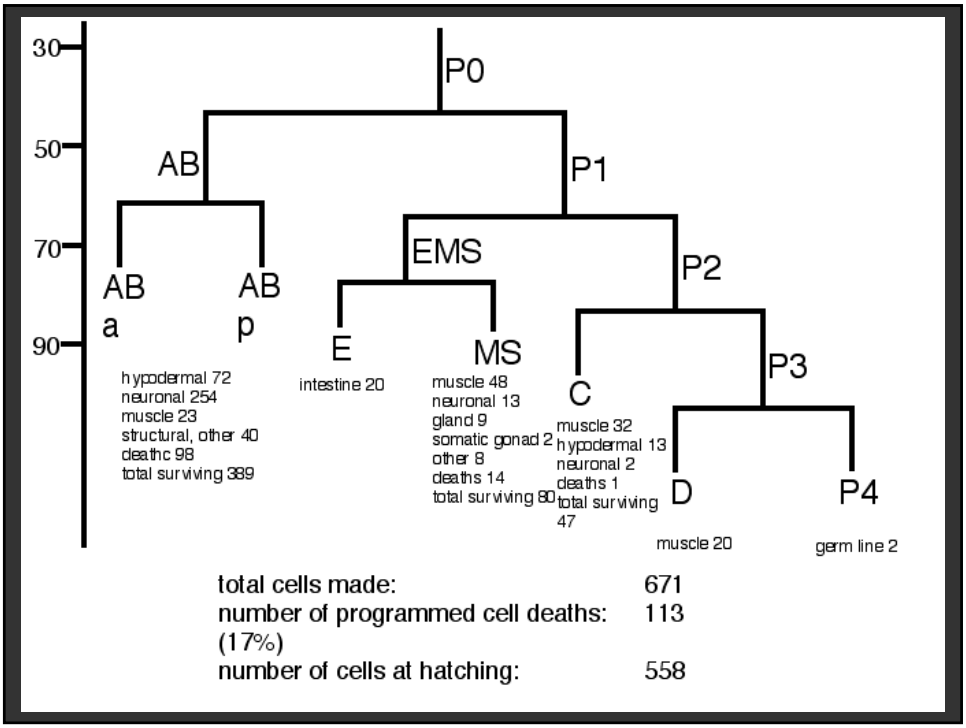


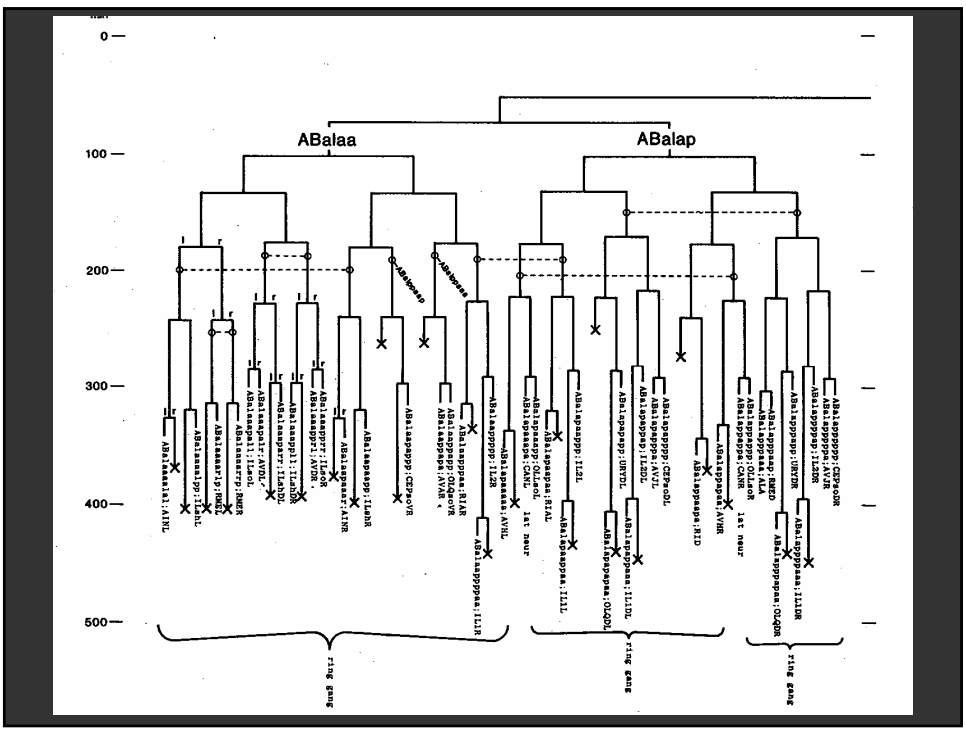


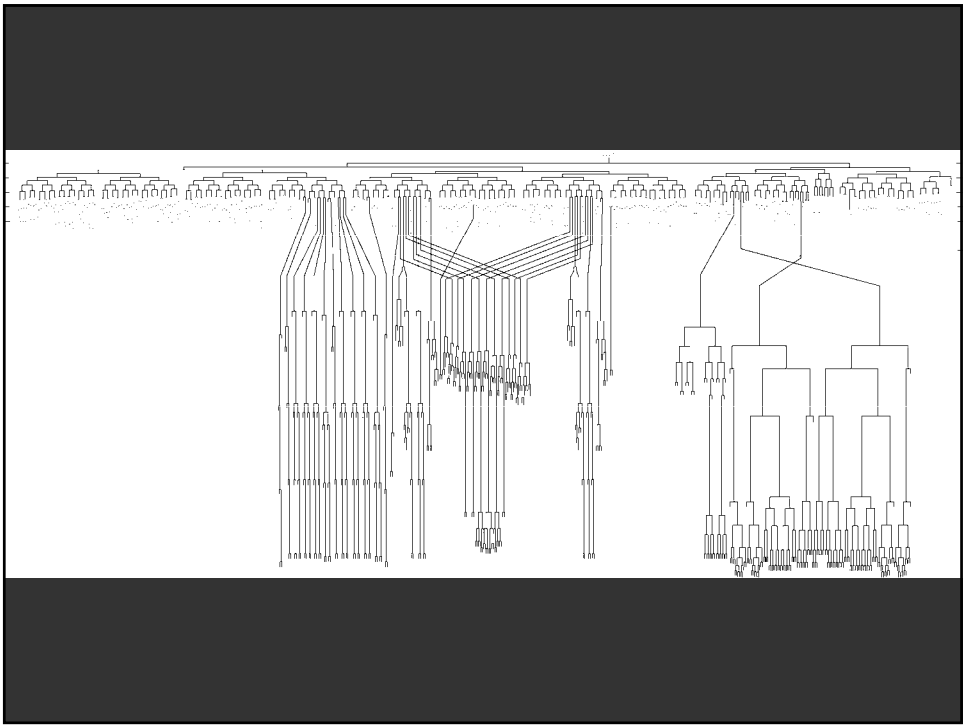
C. Elegans Cell Lineage

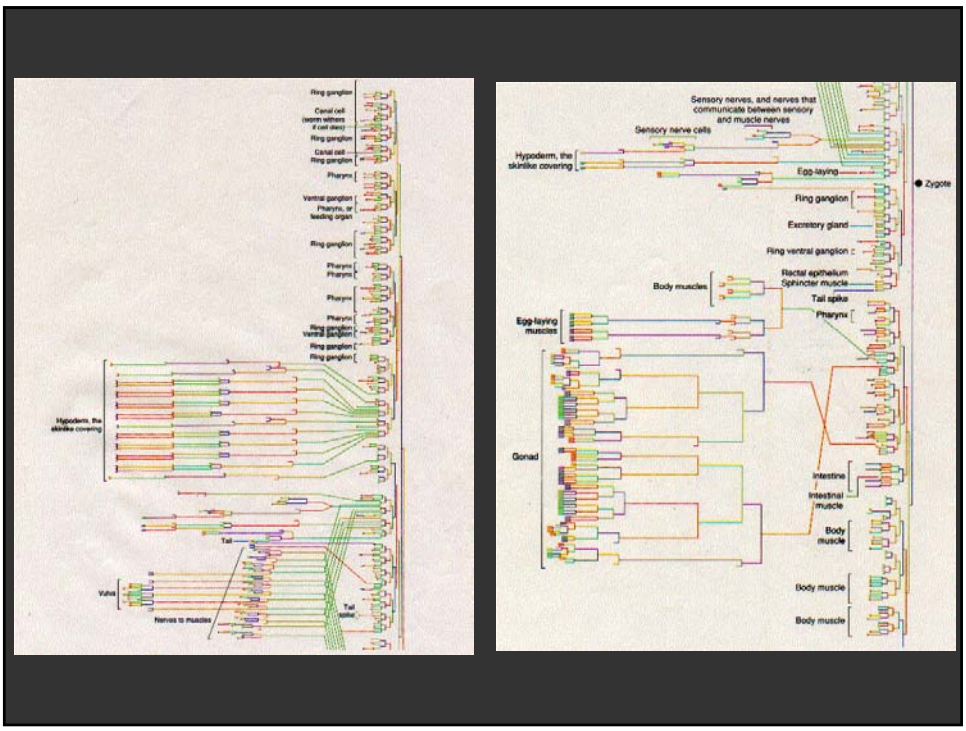


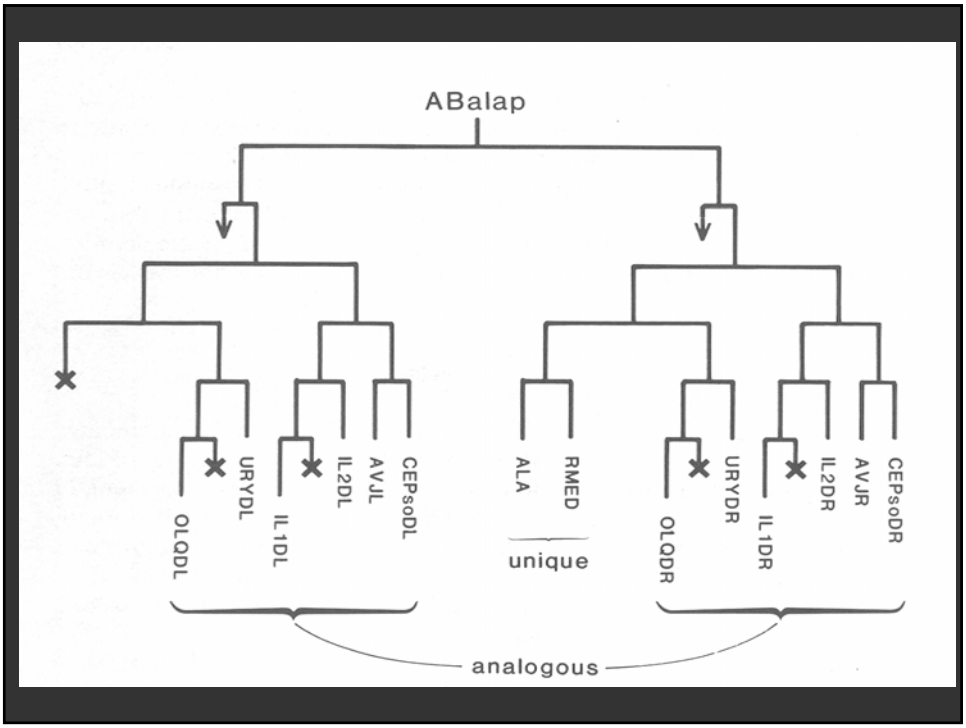
[Sulston]

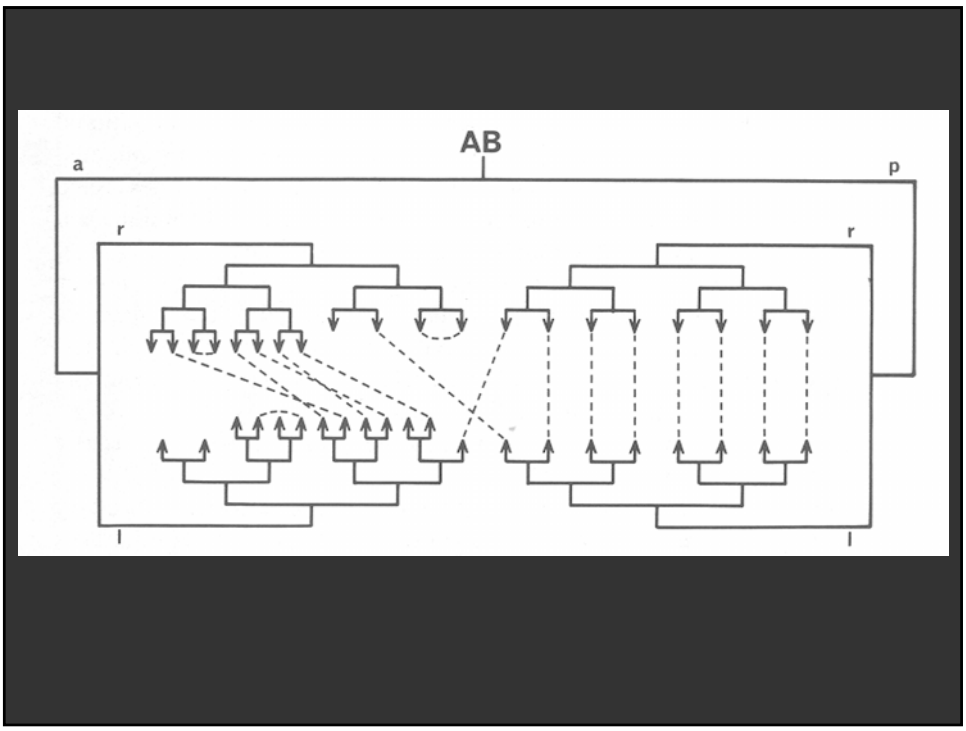

30
50
70
90
P0
AB
P1
EMS
P2
P3
C
D
P4
AB a
AB p
E
MS
hypodermal 72
neuronal 254
muscle 23
structural, other 40
deathc 98
total surviving 389
intestine 20
muscle 48
neuronal 13
gland 9
somatic gonad 2
other 8
deaths 14
total surviving 80
muscle 32
hypodermal 13
neuronal 2
deaths 1
total surviving 47
muscle 20
germ line 2
total cells made: 671
number of programmed cell deaths: 113
(17%)
number of cells at hatching: 558


0
100
200
300
400
500
ABalaa
ABalap
ring gang
ring gang
ring gang



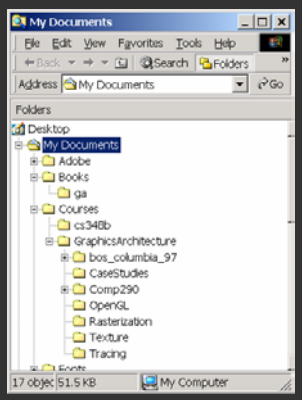

Ring ganglion
Canal cell (worm without it can starve)
Canal cell
Pharynx
Ventral ganglion
Pharynx, or feeding organ
Hypoderm, the skinlike covering
Tail
Vulva
Nerves to muscles
Tail spike
Sensory nerves, and nerves that communicate between sensory and muscle nerves
Sensory nerve cells
Hypoderm, the skinlike covering
Egg-laying
Zygote
Excretory gland
Body muscles
Rectal epithelium
Sphincter muscle
Tail spike
Egg-laying muscles
Gonad
Intestine
Intestinal muscle
Body muscle


ABalap
OLQDL
URYDL
IL1DL
IL2DL
AVJL
CEPsoDL
ALA
RMED
unique
OLQDR
URYDR
IL1DR
IL2DR
AVJR
CEPsoDR
analogous


AB
a
p
r
r
l
l

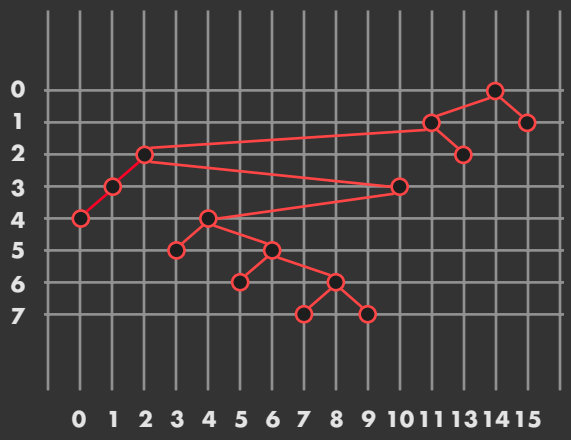
Classic Tree Drawing

Preorder or inorder traversal



	East				West			
	Coffee	Espresso	Herbal Te	Tea	Coffee	Espresso	Herbal Te	Tea

Depth-InOrder Traversal



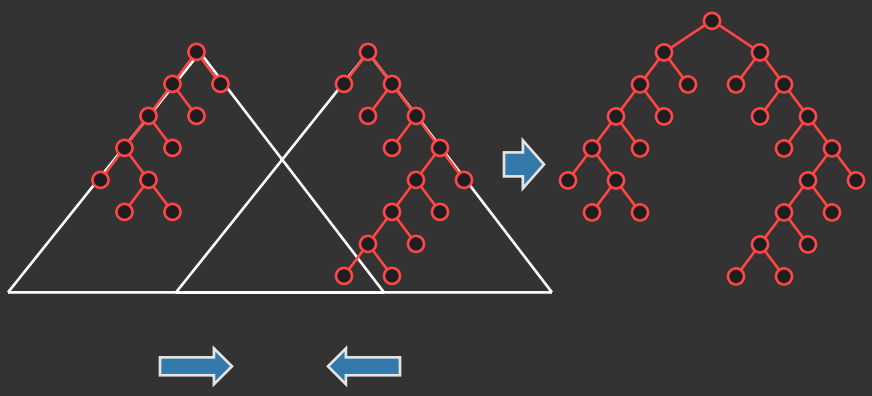
Similarly for pre-order, postorder

Note: width = n-1

Aesthetic Criteria

1. Nodes at the same levels should be aligned
2. Maintain the relative ordering of left and right subtrees
3. Parent should be centered over the children
4. A tree and its mirror image should be drawn as reflections of each other
5. A subtree should be drawn the same way regardless of where it occurs in the tree

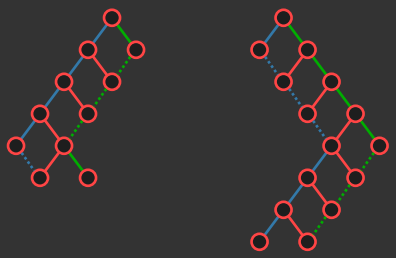
Rheingold-Tilford Algorithm



E. Rheingold, J. Tilford, Tidier drawing of trees, IEEE Trans. Software Engineering, SE-7(2), pp. 223-228. 1981

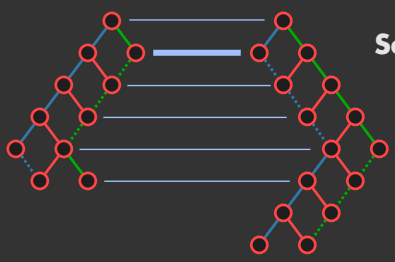
Rheingold-Tilford Algorithm

Left contour
Left threads
Right contour
Right threads



E. Rheingold, J. Tilford, Tidier drawing of trees, IEEE Trans. Software Engineering, SE-7(2), pp. 223-228. 1981

Rheingold-Tilford Algorithm

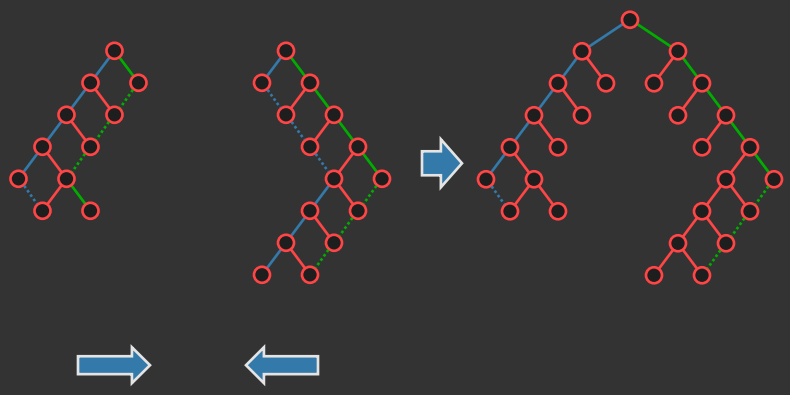


Separation = 2

E. Rheingold, J. Tilford, Tidier drawing of trees, IEEE Trans. Software Engineering, SE-7(2), pp. 223-228. 1981

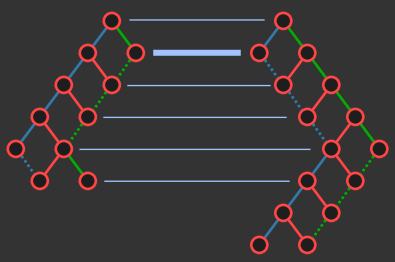
Rheingold-Tilford Algorithm

Translate by (sep+1)/2



E. Rheingold, J. Tilford, Tidier drawing of trees, IEEE Trans. Software Engineering, SE-7(2), pp. 223-228. 1981

Rheingold-Tilford Algorithm

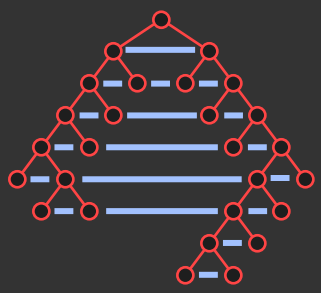


$T = \min(h(T_L), h(T_R)$

E. Rheingold, J. Tilford, Tidier drawing of trees, IEEE Trans. Software Engineering, SE-7(2), pp. 223-228. 1981

Rheingold-Tilford Algorithm

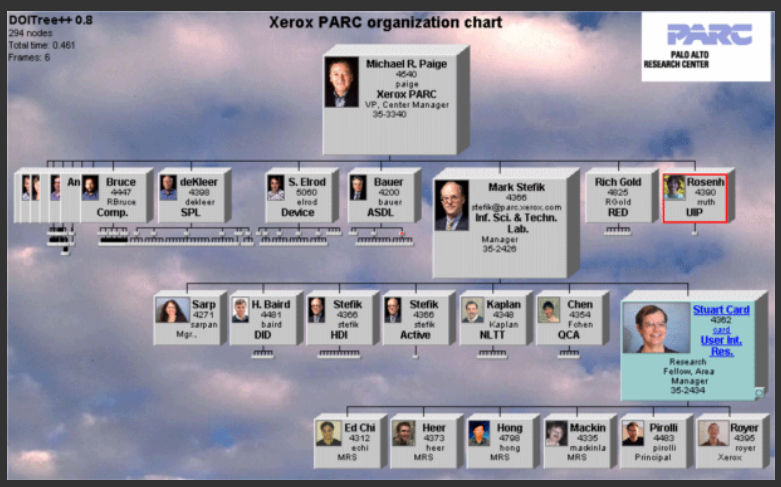
Total running time: Count –



O(n – h(T))

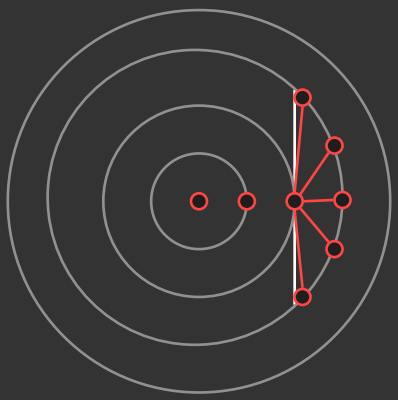
E. Rheingold, J. Tilford, Tidier drawing of trees, IEEE Trans. Software Engineering, SE-7(2), pp. 223-228. 1981

Focus+Context: DOI Tree



http://davenation.com/doitree/doitree-avi-2002.htm

Radial Layouts

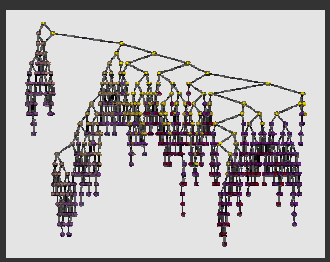


Bounded wedge

Aspect Ratio

Hierarchies are typically wide and shallow

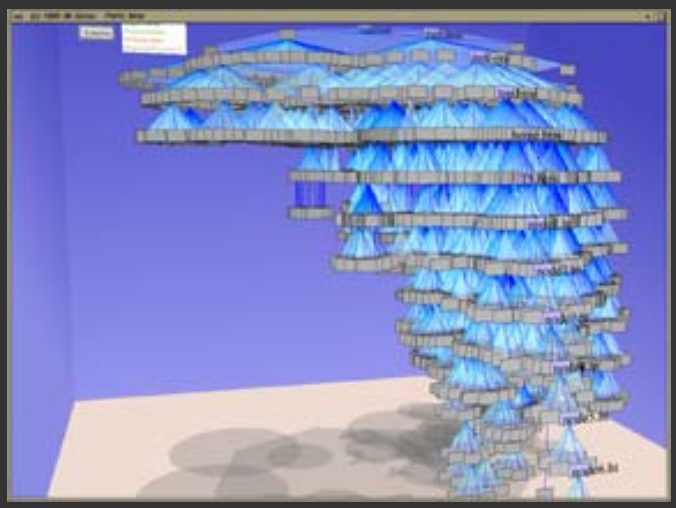
http://cgm.cs.mcgill.ca/~luc/trees.html



Strategies:

- Scrolling
- Filtering, collapsing, expanding
- Grouping and aggregating
- More space: Cone Tree, Hyperbolic Browser
- Focus + context

Cone Trees

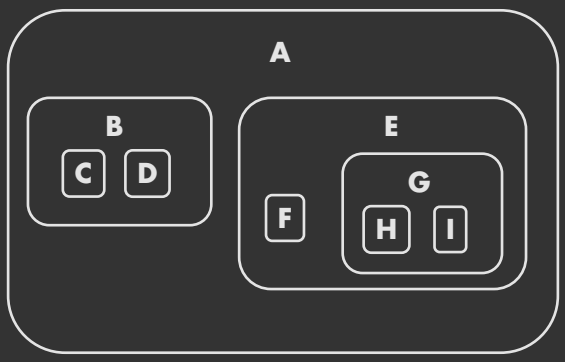


Cone Tree, Robertson, Card, Mackinlay, 1991

Hyperbolic Trees

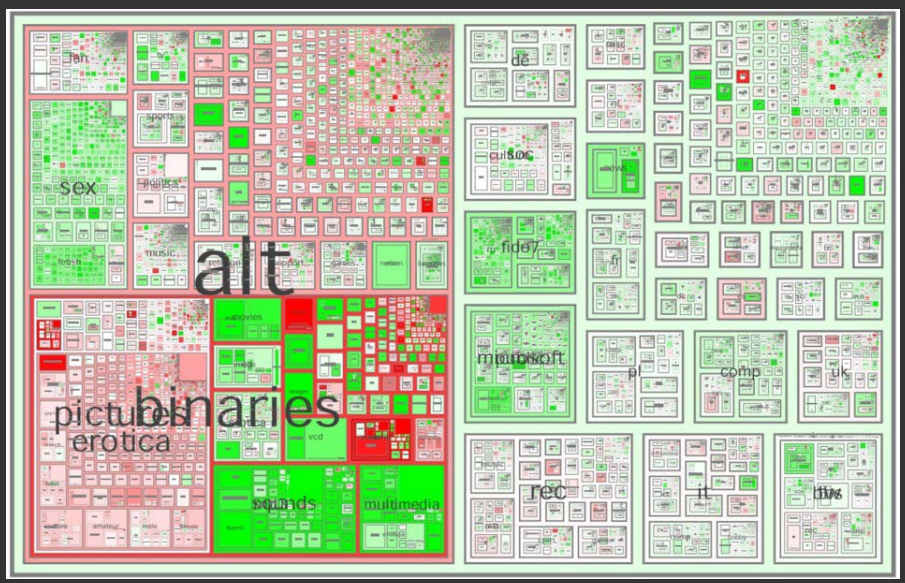
Demo on InXight

Nested/Inclusion Trees



(A (B (C D) (E (G (H I)))))

Tree Maps [Schneiderman]



http://netscan.research.microsoft.com/treemap/

Graph Drawing

Why Graphs?

- Relational databases with keys
- Complex data structures
- Social networks
- Citation networks
- Link analysis
- Computer networks
- World wide web
- Document collections
- Metabolic pathways
- ...

Aesthetic Criteria

Minimize edge crossings
Minimize area
Total edge length
Maximum edge length
Uniform edge length
Total bends
Maximum bends
Angular bends
Aspect ratio
Symmetry

G. Di Battista, P. Eades, R. Tamassia, I. G. Tollis, Graph Drawing, 1999

Algorithms

Planar drawings
Layered
Force-directed

Layered Drawing of Directed Graphs

Sugiyama et al. 1981

Dot, Gamsden et al. 1993

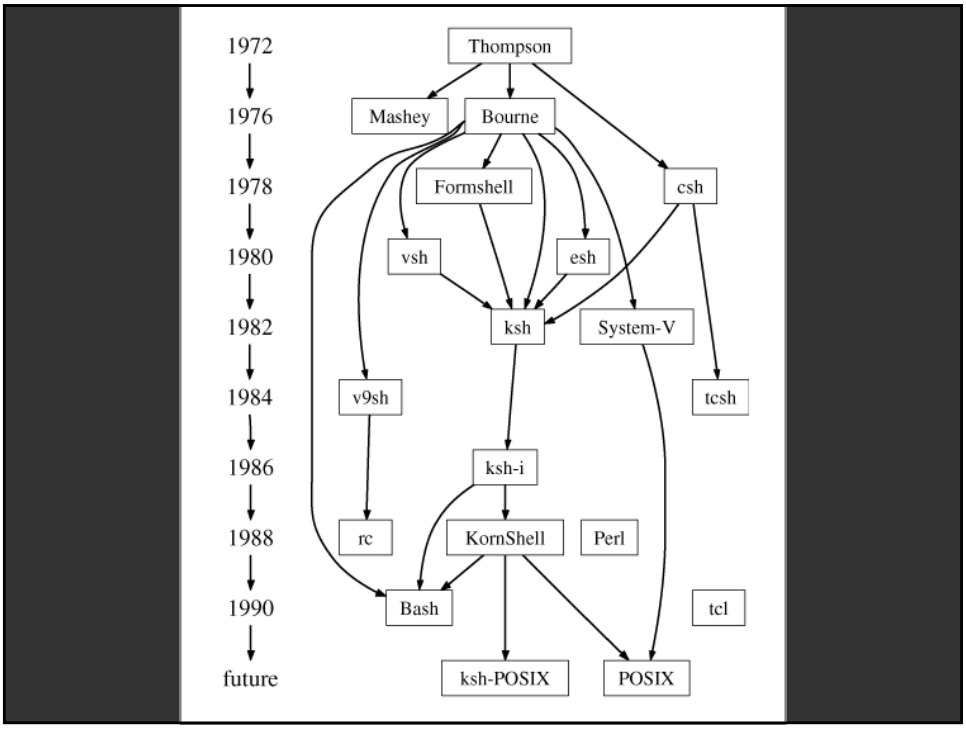
1. Layer assignment
2. Reduce edge crossings between layers
3. Position vertices horizontally

Additionally,

4. Cycle removal
5. Route edges (fit splines)

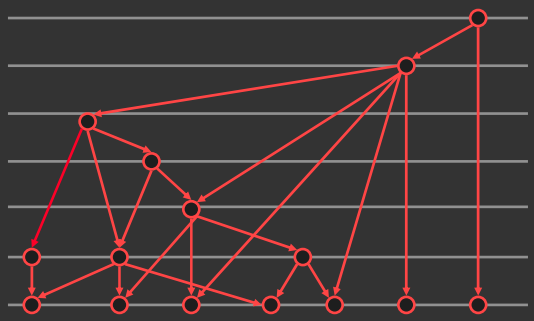
shells.dot

```
digraph shells {
  size="7,8";
  node [fontsize=24,shape=plaintext];

  1972 -> 1976;
  1976 -> 1978;
  1978 -> 1980;
  1980 -> 1982;
  1982 -> 1984;
  1984 -> 1986;
  1986 -> 1988;
  1988 -> 1990;
  1990 -> future;

  node [fontsize=20, shape = box];
  { rank=same; 1976 Mashey Bourne;}
  { rank=same; 1978 Formshell csh;}
  { rank=same; 1980 esh vsh; }
  { rank=same; 1982 ksh "System-V";}
  { rank=same; 1984 v9sh tcsh; }
  { rank=same; 1986 "ksh-i"; }
  { rank=same;
       1988 KornShell Perl rc;}
  { rank=same; 1990 tcl Bash; }
  { rank=same;
       "future" POSIX "ksh-POSIX"; }

                                    }
Thompson -> Mashey;
Thompson -> Bourne;
Thompson -> csh;
csh -> tcsh;
Bourne -> ksh;
Bourne -> esh;
Bourne -> vsh;
Bourne -> "System-V";
Bourne -> v9sh;
v9sh -> rc;
Bourne -> Bash;
"ksh-i" -> Bash;
KornShell -> Bash;
esh -> ksh;
vsh -> ksh;
Formshell -> ksh;
csh -> ksh;
KornShell -> POSIX;
"System-V" -> POSIX;
ksh -> "ksh-i";
"ksh-i" -> KornShell;
KornShell -> "ksh-POSIX";
Bourne -> Formshell;

edge [style=invis];
1984 -> v9sh -> tcsh ;
1988 -> rc -> KornShell;
Formshell -> csh;
KornShell -> Perl;
```



Layered Assignment

Longest path layering (minimizes height)



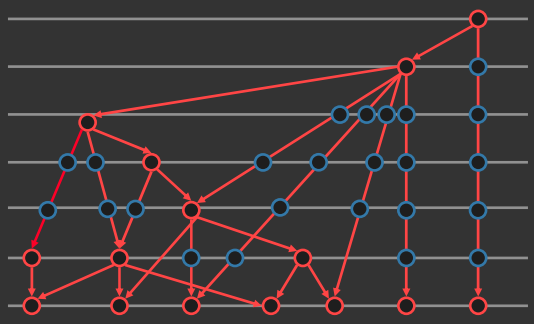
Improvements

- **Minimize width and height (NP)**
- **Approximate minimal width (scheduling problem)**
- **Minimize dummy vertices (polynomial GKNV1993]**

Layered Assignment

Longest path layering (minimizes height)

Dummy vertices



Improvements

- Minimize width and height (NP)
- Approximate minimal width (scheduling problem)
- Minimize dummy vertices (polynomial GKNV1993]

Crossing Reduction

Barycentric

Median

Improvements

- Sweep left to right flipping order to minimize crossings
- Multiple sweeps, etc.

Force-Directed Layout

1. **Energy model**

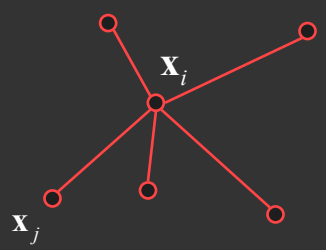
$$F_i = \sum_{j \subset V} F_{i,j} + \sum_{j \subset E(i,j)} F_{i,j}$$

2. **Minimization algorithm**

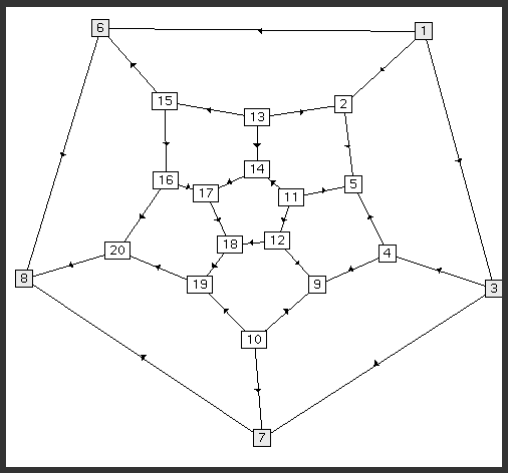
Tutte [1963]
Eades [1984]
Kamada-Kawai [1989]
Fructerman-Rheingold [1991]
Davidson-Harel [1996]

Barycentric Relaxation

$$\mathbf{x}_i = \frac{1}{\deg(V_i)} \sum_{j \subset E(i,j)} \mathbf{x}_j$$



Tutte Algorithm



Tutte proved that planar graphs will be drawn without edge crossings

Barycentric Relaxation

Fructerman and Rheingold [1991]

$$k = 0.75\sqrt{\frac{A}{n}}$$

$$F_i = -\sum_{j \subset V} \frac{k^2}{d(i,j)} + \sum_{j \subset E(i,j)} \frac{d^2(i,j)}{k}$$

Edge length

Repulsion **Attraction (Spring)**

Kamada-Kawai [1989]

$$F_i = -\sum_{j \subset E(i,j)} \frac{\left(d(i,j) - p(i,j)\right)^2}{p^2(i,j)}$$

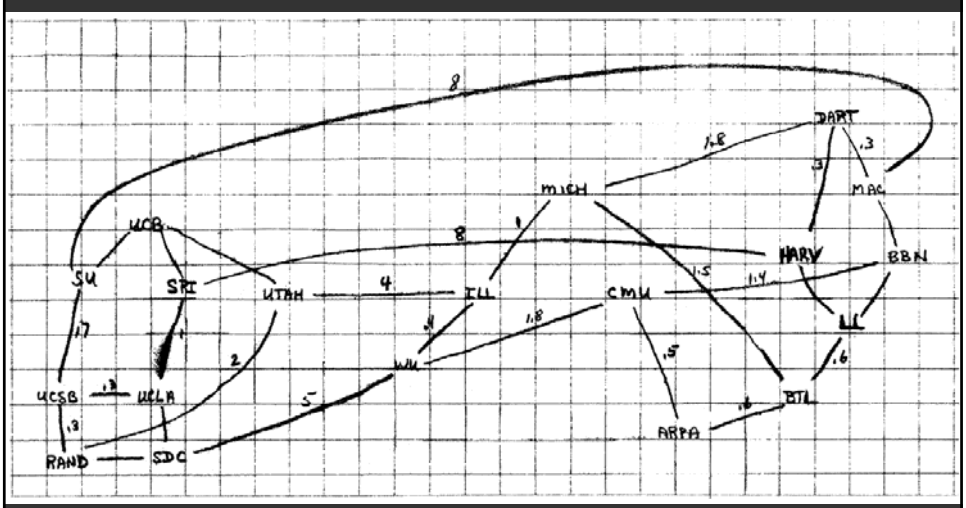
Shortest path

Force-Directed Graph Drawing Demonstrations

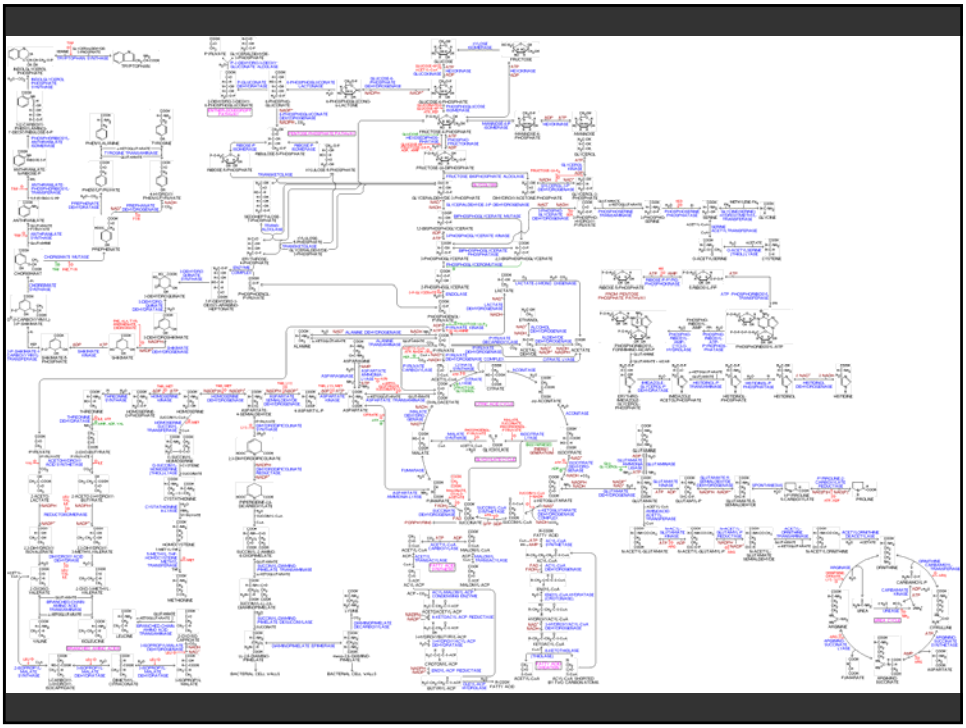
http://java.sun.com/applets/jdk/1.4/demo/applets/

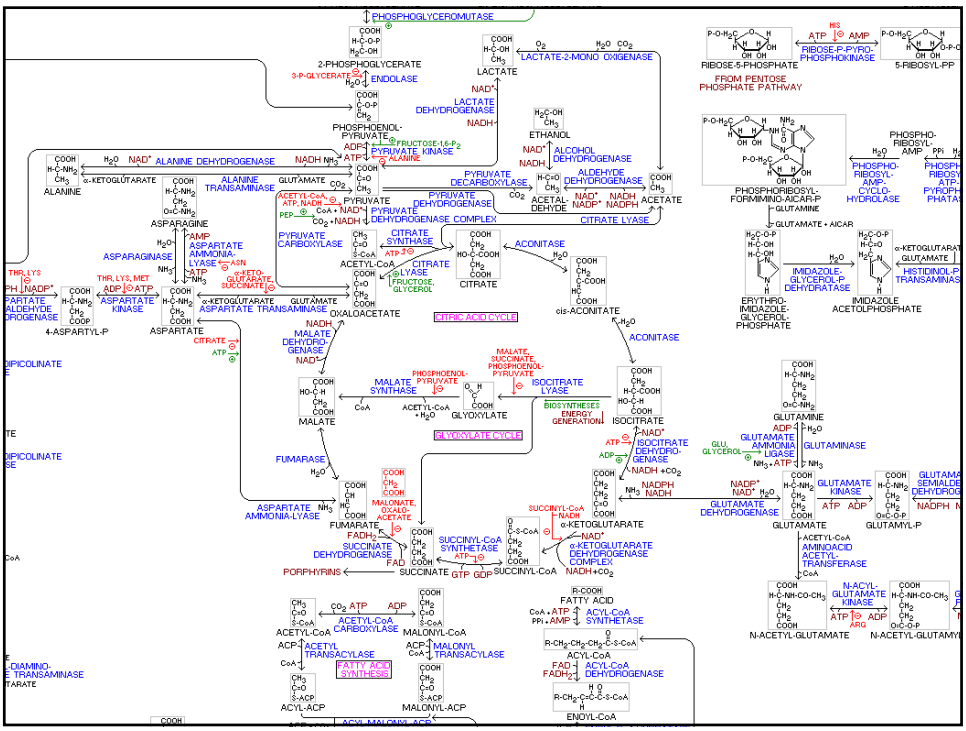
www.touchgraph.com

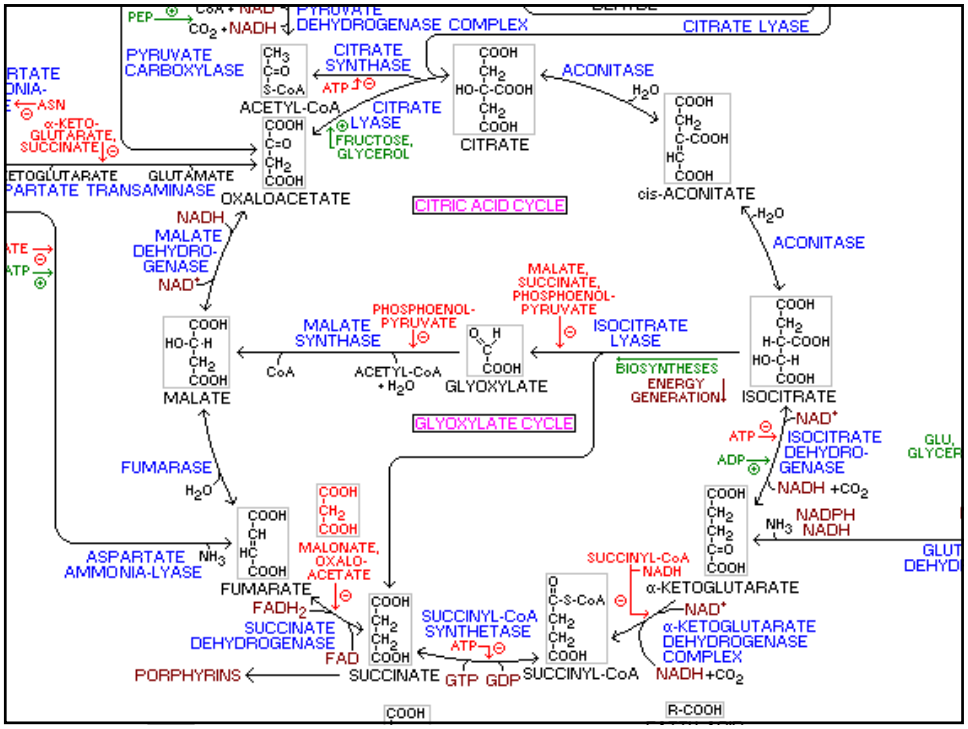
http://www.netminer.com/NetMiner/home_01.jsp



Larry Robert's Sketch of the ARPANET




PHOSPHOGLYCEROMUTASE
2-PHOSPHOGLYCERATE
3-P-GLYCERATE
ENDOLASE
PHOSPHOENOL-PYRUVATE
FRUCTOSE-1,6-P2
PYRUVATE KINASE
ALANINE
LACTATE
LACTATE-2-MONO OXIGENASE
LACTATE DEHYDROGENASE
ETHANOL
ALCOHOL DEHYDROGENASE
ALDEHYDE DEHYDROGENASE
ACETAL-DEHYDE
ACETATE
ALANINE DEHYDROGENASE
ALANINE
α-KETOGLUTARATE
ALANINE TRANSAMINASE
GLUTAMATE
PYRUVATE DECARBOXYLASE
PYRUVATE DEHYDROGENASE
PYRUVATE DEHYDROGENASE COMPLEX
CITRATE LYASE
PYRUVATE CARBOXYLASE
CITRATE SYNTHESE
ACETYL-CoA
CITRATE LYASE
FRUCTOSE GLYCEROL
CITRATE
ACONITASE
cis-ACONITE
ASPARAGINE
ASPARAGINASE
AMP
ASPARTATE AMMONIA-LYASE
ASN
α-KETO-GLUTARATE
SUCCINATE
ASPARTATE KINASE
ASPARTATE TRANSAMINASE
ASPARTATE
4-ASPARTYL-P
OXALOACETATE
CITRIC ACID CYCLE
MALATE DEHYDRO-GENASE
MALATE
MALATE SYNTHASE
PHOSPHOENOL-PYRUVATE
ACETYL-CoA
GLYOXYLATE
MALATE, SUCCINATE, PHOSPHOENOL-PYRUVATE
ISOCITRATE LYASE
ISOCITRATE
BIOSYNTHESES ENERGY GENERATION
GLYOXYLATE CYCLE
ISOCITRATE DEHYDRO-GENASE
FUMARASE
FUMARATE
ASPARTATE AMMONIA-LYASE
MALONATE, OXALO-ACETATE
SUCCINATE DEHYDROGENASE
FAD
SUCCINATE
SUCCINYL-CoA SYNTHETASE
SUCCINYL-CoA
α-KETOGLUTARATE DEHYDROGENASE COMPLEX
α-KETOGLUTARATE
GLUTAMATE DEHYDROGENASE
NADPH
NADPH +CO2
PORPHYRINS
ACETYL-CoA CARBOXYLASE
MALONYL-CoA
ACETYL TRANSACYLASE
MALONYL TRANSACYLASE
FATTY ACID SYNTHESIS
ACYL-ACP
MALONYL-ACP
FATTY ACID
ACYL-CoA SYNTHETASE
ACYL-CoA
ACYL-CoA DEHYDROGENASE
ENOYL-CoA
RIBOSE-5-PHOSPHATE
FROM PENTOSE PHOSPHATE PATHWAY
RIBOSE-P-P/RIBO-PHOSPHOKINASE
5-RIBOSYL-PP
PHOSPHORIBOSYL-FORMIMINO-AICAR-P
GLUTAMINE
GLUTAMATE + AICAR
IMIDAZOLE-GLYCEROL-P DEHYDRATASE
ERYTHRO-IMIDAZOLE-GLYCEROL-PHOSPHATE
IMIDAZOLE ACETOLPHOSPHATE
GLUTAMINE
GLUTAMATE KINASE
GLUTAMYL-P
GLUTAMATE SEMIALDEHYDE
N-ACETYL-GLUTAMATE
N-ACETYL-GLUTAMYL-P
ACETYL-CoA AMINOACID ACETYL-TRANSFERASE
N-ACYL-GLUTAMATE KINASE


PEP
CO_2 + NADH
PYRUVATE DEHYDROGENASE COMPLEX
CITRATE LYASE
PYRUVATE CARBOXYLASE
CITRATE SYNTHASE
ATP
ACETYL-CoA
CITRATE LYASE
FRUCTOSE, GLYCEROL
CITRATE
ACONITASE
H_2O
cis-ACONITATE
α-KETO-GLUTARATE, SUCCINATE
ETOGLUTARATE
GLUTAMATE
PARTATE TRANSAMINASE
OXALOACETATE
CITRIC ACID CYCLE
NADH
MALATE DEHYDRO-GENASE
NAD^+
MALATE, SUCCINATE, PHOSPHOENOL-PYRUVATE
PHOSPHOENOL-PYRUVATE
MALATE SYNTHASE
ISOCITRATE LYASE
CoA
ACETYL-CoA + H_2O
GLYOXYLATE
BIOSYNTHESES
ENERGY GENERATION
ISOCITRATE
MALATE
GLYOXYLATE CYCLE
NAD^+
ATP
ADP
ISOCITRATE DEHYDRO-GENASE
$NADH + CO_2$
NADPH
NH_3
NADH
FUMARASE
H_2O
ASPARTATE AMMONIA-LYASE
NH_3
MALONATE, OXALO-ACETATE
FUMARATE
$FADH_2$
SUCCINATE DEHYDROGENASE
FAD
SUCCINYL-CoA SYNTHETASE
ATP
GTP
GDP
SUCCINATE
SUCCINYL-CoA
SUCCINYL-CoA
NADH
α-KETOGLUTARATE
NAD^+
α-KETOGLUTARATE DEHYDROGENASE COMPLEX
$NADH + CO_2$
PORPHYRINS
R-COOH