Meshless Parameterization and Surface Reconstruction

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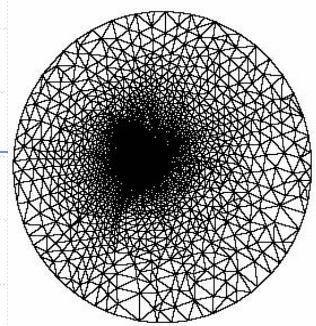
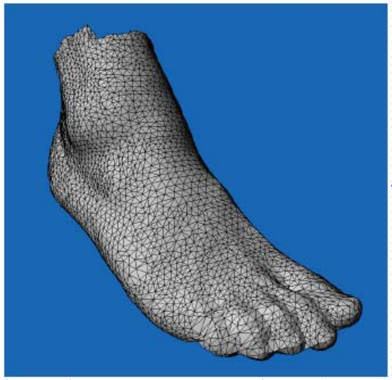
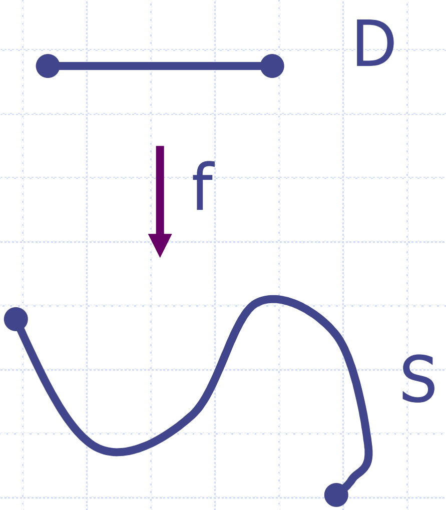
Presented by An Nguyen

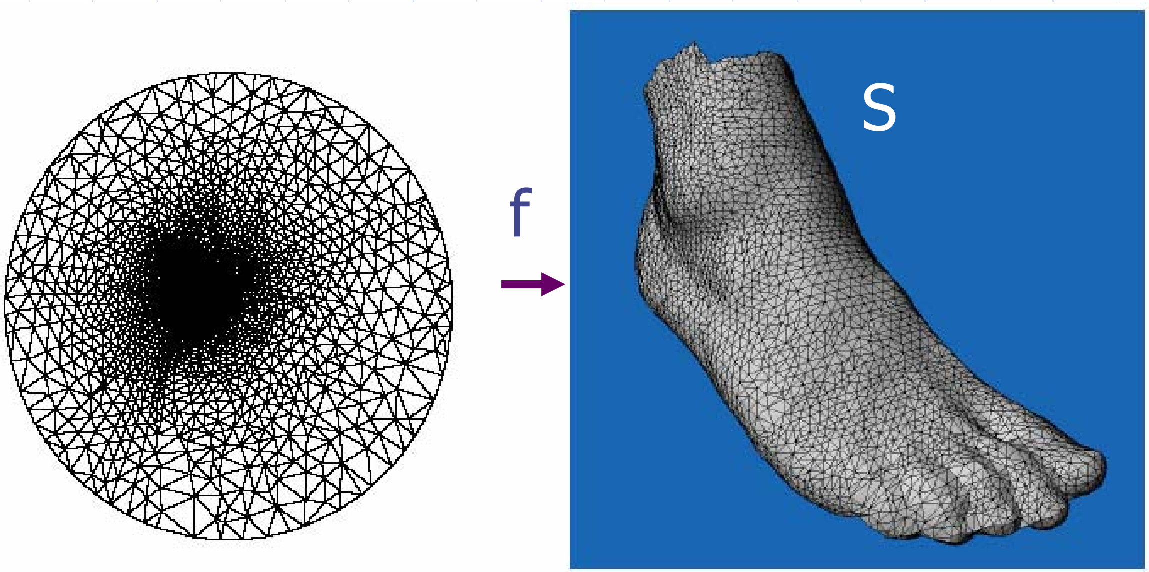




Parameterization

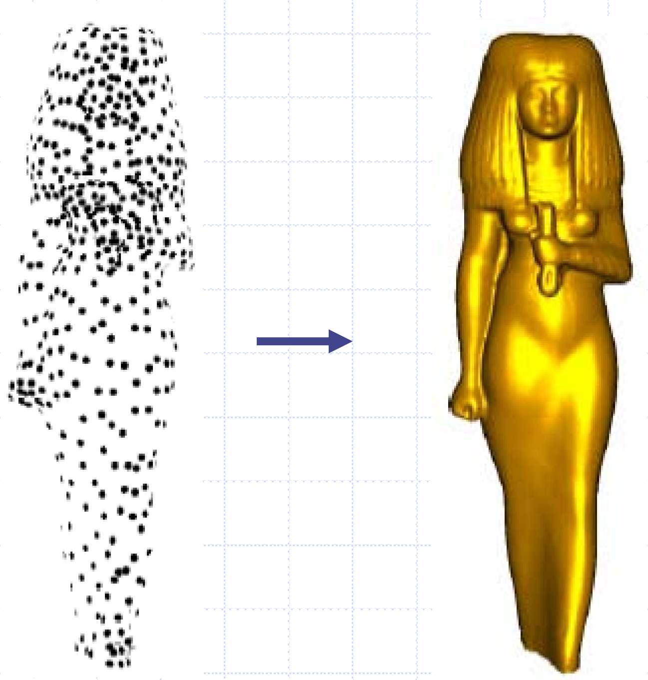
- Problem: Given a surface S in $\mathbf{R}^3$, find a one-to-one function $f : D \rightarrow \mathbf{R}^3$, $D \subset \mathbf{R}^2$ such that the image of D is S.

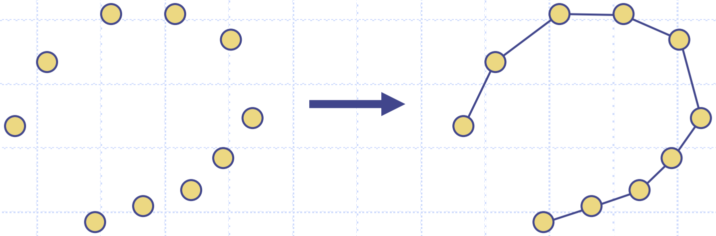




Surface Reconstruction

- Problem: Given a set of unorganized points, approximate the underlying surface



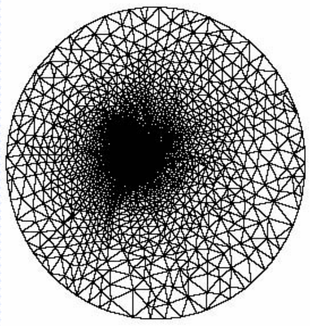


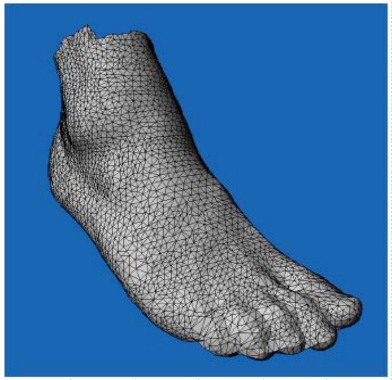
Related Works

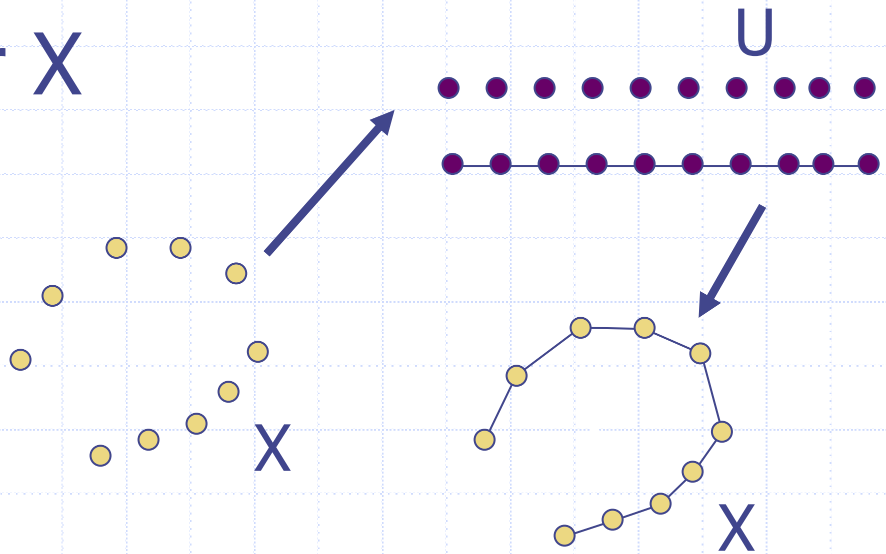
- Surface reconstruction
  - Delaunay based
  - Voronoi based
  - Implicit methods
- Parameterization for organized point set

Basic Idea

- Given $X = (x_1, x_2, \ldots, x_N)$ in $\mathbf{R}^3$, compute $U = (u_1, u_2, \ldots, u_N)$ in $\mathbf{R}^2$
- Triangulate U
- Obtain **both** a triangulation and a parameterization for X

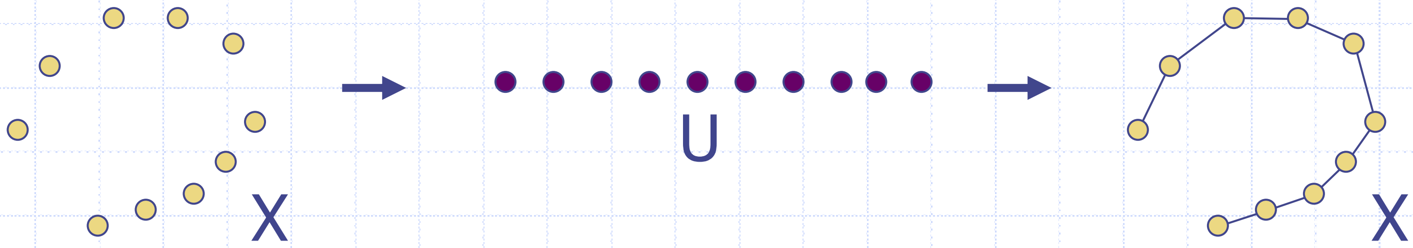




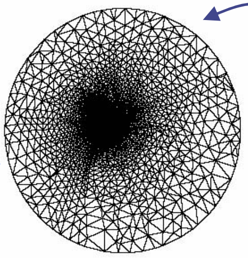


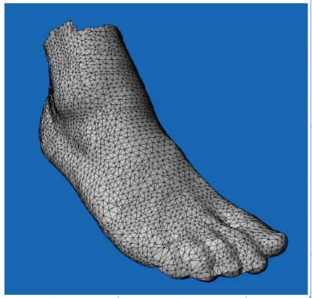
Computing U

- Assumptions
  - X are samples from a 2D patch
  - Points on the boundary of X are known
- Desirable property
  - Points closed by in U are closed by in X



Convex Constraints





- If x_j's are neighbors of x_i then we require u_i to be a strictly convex combination of u_j's

$$u_i = \sum_{j \in N_i} \lambda_{ij} u_j, \text{ where } \lambda_{ij} > 0 \text{ and } \sum_{j \in N_i} \lambda_{ij} = 1$$

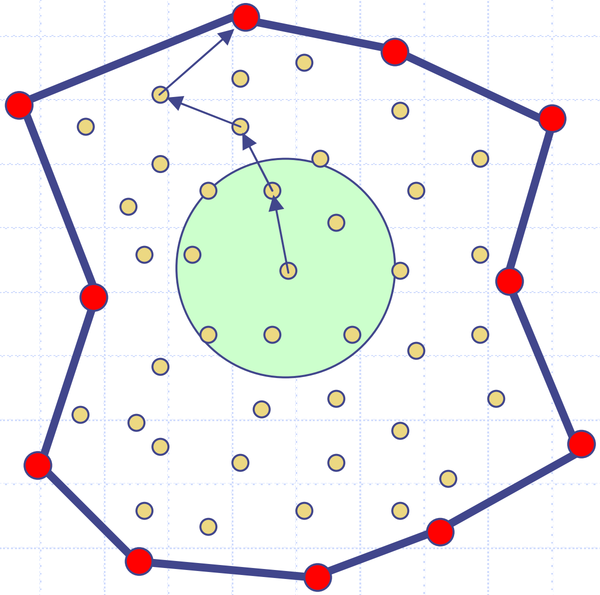
- Boundary condition: map boundary of X to points on a unit circle
- Solve resulting linear system **A u = b**
  - Use Bi-CGSTAB iterative method…

Issues

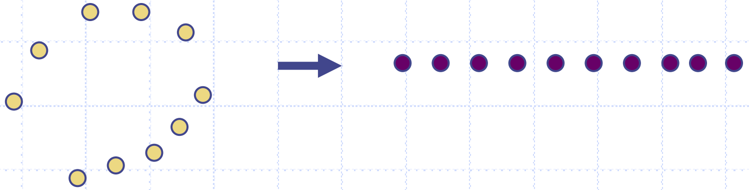
- Is the linear system $\mathbf{A}\,\mathbf{u} = \mathbf{b}$ solvable?
- How to select neighbors for a point?
- What λ_{ij} to use?

Solvability of **A u = b**

- Always solvable under a mild condition
  - Neighborhoods are large enough so that all points are boundary connected



  - Neighborhoods should not be too large…

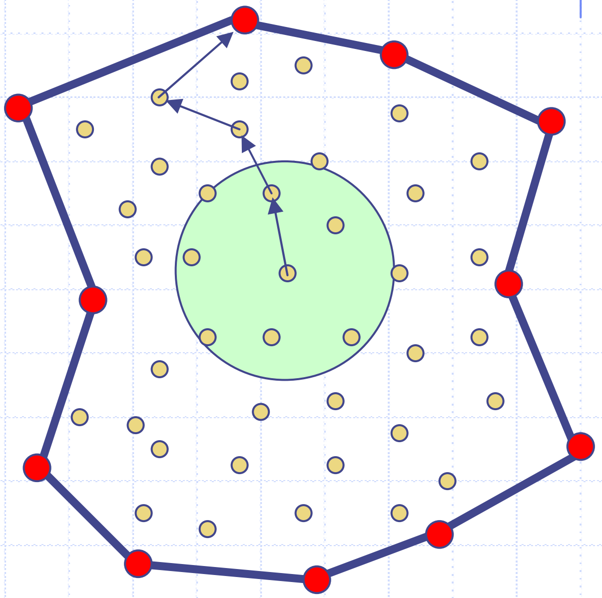


Why Solvable?

- System is decomposable, solving for x and y separately
- **A** is diagonally dominated, but not strictly dominated

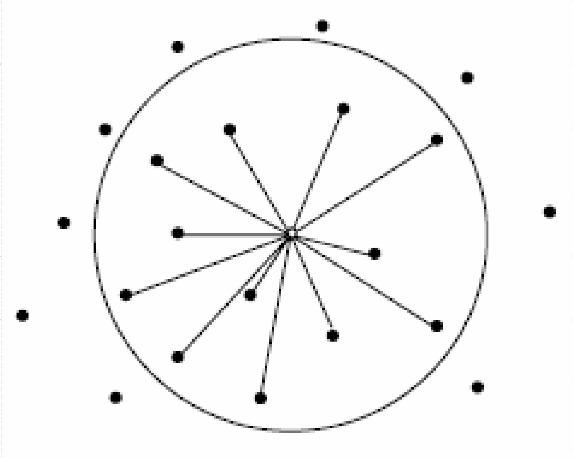
- Maximum value of **u** is on the boundary
- Minimum value of **u** is on the boundary
- When **A u** = **0, u** is **0** on the boundary

$$u_i = \sum_{j \in N_i} \lambda_{ij} u_j$$



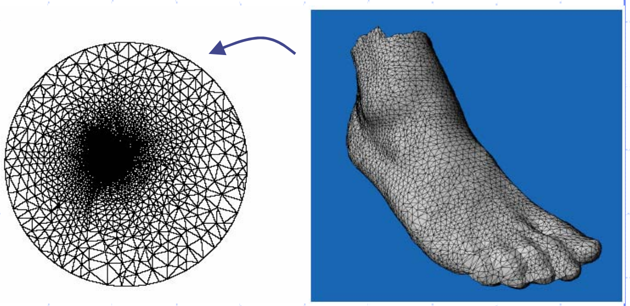
Neighborhoods and Weights

- Ball neighborhoods
  - Radius is fixed



- Weights
  - Uniform weights
  - Reciprocal distance weights
  - Shape preserving weights

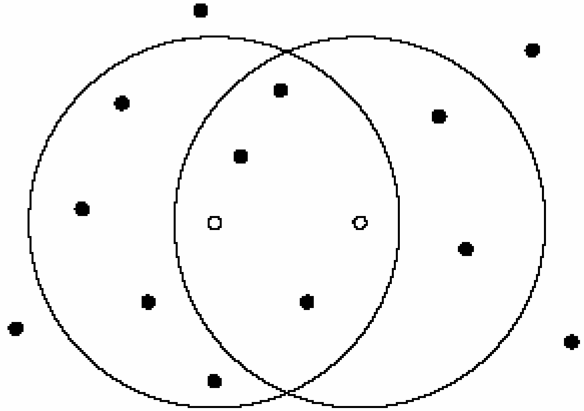
Uniform Weights



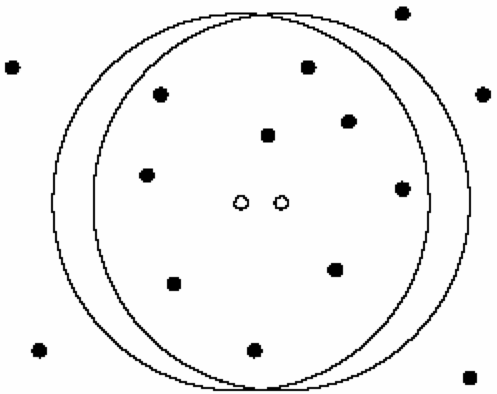
- Uniform weights: $\lambda_{ij} = 1/d_i$

(minimizing $\sum_{0<||x_i - x_j||<r} ||u_i - u_j||^2$)

- Thm: if $N_i \cup \{i\} = N_k \cup \{k\}$ then $u_i = u_k$

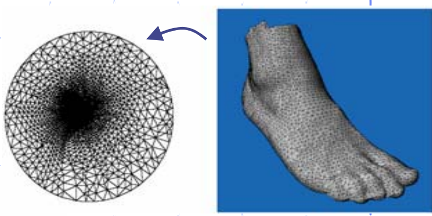


$u_i \neq u_k$



$u_i = u_k$

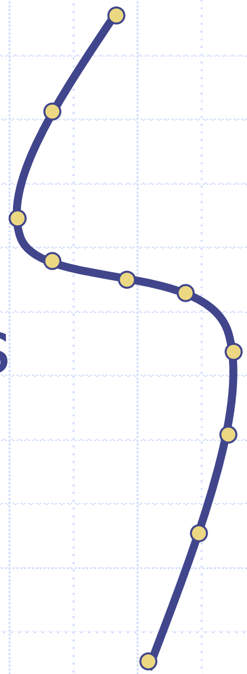
Reciprocal Distance Weights



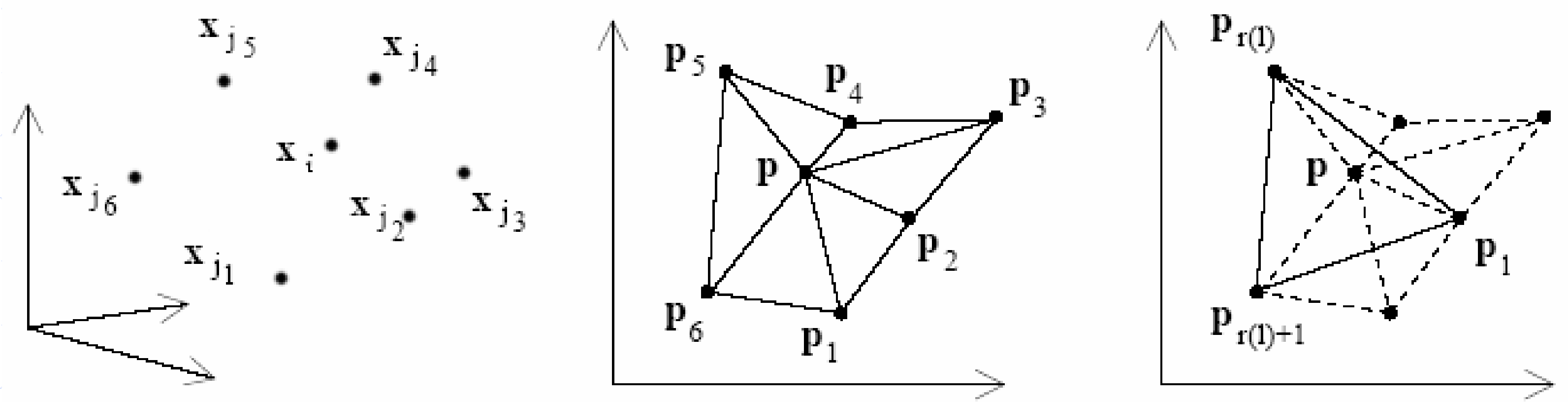
- Weight:

$$\lambda_{ij} = \frac{1}{||x_j - x_i||} \Big/ \sum_{k \in N_i} \frac{1}{||x_k - x_i||}$$

- Observation:
  - Minimizing $\sum_{0<||x_i - x_j||<r} \frac{||u_i - u_j||^2}{||x_i - x_j||}$
  - Cord length parameterization for curves
  - Distinct parameter points
  - Well-behaved triangulation



Shape Preserving Weights

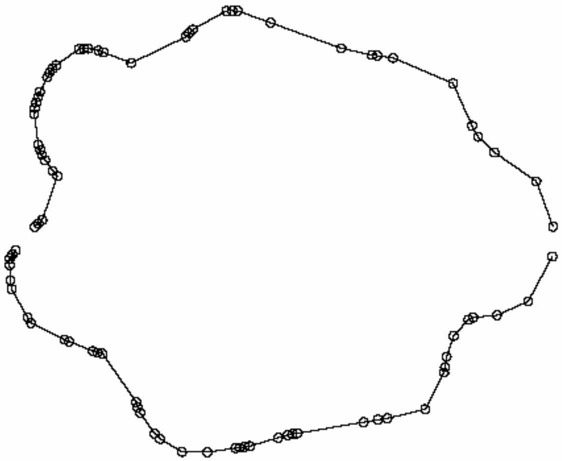


$$\mathbf{p} = \sum_{k=1}^{d_i} \mu_{k,l}\mathbf{p}_k, \qquad \sum_{k=1}^{d_i} \mu_{k,l} = 1, \qquad \mu_{k,l} \geq 0.$$

$$\lambda_{i,j_k} = \frac{1}{d_i}\sum_{l=1}^{d_i} \mu_{k,l}, \quad k = 1, \ldots, d_i$$

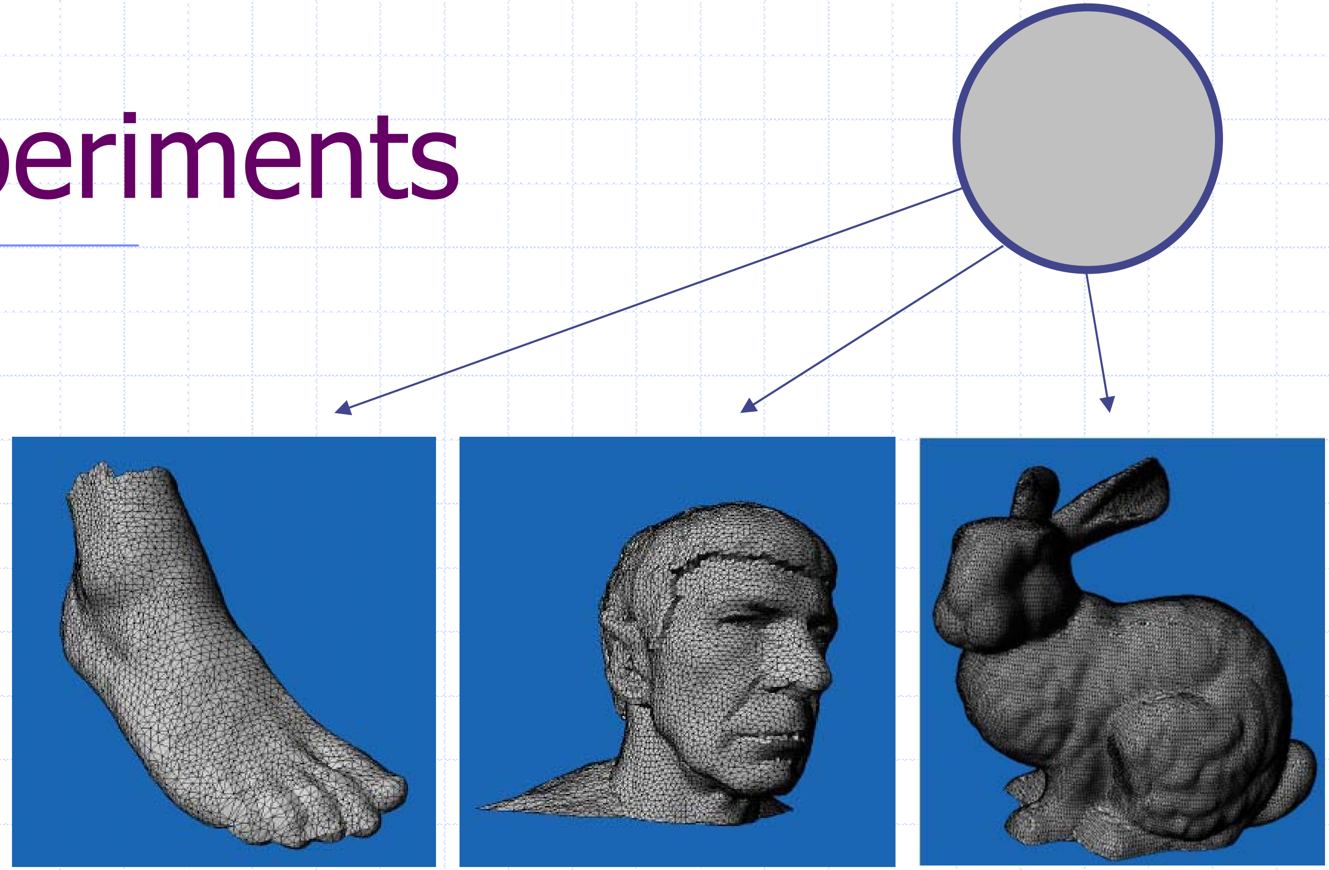
$$\Rightarrow \quad \mathbf{p} = \sum_{k=1}^{d_i} \lambda_{i,j_k}\mathbf{p}_k, \qquad \sum_{k=1}^{d_i} \lambda_{i,j_k} = 1$$

Identifying Boundary



- Use natural boundary (given as part of the data)
- Choose a boundary manually
- Compute boundary
  - Identify boundary points
  - Ordering boundary points: curve reconstruction

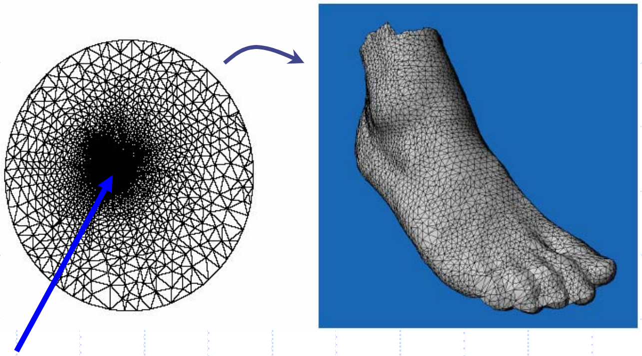
Experiments



Data set	N	x dim	y dim	z dim	r
Foot	5092	98.7	245	203	9.0
Spock	9508	0.69	0.80	0.68	0.03
Bunny	30571	0.16	0.15	0.12	0.003

Finding Parameter Points

- Uniform weight is bad
  - Foot: 5092 points ->5091 distinct points
  - Bunny: 30571 -> 30568
  - Should not be used
- Reciprocal distance & shape preserving weights
  - All parameter points are distinct
  - Considerable distortion



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Neighbor Size & CPU Usage

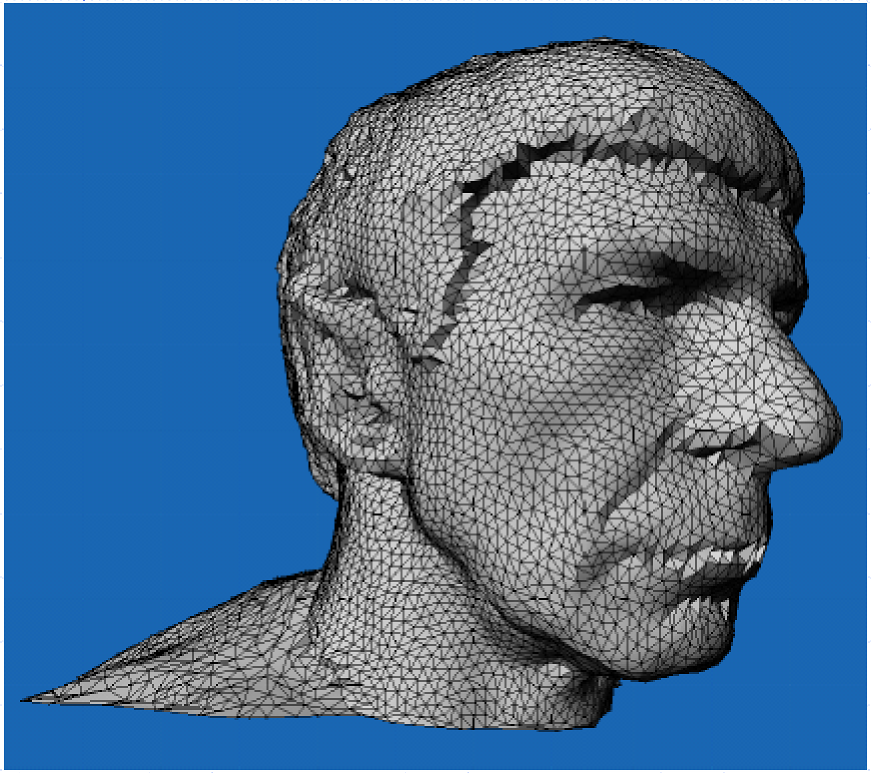
- Reciprocal distance weights

Data set	$\min_i \lvert N_i \rvert$	$\max_i \lvert N_i \rvert$	Num. iterations	CPU time
Foot	6	27	137/150	2.08/2.30
Spock	3	29	180/170	6.81/6.36
Bunny	4	24	466/406	52.71/45.92

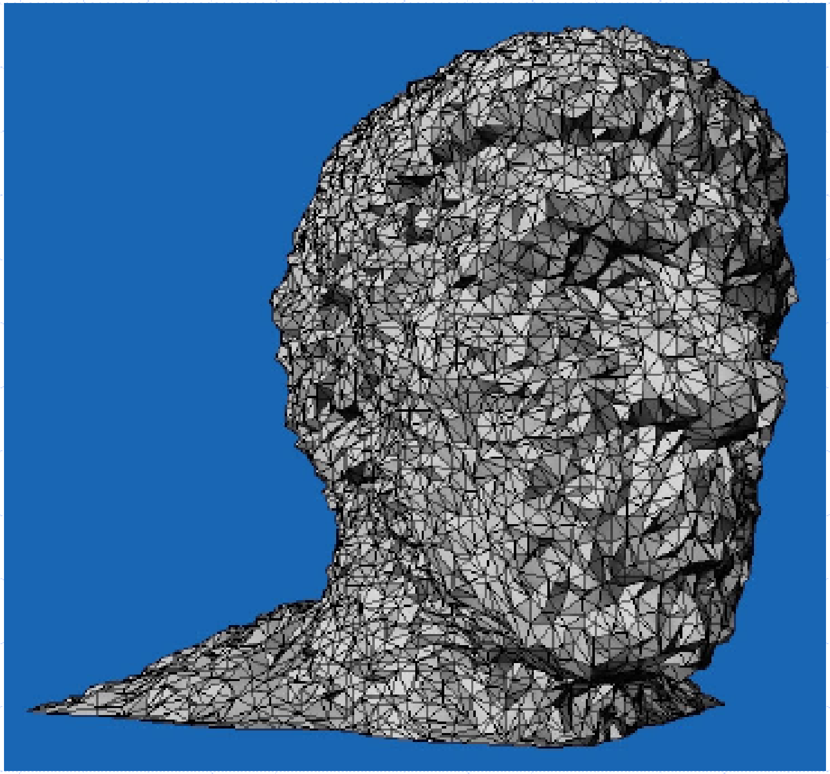
- Shape preserving weights

Data set	$\min_i \lvert N_i \rvert$	$\max_i \lvert N_i \rvert$	Num. iterations	CPU time
Foot	3	9	230/240	0.84/0.87
Spock	3	9	303/286	4.52/4.45
Bunny	3	9	635/589	40.44/37.57

Effect of Noise

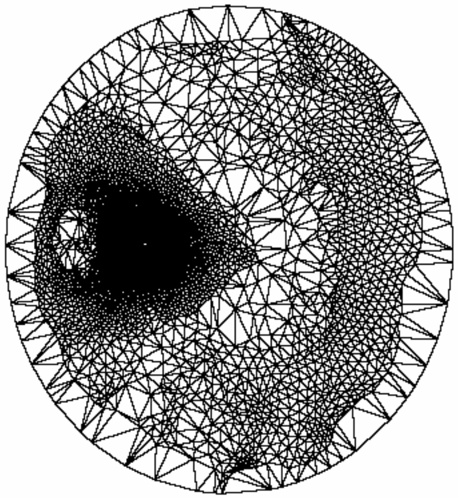


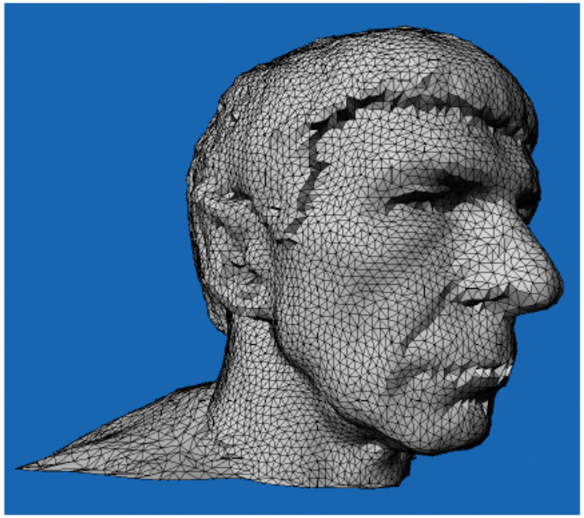
No noise



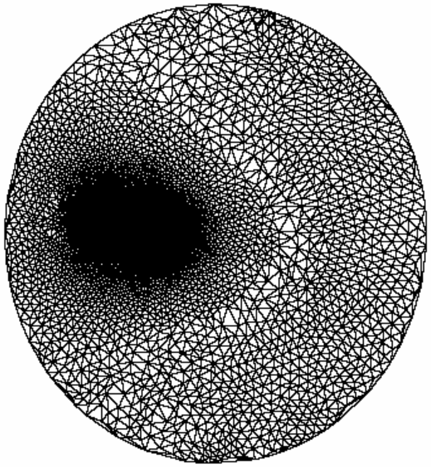
Noise added
Reciprocal distance weight

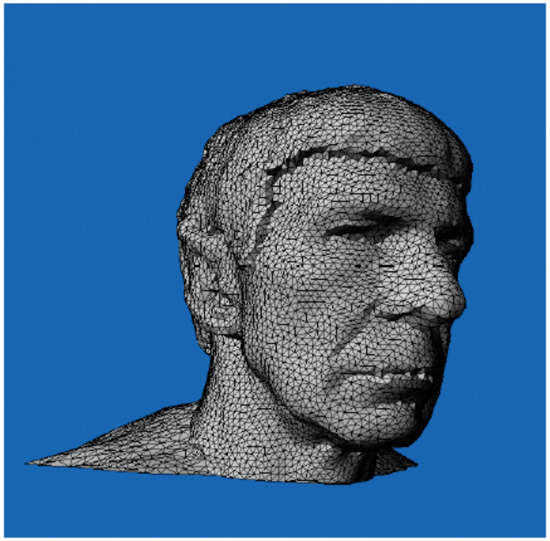
Mesh





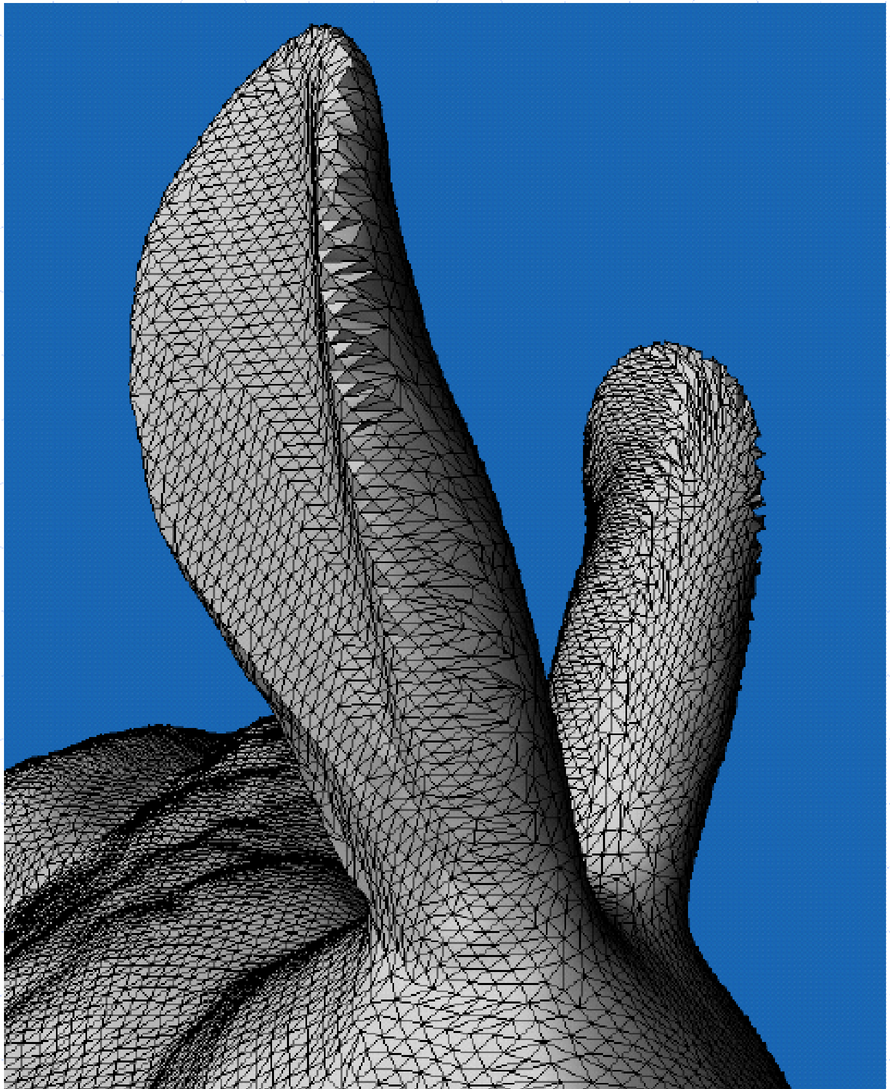
Reciprocal distance weights





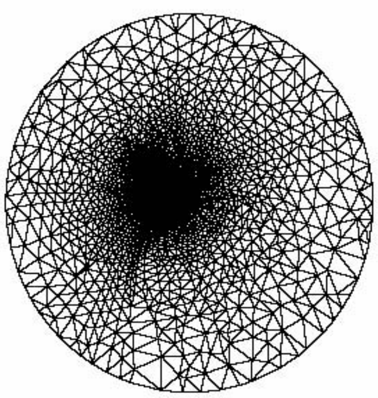
Shape preserving weights

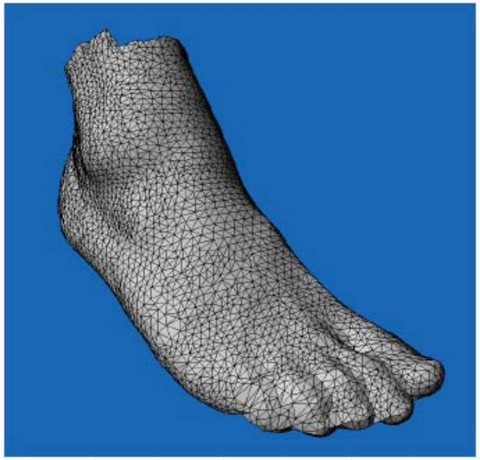
Mesh (Close Up)



Summary

- Simple, fast, and robust method
- Obtain both a parameterization and a surface reconstruction





Thank You!