CS 468 (SPRING 2013) — DISCRETE DIFFERENTIAL GEOMETRY

Course outline

1. (J) Overview of the course

2. (A) Curves

   - Define curves and related objects
   - Frenet frame and Bishop frame
   - Fundamental theorem of plane curves, turning angles

3. (J) Discrete curves

   - Choice of curvature measure
   - Turning angle etc
   - Bishop frame, rods paper
   - How to compute Frenet/Bishop frame

4+5. (A) Surface theory (embedded in $\mathbb{R}^n$)

   - Definition of surface, tangent space
   - Change of coordinates
   - Parametric vs. implicit representations Examples
   - Define what "topology" means vs. geometry; the Euler characteristic.

6. (J) Surface practice

   - Halfedge vs. implicit vs. other mesh representations
   - Surfaces via point clouds, Maya/subdivision, RBF/implicit, simulation, procedural
   - Euler characteristic

7. (A) Extrinsic curvature (second fundamental form, Gauss map, etc)

8. (J) Computing curvature.

9. (A) The intrinsic metric

   - Definition and properties
   - Length and geodesics
   - Volume and integration (w.r.t. measure)

10. (J) Geodesic computation via fast marching methods

11. (A) Derivatives

    - Definition of covariant derivative of a vector field along a curve and a surface
    - Prove relationship to second fundamental form
    - Intrinsic nature of the projected derivative; Christoffel symbols
    - Fundamental Lemma of Riemannian Geometry
    - Gradient vector field, divergence, Laplacian

12. (J) The Laplace operator of a mesh

- Cotan Laplace
- Solve the heat equation (time integration or eigenfunctions)
- HKS
- Mention that we can't do general covariant derivatives of vector fields in the same way!

13. (A) Surface deformation
    - Isometries and Gauss' Totally Awesome Theorem
    - Rigidity and the Gauss Bonnet theorem
    - Fundamental theorem of Riemannian geometry of surfaces. Motivation for what's next?
    - Smoothing and curvature flows
    - Bending/stretching energies and elasticity
14. (J) Surface deformation practice
    - ARAP
    - Time integration
    - Cloth/thin shells
    - Classification of bending/stretching energies and approximations thereof

∗ Possible section on Math 52 concepts.

15. (A) Exterior Calculus
    - Div, grad, curl
    - Define forms
    - Stokes' Theorem on a surface
    - De Rham cohomology of a surface
    - Hodge decomposition of vector fields
16. (J) Discrete Exterior Calculus
    - Integration of vector fields
    - Discrete Hodge decomposition
    - FEM interpretation
    - Relationship to cotan Laplacian
17. (A) PDEs on surfaces
    - Laplace equation, heat equation, wave equation, Schrödinger equation
    - Some analytical considerations e.g. coercivity of Laplace's equation
18. (J) Numerical solution of PDEs on meshed surfaces
    - Application papers and techniques
    - FEM approach
    - Integration, stability, convergence
    - Barycentric coordinates
19. (A+J) Conformal geometry
    - Continuous and discrete conformal geometry
    - Parameterization
    - Mapping